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G. C. McVITTIE

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Preface

The Symposium on Problems of Extra-galactic Research (I. A. U. Symposium No. 15) was held at the Santa Barbara Campus of the University of California from August 10 to 12, 1961. The first public announcement of this Symposium was made in *I.A.U. Bulletin* No. 2 (November, 1959); the Organizing Committee was V. A. Ambartsumian, W. Baade, R. Hanbury Brown, O. Heckmann, N. U. Mayall, and A. Sandage. After Baade's lamented death in June 1960, Heckmann became chairman of the Organizing Committee and A. D. Thackeray was added to its membership. In February 1961, G. C. McVittie was appointed secretary for the meeting and editor of the Proceedings.

As far as can be determined, the 129 astronomers, who participated in the Symposium represented fifteen countries. However, since the meetings were held immediately prior to the Eleventh General Assembly of the International Astronomical Union, at Berkeley, California, attendance at the Symposium was somewhat fluid. The editor, therefore, hopes that the list of participants printed at the beginning of the book is correct.

The papers read at the Symposium dealt almost entirely with observational work in extra-galactic optical and radio astronomy. Theoretical considerations entered only in so far as they were needed to elucidate the observational data. The papers are arranged in the order in which they were delivered, and each is followed by the discussion to which it gave rise.

Seven sessions were held during the three days of the Symposium. The Chairmen of these sessions were, successively; B. J. Bok, N.

U. Mayall, B. Lindblad, V. A. Ambartsumian, J. H. Oort, G. Lemaître, and R. Minkowski.

The expenses of the Symposium were defrayed by the National Science Foundation. In particular, the editor wishes to acknowledge his debt to the Foundation for making available funds that covered the cost of the secretarial and other tasks involved in the editing of this book. The editor is also indebted to the Office of Naval Research for supporting part of his own contribution to the work. Finally, it is a great pleasure to thank Mrs. Catherine B. Wyatt, who has borne all the heavy routine work connected with the editing of this volume. Without her help, it would never have seen the light of day.

G. C. McVittie
Editor

University of Illinois Observatory
October 1961

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PART ONE

Normal Galaxies as Stellar Systems

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Classification of Galaxies by Form, Luminosity, and Color

1. INTRODUCTION

The classification of galaxies can be based on a variety of criteria: morphological, photometric, colorimetric, and spectroscopic. The first three criteria are discussed in this contribution; the fourth approach will be discussed presently by W. W. Morgan.

The morphological approach, followed by E. P. Hubble [1], is based on qualitative, empirical criteria regarding the shape, concentration, and structure of galaxies visible on direct photographs in blue light. The steady accumulation of large-scale reflector photographs of bright galaxies has shown the need for a refinement of this scheme; Hubble's thinking on this subject and the results of his efforts from 1935 to 1953 were collated and interpreted by A. R. Sandage [2].

This revised system, with slight changes, was described and illustrated in an article of the *Handbuch der Physik* [3]. Over 1500 bright galaxies of both hemispheres were classified in the revised system between 1955 and 1960 [4,5]. The frequency distribution of revised types is discussed in section 2.

During the same period, E. Holmberg [6] published morphological types of 300 northern galaxies in a slightly modified Hubble system including stages Sb^-, Sb^+, Sc^-, Sc^+; recently, S. van den Bergh [7,8,9] has given modified Hubble types supplemented by luminosity classes derived from estimates of surface brightness and from structural indices which, within each of Hubble's types, Sb and Sc, appear to be correlated with absolute magnitude. Morgan [10,11] has associated to a simplified morphological classification scheme estimates of probable color class inferred from form and concentration through the empirical correlation between morphological type, nuclear concentration, and spectral type. The correlations between these various classification systems are discussed in section 3.

In all cases it is very desirable to substitute quantitative measures of objective parameters for subjective estimates of qualitative classification criteria. Thus, morphological type and luminosity class should be correlated with the luminosity and color distributions; spectral types should be correlated with color indices and with line intensities.

Surface photometry has produced few reliable quantitative results so far, notwithstanding the extensive literature on the subject [12]. In general, too few objects have been measured with sufficient accuracy to permit correlation in any detail of morphological types or luminosity classes with specific laws of luminosity distribution, with the possible exception of ellipticals. Examples of the initial results of a new program of detailed galaxy surface photometry in progress since 1957 are presented in section 4 primarily to show that a quantitative approach to the problem of galaxy classification is possible, but the results are still too limited to warrant a general discussion at this time.

Multicolor photometry in the form of integrated color indices gives less specific information perhaps, but its application to a large number of systems requires much less work. A catalogue of photoelectric integrated colors of 148 bright galaxies in the U,B,V system was published recently [13]. This catalogue and the integrated photographic color indices of 300 northern galaxies previously derived by Holmberg [6] provide sufficient material for some general conclusions and applications outlined in section 5.

2. CLASSIFICATION BY FORM

The reader is referred to the article in the *Handbuch der Physik* [3] for a detailed description of the revised classification system; additional illustrations will be found in the *Hubble Atlas of Galaxies* [2]. In brief, it includes:

Four *classes:* ellipticals E, lenticulars SO, spirals S, and irregulars I.

Two *families* of spirals and lenticulars, designated "ordinary" (SA, SAO), and "barred" (SB, SBO), with intermediate objects (SAB, SABO).

Two main *varieties* of spirals and lenticulars; the "ringed" type (r) and the "S-shaped" type (s) with transition types (rs).

Four main *stages* are distinguished along each of the four spiral sequences: SA(r), SA(s), SB(r), SB(s) denoted by a, b, c, d from "early" to "late," with intermediate stages ab, bc, cd. The transition stage towards the Magellanic irregulars Im (whether barred or not) is denoted by Sm.

Three *stages*, denoted SO^-, SO^0, SO^+, are distinguished in the lenticular class from "early" to "late." The transition stage between SO and Sa is denoted SO/a, while E^+ denotes a "late" E, the first transition stage toward SO. Non-Magellanic irregulars of structure related to the SO class are denoted IO.

Table 1 gives a summary of the absolute and relative frequencies of revised types among 1528 bright galaxies. The observed relative frequencies of $n = 10(1 - b/a)$ among elliptical objects are illustrated in Figure 1 and compared with the theoretical frequency of apparent ellipticities of randomly oriented ellipsoids whose true ellipticities are uniformly distributed from E0 to E7.

TABLE 1

Frequencies of Main Classes

CLASS	E	SO	S	Im	IO	Pec	All
n	199	329	934	39	13	14	1528
f(%)	13.0	21.5	61.1	2.55	0.85	0.9	100.0

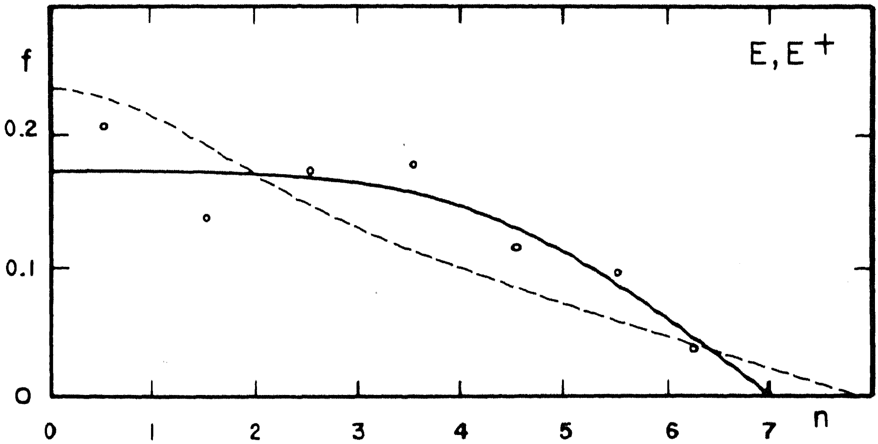

Figure 1. Relative frequencies of ellipticities f against numbers n among elliptical galaxies. Observed (continuous curve), computed (dashed curve).

The observed deficiency of E0 to E2 and excess of E3 to E6 are probably significant. Only two galaxies were classified E^+7 and they may be early lenticulars (SO^-) seem edgewise.

The lenticulars are about evenly divided between SA0 and SB0 and between S(r) and S(s), the mixed types AB and (rs) appear less frequent probably because of classification difficulties. The frequencies among the three stages SO^- to SO^+ are also about equal. About half the SO without finer notation are "spindles," i.e., edgewise objects whose presentation usually precludes identification of characteristic internal structure. Most early lenticulars (SO^-) cannot be assigned A,B, or r,s subtypes because their characteristics are not yet well differentiated at this stage.

The spirals are divided about evenly between the A, AB, and B families; if anything, the barred spirals SB are slightly more frequent than the ordinary spirals SA (the designation "normal" for this type should, therefore, be abandoned). However, there is a large and probably significant excess of (s) variety compared with the (r) variety and this excess is largest among the SA types; the ratio (s)/(r) is 2.4 among SB's and 4.5 among SA's. This may be in part an effect of resolution because the inner ring of the SB(r) type is larger in relation to the over-all diameter of the system than the inner ring of the SA(r) type, but it is doubtful if the whole of the difference can be so explained.

The apparent frequency function of the various stages of spirals of all types from S0/a to Im is shown in Figure 2. The maximum is, as usual [4,5], in the range of types Sb to Sc beyond which the apparent frequency drops rapidly mainly because of the lower absolute luminosity of the late-type systems.

A rough correction for this selection effect may be estimated by assuming that the selection rule for inclusion in the catalogue was essentially by apparent magnitude and that the mean absolute magnitudes of each subtype are given by Figure 5.

The corrected frequencies, also plotted in Figure 2, show that the true relative abundances of the late stages Scd to Im are from two to four times greater than the abundances of the earlier stages S0/a to Sc. This result is in general agreement with the similar conclusions of van den Bergh [8] on the relative space densities of galaxies of various luminosity classes.

3. COMPARISON WITH OTHER CLASSIFICATIONS

E. Holmberg [6] has published modified Hubble types for 300 galaxies from Mount Wilson photographs. The correlation with the revised types for 205 objects in common is shown in Figure 3. With the numerical coding system indicated in the caption, the average

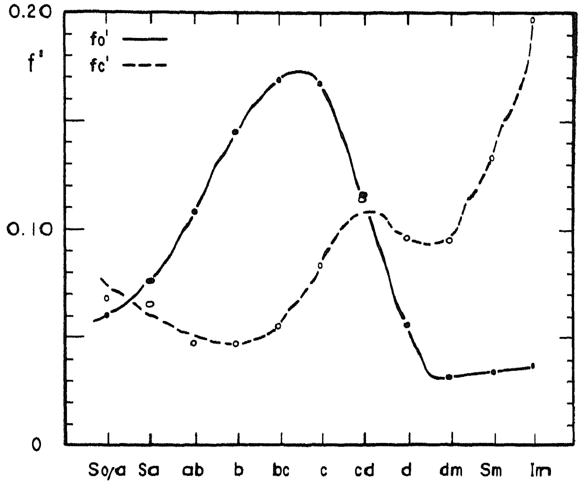

Figure 2. Relative frequencies of stages among spiral galaxies. Observed apparent frequencies f_0' (continuous curve) and corrected frequencies f_c' (dashed curve).

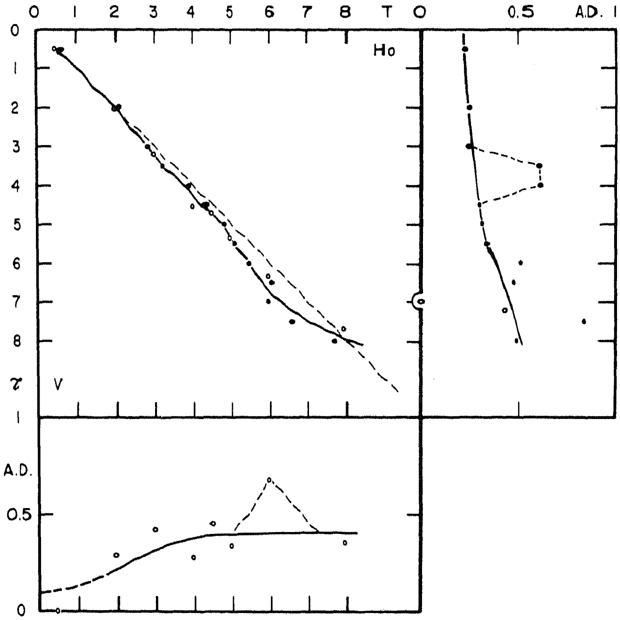

Figure 3. Correlation between revised types $\mathfrak{T}$ and Holmberg types T. The two regressions $\mathfrak{T}$ (T) and $\overline{T}$ $(\mathfrak{T})$ and the corresponding average deviations are shown by dots and circles respectively (as in **Figure 4**). Types are coded as follows—*revised types* $\mathfrak{T}$: E (0.5), SO (2), SO/a (3), Sa (3.5), Sab (4), . . . , Sdm (7), Sm (7.5), Im (8). *Holmberg type T*: E (0.5), SO (2), Sa (3), Sb$^-$ (4), Sb$^+$ (4.5), Sc$^-$ (5), Sc$^+$ (6), IrI (8).

deviation from the mean regression curve is ±0.4 interval. The main departure is at Sc^+ where the present revised system permits a finer classification of the late stages.

Van den Bergh [9] has published a reclassification of all Shapley-Ames galaxies north of declination −27° visible on the Palomar Sky Atlas prints. The modified Hubble types, including Sb^-, Sb, Sb^+, are supplemented by luminosity classes for types Sb, Sc, and Ir [7,8]. The correlation with the revised types for 838 objects in common is illustrated in Figure 4. With the numerical coding indicated, the average deviation from the mean regression curve is ±0.55. It is larger, about 0.8, for DDO types E 0-2, E 3-6, Sa, Sb^+; and smaller, about 0.3, for DDO types E 7-9 (i.e., SO) and Ir, which are more easily identified. The larger accidental error of the DDO types is obviously due to the difficulty of classifying galaxies on small-scale high contrast prints.

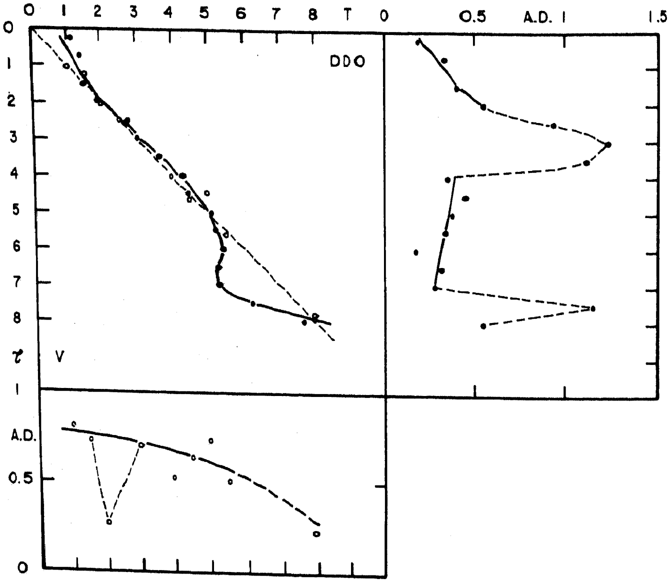

Figure 4. Correlation between revised types $\mathfrak{T}$ and van den Bergh (DDO) types T. Types are coded as follows—*revised types* $\mathfrak{T}$: EO-2 (1/4), E 3-6 (3/4), SO^- (1.5), SO, SO° (2), SO^+ (2.5), SO/a (3), . . . , Sm (7.5), Im (8). *DDO type* T: EO-2 (1), E 3-6 (1.5), E 7-9 (2), Sa (2.5), Sb^- (4), Sb (4.5), Sb^+ (5), Sc (5.5), Ir (8).

Figure 5 shows, as a function of revised type $\mathfrak{T}$, the DDO luminosity coded as follows:

$\mathfrak{L}$	1	2	3	4	5	6	7	8	9
DDO	I	I-II	II	II-III	III	III-IV	IV	IV-V	V
M_{pg}	-20	-19.5	-19	-18.5	-18	-17.5	-17	-16.5	-16

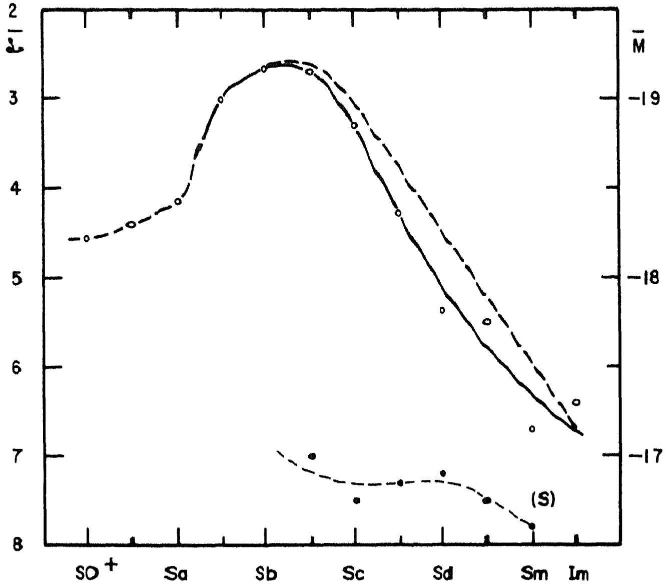

Figure 5. Mean DDO luminosity class and absolute magnitude versus revised type. The DDO types (S) are low-luminosity systems; if excluded the relation is as shown by the dashed curve.

All objects of DDO type S are low-luminosity systems of $\mathfrak{L} \approx 7.4$; the effect of removing all S types is shown by the dashed line in Figure 5. The approximate absolute magnitude calibration is from residuals of the velocity-magnitude relation assumed linear and isotropic with $H = 120$ km/sec^{-1}/Mpc^{-1} [14].

The decrease of mean absolute luminosity, $\Delta M \approx 2^m$ from Sbc to Im, indicated by Figure 5, is in good agreement with values derived directly from residuals $\Delta M = M - \bar{M}$ of the velocity-magnitude relation with allowance for the nonlinearity and anisotropy of the velocity field in the local supercluster [15].

The approximate absolute magnitude calibration in Table 2 rests on $\bar{m} = 11.3$, $m - M = 30.2$, $\bar{M} = -18.9$ for S0 to Sc galaxies in the Virgo Cluster.

Morgan [10, 11] has published two lists of galaxy types in a new Yerkes system using only broad classes E, D, B, S, I, which, except for E, have no simple correspondence with revised classes

TABLE 2

Mean Absolute Magnitudes and Dispersion for Galaxies of Known Revised Type and Radial Velocity

$\mathfrak{T}$	ΔM_m	±p.e.	$\sigma_m(M)$	n	$\bar{M}_m$	$\bar{\sigma}_m$
gE, E$^+$	+0.31	±0.11	0.65	15	−18.6	0.6
SO	+0.11	0.08	0.61	29	−18.7	0.6
SO/a, Sa	+0.19	0.08	0.43	12	−18.9	0.7
Sab, Sb	−0.26	0.15	1.03	22	−19.0	0.8
Sbc	−0.06	0.11	0.65	19	−19.0	0.9
Sc	−0.12	0.10	0.79	28	−18.9	1.0
Scd, Sd	+0.69	0.44	1.84	8	−18.5:	1.2:
Sdm, Sm	+1.13	0.22	0.98	9	−17.7:	1.4:
Im	+2.56	0.40	1.30	5	−16.5:	1.6:

and families. The color classes noted from a to k in the Yerkes system are inferred from the morphological characteristics. The mean relations with revised types for 571 and 564 objects in common with the first and second Yerkes lists, respectively, are shown in Figure 6. The mean average deviation ±0.8 indicates a probable error ≈ 0.6 interval for equivalent types derived from Yerkes color classes. A comparison of the first and second Yerkes lists for 314 objects in common gives an average deviation from the mean relation of about 0.4 for the extreme classes a and k, 0.5 for af, and 0.8 for all other intermediate classes.

4. CLASSIFICATION BY LUMINOSITY

A study of luminosity distribution in galaxies is the most direct approach to quantitative classification. Detailed isophotic maps give a complete representation of the surface-brightness distribution. For classification purposes this often complicated distribution must be summarized by a few simple parameters.

For normal galaxies of fairly regular shape the main features of the luminosity distribution function $I(x,y)$ are sufficiently described by the *luminosity profiles* $\log I(a)$, $\log I(b)$ of the major and minor axes or by the corresponding *mean luminosity profiles* $\log \bar{I}(a)$, $\log \bar{I}(b)$ (Figure 7) supplemented by the asymmetry curves $\Delta \log I(a) = \log I(a) - \log I(-a)$ and $\Delta \log I(b)$, if the asymmetry is considered significant. From the mean luminosity profiles the axis ratio $b/a = f(a)$ is derived for regularly spaced values of $\log I$. For an elliptical galaxy this gives the ellipticity curve $n(a) = 10\,(1 - b/a)$. If b/a is reasonably constant over some suitable range of a, the mean value $q = \langle b/a \rangle$ may give the inclination i of the equatorial plane of the system to the tangent plane by the relation

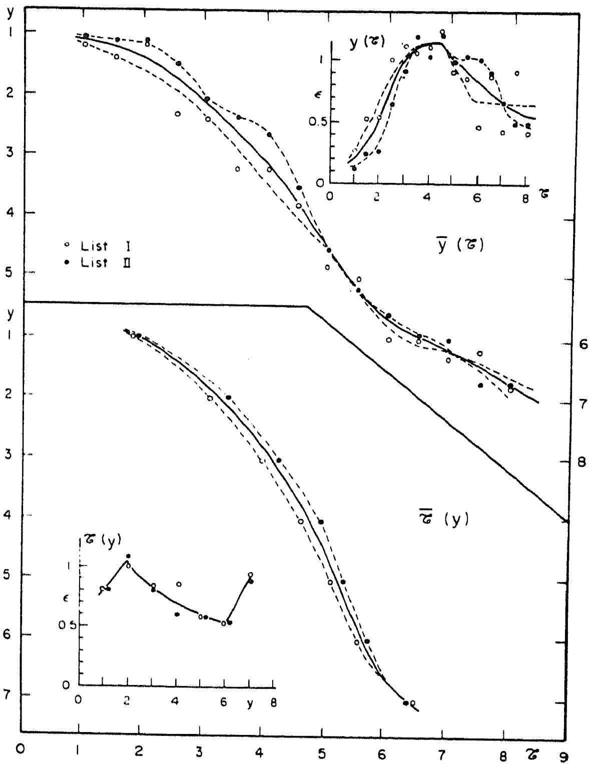

Figure 6. Correlation between revised types $\mathfrak{T}$ and Morgan (Y) color classes. *Above*: regressions $\bar{Y}$ ($\mathfrak{T}$) and average deviation ϵ; *below*: regressions $\mathfrak{T}$ (Y) and ϵ; dots: list I (1958), circles: list II (1959). Types are coded as follows—*revised types* $\mathfrak{T}$: E,E$^+$ (1), SO$^-$ (1.5), . . . , Sm (7.5), Im (8). *Yerkes types* T: k (1), gk (2), g (3), fg (4), f (5), af (6), a (7).

$\cos^2 i = (q^2 - q_0^2)/(1 - q_0^2)$, if $q_0 (\approx 0.2)$ is the true axial ratio of the spheroids approximating the isophotic surface [12]. Systems of the same type similarly oriented should give the same luminosity and ellipticity curves when suitable intrinsic units of length and surface brightness are chosen.

The intrinsic units can be introduced as follows by direct extension of characteristic parameters previously used for elliptical galaxies [12,16,17]. Let $B(r^*)$ be the surface brightness (in mag. per sq. sec. or some other convenient unit) along a given isophote of total area A (including isolated "islands," if any) and *"equivalent radius"*

$$r^* = \sqrt{A/\pi} . \qquad (1)$$

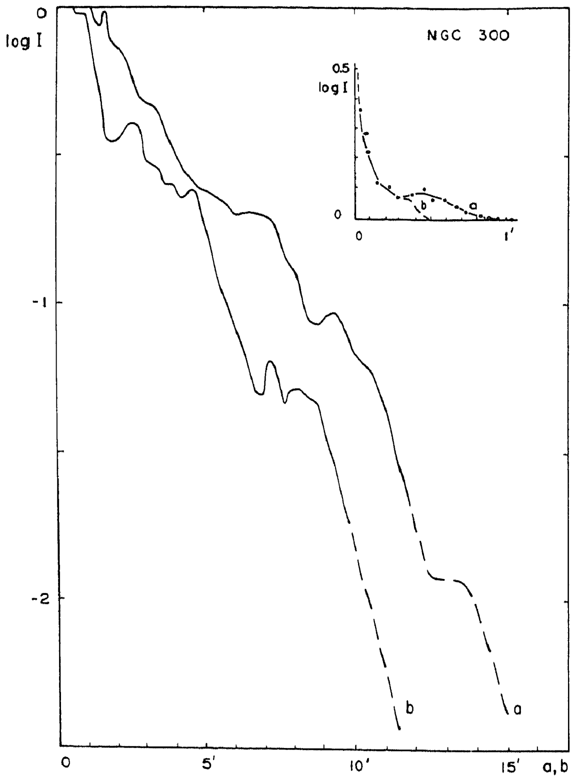

Figure 7. Mean luminosity profiles of major (*a*) and minor (*b*) axes of NGC 300 from Mount Stromlo 30-inch and 74-inch photographs. *Inset*: central region on expanded scale.

The *integrated luminosity* out to r^* is

$$L(r^*) = 2\pi \int_0^{r^*} B(r^*)\, r\, dr, \tag{2}$$

and the *total* (or asymptotic) *luminosity* is

$$L_T = 2\pi \int_0^{\infty} B(r^*)\, r\, dr. \tag{3}$$

The *relative integrated luminosity* is defined by

$$k(r^*) = L(r^*)/L_T, \tag{4}$$

as a function of r^* or, through the mean equivalent luminosity profile $B(r^*)$, as a function of B (Figure 8).

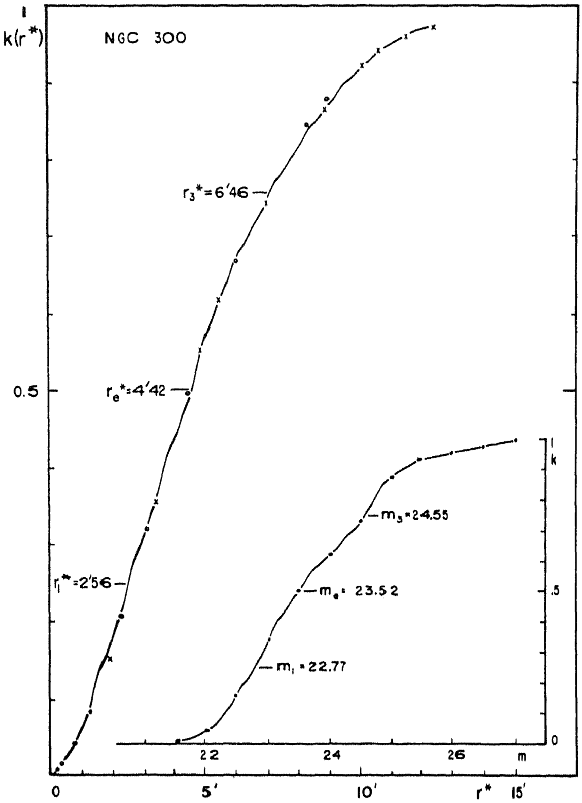

Figure 8. Relative integrated luminosity curves of NGC 300. Fraction k of total luminosity L_T versus equivalent radius r^* of isophotes and (inset) versus surface brightness (in mag./sec^2). Quartiles and effective equivalent radius and surface brightness are marked.

The *equivalent effective radius* is defined by

$$k(r_e^*) = 1/2 . \qquad (5)$$

For a circular nebula it is the radius of the circle within which half the total luminosity is emitted. The quantity r_e^* and the corresponding surface brightness B_e are intrinsic quantities independent (for a given wave length) of observing conditions and of apparent diameter as long as $r_e^* >> s_e^*$, if s_e^* is the effective radius of the instrumental scattering function determined from star images.

If m_T is the total magnitude corresponding to L_T, the effective magnitude is $m_e = m_T + 0.75$, and the effective average surface brightness is

$$\mu'_e = m_e + 2.5 \log A_e = m_T + 5 \log r_e^* + 1.99 , \qquad (6)$$

where μ'_e is an intrinsic measure of the mean surface brightness of the nebula *inside* the effective isophote; it should not be confused with the surface brightness B_e *along* this isophote.

Table 3 gives the characteristic parameters (in blue light) of some of the largest and brightest galaxies: [18,19,20,23,24].

With these parameters, the reduced luminosity, ellipticity, and integrated luminosity curves can be plotted in terms of r^*/r_e^* and B/B_e $(= I/I_e)$. These curves are independent of the choice of units for r and B. Galaxy types can be defined by their reduced characteristic curves and parameters. Thus ellipticals have been shown [12,16,21] to follow closely the reduced luminosity law

$$\log \mathfrak{B} = -3.33\ (\alpha^{\frac{1}{4}} - 1), \tag{7}$$

where $\mathfrak{B} = B/B_e$ and $\alpha = a/a_e$. The corresponding integrated luminosity law is

$$k(\alpha) = L(\alpha)/L_T = 1 - e^{-x}\left[1 + \sum_{1}^{7} \frac{x^n}{n!}\right], \tag{8}$$

with $x = 7.668\ \alpha^{\frac{1}{4}}$ and

$$L_T = 7.268 \pi B_e r_e^2 . \tag{9}$$

Significant departures from the empirical relation (7) occur only near the center $(\alpha < 0.03)$ where resolving power limitations are important [16]. Reports of a $1/r^2$ law of luminosity distribution in the outer parts of ellipticals (which would invalidate the definition of L_T) have not been confirmed by careful independent measurements of several objects by several observers [17,21].*

The ellipsoidal luminosity law seems to apply also to the spheroidal component of ordinary Sa and Sb galaxies [12,18]. Available data are still insufficient to derive characteristic luminosity laws for each revised galaxy type and to determine effects of tilt and orientation. However, an exponential law of luminosity distribution seems to be a general characteristic of spirals in their outer regions where spiral arms are marked only by minor ripples in the luminosity profile [18,19,24]. This exponential law dominates in the Sc, Sd spirals (Figure 7), and especially in the Magellanic irregulars [20] whose reduced luminosity distribution law can be written

$$\log \mathfrak{B} = -0.7290\ (\rho^* - 1)\ , \tag{10}$$

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*Among the objects observed by Mrs. Liller [17] the only significant departures from the ellipsoidal law (7) are for NGC 4350, 4417, and 4564 which have revised types SAO (sp.), SB(s)O:, and E^{+}7 or SO^{-} i.e., are not ellipticals.

TABLE 3

Photometric Parameters of Bright Galaxies

GALAXY:	*LMC*	*SMC*	*M 31*	*M 33*	*NGC 55*	*NGC 300*
$\mathfrak{T}$	*SB(s)m*	*IBm p*	*SA(s)b*	*SA(s)cd*	*SB(s)m*	*SA(s)cd*
$2a_e$	6°25	2°6	71′	24′2	(16′5)	10′7
$2a$ (25)	10°6	4°6	185′	56′3	38′:	18′8
k (25)	0.705	0.84	0.92	0.94	0.92	0.87
b_e/a_e	0.85	0.45	0.30	0.66	-	0.83
$q = \langle b/a \rangle$	0.89	0.46	0.29	0.615	0.20	0.75
i	27°	65°	77°	55°	85°	42°
r_e^*	2°87	0°92	19′5	10′0	3′32	4′42
B_e	23.68	23.11	22.62	22.83	22.1	23.5
M_T	0.63	2.79	4.36	6.27	7.9:	8.7:
$(B-V)_T$	0.55	0.50	0.91	0.55	-	–
μ_e'	13.81	13.77	12.80	13.26	13.5:	13.9:
$m - M$	18.8	18.7	24.3	24.2	26.6	26.6
M_T	−18.2	−15.9	−19.9	−17.9	−18.7:	−17.9:
log $\mathfrak{L}$	9.46	8.55	10.16	9.36	9.7:	9.4:
$m_0 - M$	18.4	18.4	23.9	23.9	26.3	26.3
log $\mathfrak{M}$	10.0 – 2	9.4	11.57	10.23	10.3	10.32
$\mathfrak{M}/\mathfrak{L}$	5.5	7±	25	7.4	4±	9.0
$H\ I/\mathfrak{M}$	0.08	0.2±	0.012	0.06	-	0.095

NOTE: $2\,a(25)$, $k(25)$ = major axis of isophote of surface brightness 25.0 mag.sec^{-2} and fraction of total luminosity emitted inside it.

m_T, $(B - V)_T$ = total (asymptotic) magnitude and color index.

μ_e' = average surface brightness (in mag min^{-2}) inside isophote enclosing 1/2 of total luminosity.

$m - M$ = apparent (photographic) distance modulus.

M_T = absolute total magnitude for adopted $m - M$.

$\mathfrak{L}$ = absolute (photographic) luminosity in solar units (not corrected for internal absorption).

$m_0 - M$ = corrected distance modulus.

$\mathfrak{M}$ = total mass (in solar units) corresponding to adopted distance.

$\mathfrak{M}/\mathfrak{L}$ = apparent mass-luminosity ratio (not corrected for internal absorption).

$HI/\mathfrak{M}$ = ratio of neutral hydrogen mass to total mass.

Units: magnitudes and surface brightnesses are accurate photoelectric values (±0.02 to 0.03) in the B system for M 31, M 33 and the Magellanic Clouds [18,19,20], approximate photographic values (±0.1 to 0.2) in the IP_g system for NGC 55 [23] and 300 [24].

if $\rho^* = r^*/r_e^*$. The corresponding integrated luminosity law is

$$k(\rho^*) = 1 - (1 + x)e^{-x}, \tag{11}$$

with $x = 1.6785\ \rho^*$ and

$$L_T = 3.803\pi B_e r_e^*. \tag{12}$$

It is possible that a combination of the luminosity laws (7) and (10) will be found to describe adequately the main characteristics of several intermediate galaxy types; if so, an approximate estimate of the relative contributions k_1 and k_2 of the flat and spheroidal components to the integrated luminosity of complex systems may become possible; for example, in the nebula Andromeda $k_1 \approx 0.76$, $k_2 \approx 0.24$ in the B system [18].

5. CLASSIFICATION BY COLOR

Studies of the integrated colors and, more recently, of the color distribution, supply other useful indices for quantitative classification. Data on the detailed color distribution are still few and their accuracy is low when derived from separate measurements of the luminosity distribution in two colors. Integrated colors are more reliable; in a circular object, if $C(r)$ is the average local color at a given radius r (r^* in a complex object) the integrated color out to r is

$$C_r = \frac{\int_0^r C(r)\, I(r) r\, dr}{\int_0^r I(r) r\, dr}, \tag{13}$$

and it tends toward the total (or asymptotic) color C_T when $r \longrightarrow \infty$.

E. Holmberg [6] has derived total color indices $C = m_{pg} - m_{pv}$ for 300 galaxies by photographic photometry. As expected, color depends statistically on red-shift and galactic latitude; colors reduced to zero velocity and to the galactic poles depend on morphological type and, for type Sa to Sc$^+$, on the inclination of the equatorial plane of the system because of internal absorption. The observed color C may be written

$$C = C_0' + \alpha(\operatorname{cosec} b - 1) + \beta 10^{-3} V + \gamma(1 - q) \tag{14}$$

where C_0' is the corrected (face on) color, V the velocity in km sec^{-1}, b = galactic latitude, q = axis ratio; in the color system of Holmberg $\alpha = +0.062 \pm 0.007$ (m.e.), $\beta = +0.018 \pm 0.011$ (m.e.) for all types. The tilt parameter γ and mean corrected color $<C_0'>$ vary with morphological type. The mean corrected colors of the revised types $\mathfrak{T}$ are shown in Figure 9. The dispersion about the mean is highest ($\approx 0^{\mathrm{m}}.10$) at Sbc where the color changes most rapidly with type, lowest at Sd-Sm ($\approx 0^{\mathrm{m}}.05$) where it is stationary. Types and mean colors are essentially identical in both systems at E, SO, and Im. The systematic color changes from Sc to Sm support the introduction of these stages. The color change is slow from SO/a to Sb and rapid between Sb and Sc.

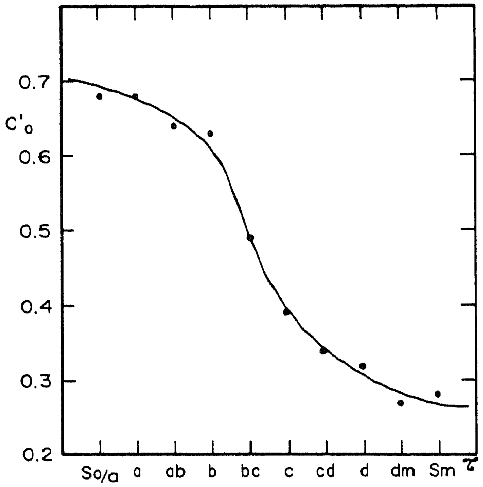

Figure 9. Mean corrected colors C'_0 of revised spiral types.

Integrated colors C_r are best measured and most easily secured by multicolor photoelectric photometry. Integrated colors in the U, B, V system of 148 galaxies have been published recently [13]. The observed colors *B-V, U-B* can be written in a form similar to Eq. (14) with $\alpha = +0.045 \pm 0.022$ (m.e.), $\beta = +0.012 \pm 0.003$ (m.e.) in both colors, in good agreement with Holmberg. The tilt coefficient γ in U-V (of which 40 per cent is in B-V and 60 per cent in U-B) is as follows, and is in good general agreement with Holmberg [6].

TABLE 4

Color Coefficients in U – V.

$\mathfrak{T}$	E Im	SO Sdm, Sm	IO, SO/a Scd, Sd	Sa Sbc, Sc	Sab Sb
γ	0.0	0.1	0.2	0.3	0.4

The mean corrected colors depend on type $\mathfrak{T}$ and on the ratio $A/D(0)$ of the field aperture A to the face-on diameter $D(0)$ in a standard system [13]. The standard color-aperture relations for revised galaxy types E to Im are shown in Figure 10. The main points of interest are the quasi-identity of the color-aperture relations for stages Sa, Sab, Sb, the near discontinuity between Sb and Sbc, Sc and the rapid reversal of the color gradient between stages Sc and Sm. All normal galaxies of types E to Scd are redder in the center while the Magellanic irregulars Sm, Im are bluer in the center. This reversed color trend confirms the earlier results on the Magellanic Clouds [20]. M 33, at revised type SA(s)cd, shows only a very slight bluing outwards [19].

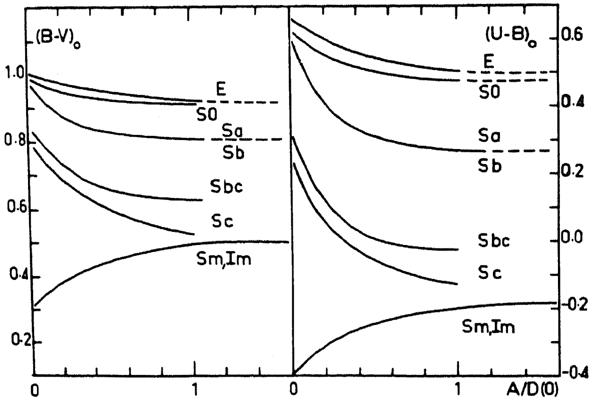

Figure 10. Standard color-aperture relations for normal galaxies. Mean colors $(B\text{-}V)_0$, $(U\text{-}B)_0$ corrected for tilt-reddening versus ratio $A/D(0)$ of aperture diameter to galaxy face-on diameters. Reduction to outside galaxy and zero red-shift ≈ -0.07 in both colors.

Color anomalies are correlated with line-emission intensities and other peculiarities. An important correlation for E and SO galaxies is between colors and absolute magnitude, an effect also noted by Baum [22].

The reader is referred to [13,25] for details and other applications to classification problems.

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DISCUSSION

Vorontsov-Velyaminov: For his classification, Hubble had at his disposal some 400 galaxies, of which many were ellipticals. The later classifications based on somewhat larger material adopted the principles of Hubble's scheme. A unique correlation was adopted between the properties of the nuclear region and of the spiral patterns. Yet the Palomar Sky Atlas reveals more than 10,000 galaxies up to 16^{m}. They show the enormous variety of combinations of disks, rings, spiral arms, and bars and of the degree of resolution, of patchiness, of these patterns. This enormous variety of morphological types of tens of thousands of galaxies picked up from the Palomar Atlas cannot be reconciled with the existing classifications. A satisfactory classification of galaxies that embraces all types and does not distort the true picture cannot be given at present. Therefore, for the *Morphological Catalogue* of galaxies, I decided to give a more or less detailed description in each case instead of trying to push every galaxy into some inappropriate class.

De Vaucouleurs: The revised classification systems are usually based on an examination of some 1000 to 2000 bright galaxies on large-scale reflector plates. About 95 per cent of these galaxies can be fitted without violence among the 50 to 100 types and subtypes provided. It is true that a classification of this kind does not fully replace a detailed description, but the same problem of exceptions arises also in botany and zoology without vitiating the classification of species. It should be noted also that it is difficult to classify galaxies on small-scale plates and especially on high-contrast prints such as the Palomar Sky Atlas. Too often, low resolution and overexposure conceal the significant details of the inner structure. Certainly, an examination of tens of thousands of galaxies will disclose new rare classes or types of abnormal systems.

Baum: The relationship between the color indices of ellipticals and their absolute magnitudes was discussed in the H-R diagram symposium at the 1958 Moscow meeting of the I.A.U. As shown in Figure 11, the color indices of dwarf systems are similar to those of globular clusters, whereas the giant ellipticals are $0^m.3$ redder.

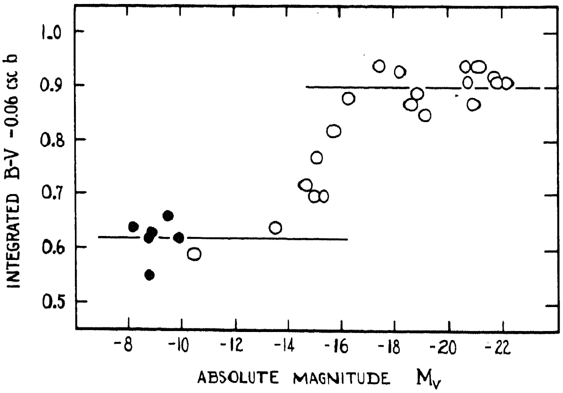

Figure 11. Color indices of galaxies.

In making use of colors in the transition region to estimate absolute magnitudes, I feel slightly less optimistic than Dr. de Vaucouleurs about the precision that can be assumed.

De Vaucouleurs: The findings of Dr. Baum are noted in [13] and in the text of my communication. The probable error of $\pm 0^m.2$ (exclusive of uncertainties in the distance scale itself) applies to absolute magnitudes derived from the long color base $U - V$ (not $B - V$).*

Whitrow: What are the main differences between the distribution of types and forms of galaxies in the Local Group and those in a much larger volume of space?

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*See *Ap. J. Suppl.*, *5*, 233, 1961.

De Vaucouleurs: The Local Group does not include giant ellipticals, lenticulars, or Sa spirals – either ordinary or barred.

Elsässer: Concerning the color distribution in irregular systems, it is true that the central parts of the SMC are bluer than the outer ones, but does the LMC also become redder with increasing distance of the nucleus?

De Vaucouleurs: Yes, the outer regions of the Large Magellanic Cloud are redder than the central regions although the variation is less than in the case of the Small Cloud. This was shown by the photoelectric observations of both Dr. Eggen and me, and of Dr. Elsässer himself.[†]

[†] See *Ap. J. 131*, 265, 1960.

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Spectral Types of Galaxies

GENERALITIES

The spectral classification of composite sources is considered at the present time from a somewhat different point of view from that of a few years ago. The most useful preliminary approach now seems to be a separation of spectra of galaxies into three groups: (1) spectra showing absorption or emission lines of He I in the blue or violet regions; (2) spectra giving evidence of an extremely high degree of compositeness, with lines in the violet region normally associated with spectral types ranging from A to K; (3) spectra of class K in the violet region. Spectra intermediate between these groups are classified into subgroups.

When the resolving power is too low to show the He I lines of Group I, the general spectroscopic appearance can be used for classification purposes; galaxies having spectra similar to NGC 4214 can be classified on low dispersion by means of the emission lines of H, [O III], and a general similarity to the spectrum of the inner part of the Orion nebula. The spectrum of such galaxies seems to resemble that of the nuclear region of O-associations; such spectra are strikingly different from those of planetary nebulae and the Seyfert galaxies.

In Group II, a subdivision by spectral type appears to be worth while, together with a parameter indicating the degree of compositeness. The relative prominence in the violet of the absorption lines of hydrogen and the G-K metallic absorptions gives a simple criterion for a classification by spectral type; the relative prominence of the G-band and the hydrogen lines in the blue region can also be used. The variation of spectral type with wave length gives an indication of the degree of compositeness.

The pure – or almost pure – K-type spectrum observed in the violet region for spectra in Group III provides a simple criterion for classification.

SPECIAL CASES

There are certain situations where conclusions of a relatively definite nature can be obtained; these result from the observed presence or absence of specific spectral features indicative of the presence or absence of certain kinds of stars. The principal cases are listed below:

(a) The He I line at λ 3819. This line, which has been observed in certain galaxies by de Vaucouleurs, G. Münch, and Mayall and Morgan, gives conclusive evidence for the presence of large relative numbers of early B-stars.

(b) The CN bands near λ 4200 and λ 3880. On spectrograms of low dispersion, these bands are observed only in yellow giant stars (G5 - K4). Their presence in the integrated light of certain galaxies gives definite proof of large percentages of yellow giant stars.

(c) The MgH band near λ 4800. The behavior of this band has been described by Öhman [1]. This band is observed in considerable intensity in late K and early M-dwarfs; it is absent or weak in giants. While the MgH band near λ 5200 is complicated by blends and variations in emulsion sensitivity, the band near λ 4800 furnishes a clear and unambiguous indication of dwarf characteristics. In later M-type subdivisions it may be blended with a band of TiO; but in K and early M-dwarfs its appearance is unmistakable. The absence of this band in the integrated spectrum of the inner part of M 31 furnishes strong evidence against the presence of appreciable contributions of dwarf K - MO stars to the light of M 31 in the blue-green region.

(d) The TiO bands in the red and infrared spectral regions. These bands furnish definite evidence for the presence of large relative numbers of M-type stars in the nuclear region of certain galaxies.

POSSIBLE AMBIGUITY IN INTERPRETATION OF COMPOSITE SPECTRA

The use of spectral features observed over considerable ranges of spectral type and absolute magnitude can be dangerous; in particular, low-excitation features which increase in intensity with decreasing temperature – and which also increase in intensity with decreasing luminosity – may be especially difficult to interpret.

An interesting example of this kind is the resonance line of Ca I at λ 4226. This line increases markedly in intensity on passing from early K-stars to those of later type; it also increases in intensity on passing from giants to dwarfs of type K. A comparison of the spectrum of the inner part of M 31 with the spectrum of the KO giant δ Tau suggests that the Ca-line is considerably stronger in M 31 than in the comparison star. From this, it might be concluded that the blue light from M 31 is due principally to dwarf K-stars. That this conclusion is erroneous is shown by the presence of CN bands in the same spectral region; and the behavior of Ca 4226 is probably explainable in terms of contribution to its intensity by later K-type *giant* stars.

EVIDENCE OBTAINABLE FROM SPECTROSCOPIC AND PHOTOMETRIC OBSERVATIONS

The peculiar advantage of spectroscopic observation of galaxies lies in the uniqueness with which certain conclusions can be drawn. Observation of the bulge region of the Galaxy furnishes a good example of this uniqueness. Because of uncertainties in the correction for interstellar reddening of photometric observations of the galactic nuclear region [2], it is not possible to draw firm conclusions from the photometry alone as to the nature of the stars contributing principally to the observed light. Spectroscopic observations, however, show that in a transparent region approximately 3° from the galactic nucleus the light in the ordinary photographic region comes principally from K-type stars – and in this respect, the stellar population of our galactic nucleus seems to resemble that of the inner part of M 31. In addition, the great number of M-type giants discovered by Nassau and his associates in the galactic nuclear region suggests a parallel to the observation of TiO bands by Mayall and Morgan in the nuclear part of M 31.

There are certain other instances where spectral type and color are discordant; cases like M 82 and NGC 5195 are especially promising for further investigation.

Accurate photometric observations, such as the classical results of Baum [3], are, of course, of the greatest value; and the maximum information obtainable for distant galaxies will result from a combination of the spectroscopic and photometric methods.

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DISCUSSION

Spinrad: The line strengths of the Na D lines, the Mg I 'b' triplet and the Ca I λ 6162 line in the K-nuclei galaxies show that part of the yellow and red light must originate in K and M dwarf stars. The D-line strength is not likely to be produced in interstellar matter; the strong Mg lines originate 2.7 ev above ground level and cannot be interstellar. Both features are luminosity sensitive; they are too strong in galaxies such as M 31, M 81, NGC 4594, 3379, and 4472 to be due to normal K-M giant stars. An approximate model for the nucleus of M 31 has been made to fit EW measures of the D lines and Ca I λ 4226 and the six-color photometry of Stebbins and Whitford. The model predicts 40 per cent of the yellow light from dM1 stars; in the blue, the light originates in gKO stars. Thus, the model is compatible with the results of Morgan in the blue. The dwarf luminosity characteristics are related to the absolute magnitude and total mass of a galaxy; the brighter the galaxy, the stronger the Mg I lines are than the Na D lines. All galaxies with $M > 8 \times 10^{10} M_\odot$ have strong dwarf appearance in the yellow-red.

Arp: If the apparent strength of the D lines depends on titanium blanketing, then would it not be possible for an increase in metal abundance to give an apparent decrease in D lines (since Ti O goes twice as fast as the single metal atom D lines)? This would make it possible for a change in metal abundance to be erroneously interpreted as a change in luminosity.

Spinrad: The strength of the D lines depends mostly on pressure in the stellar atmosphere, and partially on Ti O blanketing, that accounts for the large EW difference between K5V and K5III stars; these types are too hot for Ti O to matter.

However, I am sure that the D lines in galaxies do not become strong when the Ti O is weak, as Dr. Arp implies. NGC 4472 and 3379 could thus have quite strong Ti O at λ 6200; this can only be interpreted as coming from strong line stars. The D lines in both NGC 4472 and 3379 are strong also.

Oke: There is a possibility that the strong D lines are produced by interstellar gas in the galaxy. The problem is one of radiative transfer and has not yet been analysed. If the gas and stars are favorably distributed, and if the gas has a velocity dispersion of 150 to 200 km/sec, then the D lines can be produced by between 10^7 and 10^8 solar masses of gas with the normal chemical abundance.

Spinrad: The numbers quoted by Oke are certainly conceivable; however, somewhat different initial assumptions as to the physical conditions in the nuclei in a calculation I have made lead to an interstellar D contribution that is quite small.

The green Mg I triplet, strong in galaxies with strong D lines originates about 2.7 volts above ground level; it cannot be interstellar. The correlation of the Mg I and D lines is so good that a common origin for the two features seems implied.

Baum: As Dr. Morgan has indicated, spectroscopy and multicolor photometry have both been examined previously in efforts to infer the stellar content of elliptical galaxies. Dr. Spinrad is now resurrecting the old belief that late type dwarfs are present in such vast numbers as to contribute substantially to the integrated light of an elliptical. This is definitely not in agreement with the results of multicolor photometry, which, like Morgan's spectra, are much better fitted by a predominance of late giants than by a predominance of late dwarfs.*

van den Bergh: Have you detected weak-line characteristics in the spectra of any galaxies?

Morgan: It is possible that the metallic lines are weakened in the spectra of the edge-on, rapidly rotating galaxies NGC 4111 and NGC 4762; however, better spectrograms are needed to make certain.

Thackeray: When sufficiently high-dispersion is used, the D lines show very little sensitivity to luminosity in K stars. Any effect observed on low-dispersion is probably due to superposed Ti O absorption. In contrast, the Mg 'b' lines in KO stars are very sensitive to luminosity.

Spinrad: The big difference in EW of the D lines between III and V comes in stars later than K2.†

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*Baum, W. A., *P.A.S.P. 71,* 106, 1959.

†See Griffin, R. F., *M.N.R.A.S.*, *122*, 181, 1961.

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Distribution of Luminosity, Color, and Spectral Energy in Bright Galaxies

LUMINOSITY DISTRIBUTION

The distribution of luminosity in a galaxy is one of its fundamental characteristics, a basic observable quantity to which models representing the spatial distribution of mass or the internal motions in a stellar system must conform. The luminosity distribution function must also enter any satisfactory operational definition of the angular diameter of a galaxy. Likewise, the integrated magnitudes used in calculating the absolute magnitude of a galaxy, or in testing the red-shift-magnitude relation, must be carefully defined in terms of the law of luminosity distribution.

In 1930 Hubble [1] published a pioneer investigation of the luminosity distribution in elliptical galaxies and found a good fit with the distribution law

$$I(r) = \frac{I_0}{(1 + r/a)^2} \qquad (1)$$

where I_0 is the central intensity and the scale constant a is the radius at which the surface brightness has fallen off to 1/4 of that at the center. Later investigations by R. O. Redman and E. G. Shirley [2], G. de Vaucouleurs [3,4], E. W. Dennison [5], and M.

Liller [6] have carried out further studies by photographic methods similar to those used by Hubble. The latter two authors used an isophotometer to draw contours of constant brightness. The intensity measurements were carried down to only a few per cent of the brightness of the sky. Both de Vaucouleurs and Dennison found that the intensity falls off more rapidly in the outer portions than would be predicted by Hubble's law, as indeed it must in order to avoid an infinite integrated luminosity. De Vaucouleurs found that the brightness B in elliptical galaxies could be represented by an equation

$$\log B = -Ar^{1/4} + b \tag{2}$$

where A is the scale constant and b determines the zero point of intensities.

For the most part these studies have relied upon sensitometer spots for their calibration. M. Liller's work was based upon a zero point determined from photoelectric measurement of the sky background. Recently, C. J. van Houten [7] has published an extensive study of surface brightness in galaxies of both the elliptical and spiral type. The intensity at selected points in the galaxies studied was directly measured photoelectrically at the telescope and the photographic brightnesses were interpolated between these standard points. The photographic calibration depended also on extrafocal star images, a technique used very successfully by E. Holmberg [8,9]. Surface brightnesses fainter than the night sky were not measured. Measurements by other authors down to this limit were carefully compared with the new results.

Since the disk population is the dominant source of light in spiral galaxies, interpolation formulae of the type already cited for ellipticals may be used to express the rate at which the surface brightness falls off in the outer parts. Holmberg [8] found that only in the Sc galaxies do the spiral arms contribute appreciably more than 10 per cent of the total light. The structural form of early type spirals may, however, require separate treatment for the bulge and disk portions, as was done by van Houten [7] for S0 and Sa systems. For the outer portions of spirals, de Vaucouleurs [4] gives the luminosity law

$$\log B = A - Kr \tag{3}$$

where A and K are respectively the constants for zero point and angular scale.

Holmberg [9] showed that the surface brightness of inclined spirals is systematically lower than that of similar systems seen face-on, an effect ascribed to internal reddening. The effect is proportional to the cosecant of the inclination angle, and is largest in Sa and Sb systems, and smaller in Sc's; for the former, the reduction in surface brightness for unit increase in the cosecant was found to be $0^{\rm m}.43$.

Holmberg's [9] extensive catalog of the integrated photographic and photovisual magnitudes of galaxies is based on the asymptotic total brightness for each system. The limiting measurable surface brightness was stated to be $26^{\rm m}.5$ per square second for photographic light. The integrated magnitudes were in good agreement with J. Stebbins and A. E. Whitford's [10] photoelectric measures for comparable diameters. Humason, Mayall, and Sandage [11] adopted, as a working definition for the diameter, 2.5 times the "visual inspection diameter" found on the plates of the Palomar Sky Atlas; from Dennison's [5] radial intensity function for NGC 3379 they derived a photographic surface brightness of $22^{\rm m}.6$ per square second for the visual inspection isophote and $25^{\rm m}.1$ per square second at the adopted limit for the total diameter. After an intercomparison of various visual inspection limits de Vaucouleurs [4] defined a "micrometric diameter" based on the Reinmuth-Heidelberg system, and found that, for ellipticals, the photographic surface brightness was $23^{\rm m}.0$ per square second for this limiting isophote. For a galaxy of type E0, there is of the order of $0^{\rm m}.5$ difference in the integrated magnitude between that for the visual inspection diameter and that for Holmberg's asymptotic diameter.

Although the law of luminosity distribution is well enough known to serve as an interpolation formula for correcting integrated magnitudes to a standard system, the measured absolute values of surface brightness in the same galaxy differ as much as $0^{\rm m}.20$ between different observers, principally because of a poorly determined zero point. Direct photoelectric measurements of many points in a galaxy should yield more accurate data. De Vaucouleurs [12] has reported on such a survey of M 31. Extension of the technique to more distant systems would be profitable. A large reflector with provision for offset guiding and for frequent sky monitoring would be required.

COLOR

The color of a galaxy is closely related to its stellar content, and a change in color over the various parts indicates differences in stellar population. The most extensive studies of the integrated colors of galaxies have been those of Holmberg [9], by a photographic method, and de Vaucouleurs [13], by direct photoelectric photometry. In order to obtain intrinsic colors of galaxies it is necessary to eliminate, first, the effect of galactic absorption, and, second, the inclination effect caused by internal absorption in the distant system. The correction to the measured color to obtain the color at the galactic pole is of the form

$$\Delta C = -\beta\ (\mathrm{cosec}\ b - 1) \tag{4}$$

where b is the galactic latitude and β is a coefficient which de Vaucouleurs found to be equal to $0^{\mathrm{m}}.044 \pm .017$ on the B,V system. Holmberg found the coefficient to be $0^{\mathrm{m}}.062 \pm .007$ on the P_g,P_v system, a value consonant with the traditional value of $0^{\mathrm{m}}.25$ for the total photographic absorption at the galactic pole as determined by Hubble [14] from counts of galaxies. Holmberg's [9] photographic photometry contained adequate material to give a separate solution for the inclination effect in each type of galaxy. The correction to face-on color was found from the relation

$$\Delta C_0 = -\gamma\ (1 - b/a) \tag{5}$$

where a and b are the major and minor diameters of an inclined system. The tilt coefficient γ was found to be zero for ellipticals and Magellanic irregulars, and reached a maximum value of $0^{\mathrm{m}}.25$ for spirals of type Sa and early Sb. The colors of galaxies on the U, B, V system, published by de Vaucouleurs [13], give a similar tilt coefficient in the B, V index, though the maximum effect is not as great.

That the spiral arms of galaxies are bluer than the nuclear region has been known for many years; F. H. Seares [15] called attention to the difference in 1916. Holmberg [9] found that if the nuclear region is excluded in a prescribed manner, the integrated color of the remainder of a spiral galaxy (called the "main body") became systematically bluer by an amount ranging from $0^{\mathrm{m}}.04$ to $0^{\mathrm{m}}.10$ with the maximum effect occurring for early Sb systems. No solution was attempted for ellipticals and irregulars. From a comparison of the effect of inclination on both surface brightness and color, it was possible to show that the ratio of total photographic absorption to selective absorption was close to the value of 4.0 generally found for interstellar material in the galaxy. For a medium of interspersed luminous and dark absorbing material, the ratio need not be the same as for a dark cloud of the same composition in front of a single star. Accepting the empirical result, however, Holmberg derived intrinsic colors, corrected to outside the galaxy, for the various types. For Magellanic irregulars the intrinsic color (on the P_g,P_v system) was found to be $+0^{\mathrm{m}}.12$ while for ellipticals it was $+0^{\mathrm{m}}.77$. The spirals were intermediate. For the spiral arm portion of late Sc's, the color came out as $+0^{\mathrm{m}}.12$, the same as for the irregulars; for late Sb's, the spiral arm color was $+0^{\mathrm{m}}.31$, as compared to an estimate of $+0^{\mathrm{m}}.28$ for the stellar content of the solar neighborhood.

Photoelectric measurements through apertures of increasing diameter were used by de Vaucouleurs [13] to derive empirical color-aperture relations for each type of galaxy. The centers of Magellanic irregulars were found to be as much as $0^{m}.2$ bluer than the integrated B,V color for the whole system. All other types were redder at the center. The difference amounted to $0^{m}.28$ for the Sc's and to $0^{m}.08$ for the ellipticals.

Studies of the color of galaxies which show only the integrated radial variation of color with distance from the nucleus must, of necessity, smooth over the detailed variations which are associated with the spiral arm structure. F. Zwicky's [16] method of composite printing gives an easily read, graphic representation of the large and striking differences that exist. W. Fricke [17] evaluated the color variations in M 31 quantitatively by a three-color photographic method and correlated the results with the spiral structure. B. E. Markarian [18] mapped the brightness and color in M 101 and M 51. A. K. Kalloglian [19] studied barred spirals and found the light of the bars bluer than that of the nuclei in the same systems.

The effect of the absorbing material in spiral systems seen nearly edge-on was intensively investigated by van Houten [7]. The surface brightness of the dark lane in NGC 4594 was found to be similar to that of the Cygnus Rift in the Milky Way. A study of the color of the emergent light in this dark band was compared with the predictions of two models assumed for the optical properties of the scattering particles; the accuracy of the measurements was not high enough to discriminate between scattering particles with low albedo and isotropic scattering and particles with high albedo and a forward-throwing phase function.

ENERGY CURVES

The color index of a stellar system is, of necessity, a blended characteristic since stars of widely differing temperatures contribute to the total light. The composite character of the energy curve was demonstrated in the six-color measurements of Stebbins and Whitford [20] in 1948. Only eight galaxies in the local group were measured, and in most cases the aperture included only the central portions. Since this early work there have been additional multicolor filter measurements by A. E. Whitford [21], W. G. Tifft [22], W. A. Baum [23], and by Whitford and Sears [24]. While most of these results have not yet been published in full, it is clear that the wide range from 3500 A to 11,000 A is of considerable aid in separating the components of the stellar population that make the dominant contribution to the energy curve in each part of the spectrum. All measurements agree in the strong contribution of a source of radiation at very low temperatures to the infrared point of the energy curve. Baum [25] has suggested that the temperature of this component may be as low as 1500°K.

Because the shape of the energy curve in the blue, violet, and near ultraviolet has a determining influence on the magnitude corrections to be applied to a galaxy with a large red-shift, particular attention has been given to this part of the spectrum. When the energy curve of the source has a reasonably smooth character, then the concept of effective wave length for a wide filter band can, as shown by B. Strömgren [26], be used to derive a monochromatic intensity. That the violet portion of the energy curve of the sun is unsatisfactory from this viewpoint was first pointed out by de Vaucouleurs [27] in 1948. In K giants, which were shown by Morgan and Mayall [28] to be the dominant contributors to the violet part of the spectrum of giant ellipticals, the dense blanketing by numerous metallic lines in this region produces an even greater departure from the Planck curve than that present in the sun. This renders the use of wide-band filters even more doubtful in deriving an energy curve. In the green, red, and infrared, where the curve is smooth and the blanketing small, wide-band filters are well adapted to measurements of energy and color.

The first narrow-band comparison of galaxies with stars of known spectrophotometric distribution was carried out by A. D. Code [29], using a photoelectric scanning spectrograph with a resolving power of approximately 40 A. Owing to the construction of the spectrograph, only the brightest portion of the nuclear region of M 32 could be included in the entrance slit; the area was approximately 120 square seconds of arc (Code [30]). The steep descent in the violet and near ultraviolet was, however, clearly shown at the spectral resolution employed, and closely resembled that found in late-type giants. In ellipticals, where the radial variation of color is small, the use of so small a part of a galaxy may be safe for comparison with energy curves of distant galaxies, where the seeing and guiding difficulties (not to mention the limited amount of light) force the observer to include the whole galaxy.

Further surveys of the energy curves of elliptical galaxies have been continued by J. B. Oke [31]. In future investigations, considerable thought should be given to optical arrangements which do not limit the size of the entrance slit of the scanning spectrograph to a small angular diameter around the nucleus. These studies will prove valuable not only in evaluating red-shift corrections, but also in analyzing the stellar content of unresolved galaxies. However, the energy curve alone may not give an unambiguous determination of the population; spectra covering a similar wide range of wave lengths are needed to provide additional information.

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Discussion of this paper is included in the discussion of the paper following.

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Absolute Energy Distributions in the Spectra of Galaxies

The observation of galaxies by photoelectric photometry is much more complicated than similar observations of stars for a number of reasons. (1) Galaxies exhibit a very large range in angular diameter. One cannot observe a similar portion of a sequence of galaxies with a large telescope. For nearby galaxies, one observes at one time only a minute fraction of the total area; a distant galaxy is nearly starlike, and one must observe the whole object. (2) The spectrum is composite, the light coming from a wide variety of stars. The nature of the radiation may differ in various parts of the galaxy. (3) The energy distribution is velocity shifted by sizable amounts in different galaxies.

The first of these complications is not fundamental and will not be discussed. The second and third complications are inherently rather more difficult to cope with if one is restricted to broad-band photometry. Three-color observations can yield little information about the constituent stars of a galaxy. Multicolor methods can yield much more information, particularly since they usually cover a very broad spectral range. Ideally, a very large number of measures in the spectrum should be obtained to solve satisfactorily the problem of the composite spectrum.

In dealing with observations of galaxies with considerable redshifts, it is essential that the absolute energy distribution in the spectrum of a typical galaxy be known. This was first determined for M 32 by Stebbins and Whitford [1] using their six-color system. In many respects such absolute calibrations are inadequate since one must characterize the complete spectrum by only six points. The effective wave lengths of these points cannot always be chosen ideally. As a result, striking irregularities, such as the sudden drop of intensity to the violet of λ 4000, are not usually detected. If such a feature is not allowed for, as it shifts through various filter bands it can be expected to alter colors in an unpredictable way. In spite of these difficulties multicolor broad-band photometry has been used very successfully for galaxies by Stebbins and Whitford [1], Baum [2], and Tifft [3].

It was decided to use the photoelectric spectrum-scanning instrument at the Mount Wilson and Palomar Observatories to obtain absolute energy distributions of a selection of galaxies, employing a wave-length resolution intermediate between that of spectroscopy and broad-band photometry. Some observations of this kind had already been made by Code [4]. It is expected that such observations can solve a number of problems which cannot be investigated by broad-band photometry or slit spectroscopy. The results should also make it possible to interpret broad-band results more easily and accurately.

OBSERVATIONS

The photoelectric spectrum scanners which now exist were designed to be used in slitless form to observe stars. The maximum resolution is set by the optics of the instrument and the seeing disk of the star and is typically about 5 angstroms. Such instruments have been used very successfully to measure absolute energy distributions in the spectra of stars. When employed to observe extended objects of low surface brightness, one must, on the one hand, use a slit or aperture which allows a reasonable amount of light into the scanner. If the surface is not uniform, the seeing must also be taken into consideration when choosing the size of aperture. On the other hand, if the aperture is too large, the spectral resolution becomes extremely poor. For the present observations, a circular diaphragm, about 2.7 mm in diameter, was used. The exit slit was set at 50 angstroms in the spectrum. This combination yields a final resolution of about 65 angstroms.

In the case of bright nearby galaxies, the light signals were sufficiently large that the spectrum could be scanned continuously at a rate of 200 A/min. The signal in this case was amplified and recorded by the usual direct-current techniques. For fainter objects, a charge-integration method was employed. Successive 50 angstrom bands were observed one by one throughout the spectrum. Most observations have been made with a 1P21 photomultiplier between λ 3400 and λ 6000. A few measures out to λ 7500 have been made with a red photomultiplier tube.

It was decided that an initial program should include the easily observed objects such as M 31 and M 32, and then predominantly giant-elliptical and S0 systems in clusters. At the present time, only the nuclear regions of galaxies have been observed. In the case of M 31 and M 32, scans with a resolution of 20 angstroms have been made. Scans were also made of these objects out to λ 7500.

The galaxies which have been observed are listed in Table 1. Most of the observations were made with the 100-inch telescope; the two Coma Cluster members were observed with the 200-inch telescope.

For most galaxies a complete set of observations requires several hours. It is essential, therefore, that precautions be taken to detect systematic errors caused by sky-brightness variations and extinction changes. At each wave length an integration of the light from the galaxy was followed or preceded within a minute or two

TABLE 1

Observations of Galaxies

Galaxy	*Class*	*Spectrum*	*Energy Distribution Type*	*Remarks*
NGC 221	E2	G3	0	M 32
NGC 224	Sb	G5	2	M 31
NGC 3379	E0	G7	4	
NGC 4278	E1	G5	5	
NGC 4494	E1	G7	4	
NGC 5194	Sc	F8	-	M 51, Partly observed
NGC 5866	S0	G2	7	
NGC 5907	Sb	G3	-	Partly observed
NGC 4374	S0	G5	7	Virgo Cluster
NGC 4406	E3	G7	-	Virgo Cluster
NGC 4429	S0	G3	6	Virgo Cluster
NGC 4472	E1	G7	7	Virgo Cluster
NGC 4486	E0	G5	4	Virgo Cluster, M 87
NGC 4594	Sb	G3	7	Virgo Cluster
NGC 4636	E0	G2	7	Virgo Cluster
NGC 4649	E2	G7	6	Virgo Cluster
NGC 4874	S0	G7	4	Coma Cluster
*	E	-	3	Coma Cluster

* 46″ South and 19″ West of NGC 4874.

by an integration of the sky background. Except at wave lengths where emission lines occur, the sky was usually very faint.

Early observations were made by integrating at even 100 angstrom settings from red to violet. Immediately afterward integrations at the 50 angstrom settings were made from violet to red. No systematic differences were normally observed after extinction corrections had been made. In later observations, one band, usually the one at λ 4700, was monitored approximately every twenty minutes during the observations. All observations were corrected for extinction using a standard curve. The variations of the λ 4700 readings could be used to make any further non-wave-length dependent corrections. On most nights these last corrections were very small. Occasionally they were as high as $0^{m}.15$ early in the evening or before dawn. Spectrophotometric standard stars [5] were scanned at various zenith distances during the course of the night. These were used to check the extinction law and to put the galaxy measures on an absolute-energy system. The final energy distributions are expressed as *const* − 2.5 log F_{ν}, where F_{ν} is the flux per unit frequency interval.

In Figure 1 are shown results for the two extreme cases observed: M 32 (the bluest), and NGC 4374 (one of the reddest). All galaxies observed were "classified" by their approximate location between

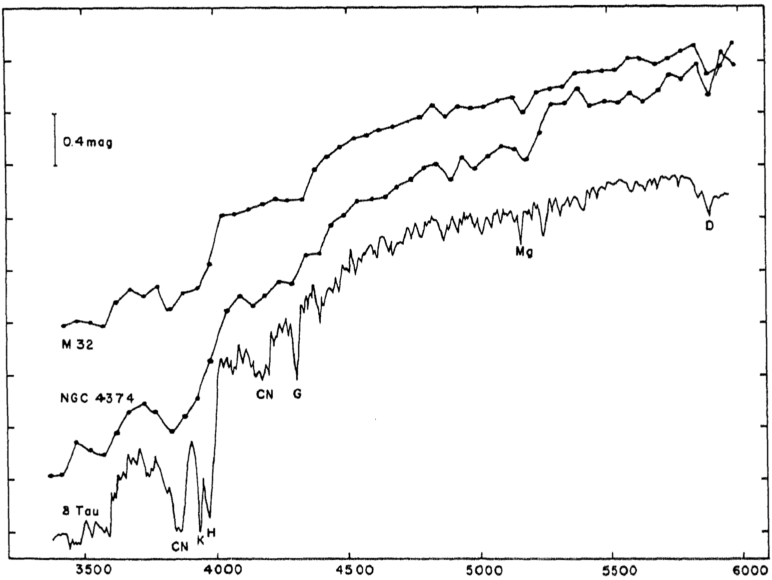

Figure 1. The absolute energy distributions (in magnitudes per unit frequency interval) in the spectra of M 32, NGC 4374, and δ Tauri.

these limits. M 32 is called "0" and NGC 4374 is "7." The "classes" are listed in Table 1 and approximately indicate the energy distribution in the galaxy. It is noted that M 31 and M 32 are both blue, but not identical. All the Virgo Cluster members are at the red extreme except NGC 4486 for which the observations are a little uncertain. The results for the Coma Cluster galaxies are also a little uncertain because of occasional clouds which were present while the observations were being made.

As a basis of comparison, the absolute energy distribution in the spectrum of δ Tauri, a KO III star, is shown in Figure 1. The resolution on the original scan is about 10 angstroms. The over-all energy distribution is rather like that of M 32. The absolute energy calibration beyond λ 5850 is rather uncertain because of the low sensitivity of the 1P21.

The detailed features in the spectrum as exhibited by δ Tau are in general observed in galaxies. Most galaxies show a depression at the sodium D-lines. The lines are seen easily on the higher resolution scans of M 31 and M 32. The sensitivity of the 1P21 is very low, however, and the existence of strong D lines in the fainter galaxies is still uncertain. The absorption in the spectrum near λ5170 caused mainly by Mg I lines is usually observed. The G-band can usually be detected. The cyanogen absorption at λ 4215, which is a characteristic of giant stars, is seen in the majority of the galaxies. It is conspicuously weak, however, in M 31 and M 32. This may be a result of the higher characteristic temperature of the radiation. The cyanogen band at λ 3889 is badly blended with other strong metallic lines. The higher-resolution scans of M 31 and M 32 show it to be rather weak. The most conspicuous feature is the sudden drop of intensity at the H and K lines of Ca II. Figure 1 suggests that these lines are weaker in M 32 than in NGC 4374, again perhaps a consequence of higher temperatures. The location in wave length of this sudden drop of intensity at the H and K lines can easily be measured for very faint galaxies. It, therefore, provides a method of measuring radial velocities of very distant galaxies. The advantage that it has over Baum's method [2] is that no important assumption has to be made about the over-all absolute energy distribution. It might, however, require a large amount of observing just to locate the approximate spectral region to observe. This method would be best used as a check of the velocities measured by Baum.

One rather disconcerting result of the measures discussed here is the lack of satisfactory correlations with the various colors determined by wide-band photometry. Correlations of different wideband colors are also rather poor. One difficulty arises from the fact

that observers use different apertures. Any variation of color with distance from the center of the galaxy can cause discrepancies. At the present time no other explanation can be suggested. A program will be undertaken with the scanner to check the accuracy of the energy distributions already obtained. Measurements will also be made at various distances from the nuclei of M 31 and M 32 and, if possible, for a Virgo Cluster member.

VARIATION OF COLOR INDEX WITH RED-SHIFT

The results already obtained can be used to compute the change of color index of a galaxy with red-shift. This was done initially by Stebbins and Whitford [1] and by Whitford [6] using the absolute energy distribution of M 32 derived from six-color measures. The observed colors were found to be much redder than those predicted. Whitford and Code repeated the calculations using a detailed energy distribution of M 32 determined by scanner observations. They reported [7] that the calculations virtually eliminated the Stebbins-Whitford effect.

The average absolute energy distribution of four Virgo Cluster members, (NGC 4374, 4429, 4594, and 4649) was used to recompute the color effect using the sensitivity functions of Stebbins and Whitford [1]. A similar calculation was also made for the M 32 energy distribution shown in Figure 1. The observed and computed color differences are given in Table 2. These figures suggest very strongly that the Stebbins-Whitford effect does in fact exist. The effect could, however, be made considerably smaller if the absolute calibration of α Lyrae in the ultra-violet were incorrect.

TABLE 2

Change of Color Index with Red-Shift

v (km/sec)	ΔCI (S-W obs)	ΔCI (Virgo)	ΔCI (M 32)
0	0.00	0.00	0.00
20,000	0.26	0.15	0.15
40,000	0.50	0.29	0.25
60,000	0.68	0.40	0.26

CONCLUSIONS

A preliminary outline has been given of the types of problems which can be solved with photoelectric observations. Observationally, energy distributions should be measured for many more elliptical and S0 systems and also for spirals. The measures should be extended, where possible, to λ 10,000. It is planned to observe more distant objects to see if the intrinsic energy distributions change with distance. Scans have been made of the various kinds of stars which one can expect in the central regions of galaxies. An attempt will be made to use these to synthesize the observed energy distributions.

The author is indebted to Miss M. Kaufman who has performed most of the reductions and calculations.

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DISCUSSION

Bok: What are the possibilities for narrow-band photometry for the study and classification of galaxies?

Whitford: Narrow-band photometry certainly has promise. The calibration of sensitive features in late-type stellar spectra will be important. In principle it should be possible to find indices for temperature, stellar luminosity, and metal content.

Sandage: Would Dr. Whitford comment on the presence of the Stebbins-Whitford effect found by Oke?

Whitford: Oke's conclusion that there is a real and large effect is surprising to me. The decline of the energy curve of the Virgo ellipticals found by Oke is not as steep as that found by Code for M 32. It was this near discontinuity, previously smoothed over by broad filter bands, that wiped out the effect. A multicolor filter comparison of Virgo ellipticals and distant systems which I made in 1955 showed a negligible effect when the absolute energy calibration was via Spica rather than the sun. The latter conclusion requires more analysis.

Oke: There is a possible uncertainty of $0\overset{m}{.}1$ in the ultraviolet part of the absolute energy curve for Vega, used in my scans. An error in the right direction would decrease the effect considerably. Code is recalibrating Vega.

Baum: Oke's conclusion does not seem consistent with the fact that red-shifts derived from multicolor photometry agree with the red-shifts obtained spectroscopically. Moreover, the reduction of the multicolor results to an absolute energy scale was based upon Oke's calibration of HD 182487. A possible explanation is that the photometrically derived red-shifts are based upon fitting a mean curve to a complete set of points across the spectrum, whereas Oke's predicted color indices are based on only two points which will be very sensitive to the wave-length response profile assigned to them.

Oke: Baum's explanation may very well be correct. For velocity shifts of less than 60,000 km/sec only the blue filter is involved in producing a reddening effect. This one point would affect Baum's velocity determination by only a small amount.

King: With reference to Whitford's paper, de Vaucouleurs' law is objectionable because it extends to infinity. We know that stellar systems do not extend to infinity, and that a cutoff is necessary with any law. The infinite mass in the Hubble law then disappears, and it applies equally well to globular clusters, to low-surface-brightness spherical galaxies, and to ordinary spherical galaxies.

De Vaucouleurs: Difficulties with infinity occur outside the range of possible observation and the question is one of fit over the observable range. Unlike the Hubble law, the $r^{1/4}$ law has no arbitrary parameter in it which must be adjusted to obtain a straight-line relation. Both old and new data fit the $r^{1/4}$ law very closely when applied to the major axis of elliptical galaxies. The departures that have been found were for SO types, or for the minor axes of galaxies of various types.

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Stellar Content of Galaxies

Accurate three color observations of faint stars in globular clusters have recently been made [1,2]. Metallic-line-blanketing corrections are now known well enough to transform the resulting main sequence into the M_{bol}, log T_e diagram [3]. Then it is possible to fit the evolutionary models computed by Haselgrove and Hoyle [4]. A fit to a particular evolutionary track requires knowledge of the hydrogen, helium, and metal proportions in a globular cluster main sequence. These proportions are quite uncertain, however, and, therefore, the derived ages of the globular clusters can range between 25 and 15 billion years.

Nevertheless, galactic clusters are generally much younger than this, and it has been known for some time that they are also much more metal-rich than the globular clusters (5). This circumstance implies that stars formed in the early stages of the formation of a galaxy are more nearly pure hydrogen, whereas subsequent star formation takes place in an increasingly metal-rich interstellar medium.

Because conditions at the time of formation of the galaxy and subsequent evolution have important cosmological consequences, it would be extremely useful to investigate in detail the relation between chemical composition and time of formation. Of course, as soon as stars are characterized by these two parameters, the concept of stellar populations is modified and extended. The original terminology of "globular and galactic clusters" and "population I and II" becomes less exact than is the term "star cluster" of a certain structure containing stars of a certain age and chemical composition. Regardless of terminology, however, these two parameters are necessary for an accurate description of the stellar content of a galaxy.

To obtain these parameters, extensive observations of the evolutionary turn-off points in the main sequence of color-magnitude diagrams for star clusters are required. This furnishes ages. The lack of metal lines in a star's spectrum allows an abnormal amount of light to be radiated in the ultraviolet. Three-color measures of this ultraviolet excess can be very easily calibrated in terms of metal abundance in main sequence stars [6]. Ultraviolet excess can also be measured in the red giants. Near $B-V=1\overset{m}{.}0$ all luminosity classes have the same intrinsic $U-B$, $B-V$ relation and the ultraviolet excess measured at this point therefore reflects only the metallic line-blanketing effects. The ultraviolet excess at this point in the giants has been calibrated against the excess on the main sequence between $B-V=0\overset{m}{.}4$ and $0\overset{m}{.}7$ where the conversion to metal abundance has been established. Red giants of $B-V=1\overset{m}{.}0$ can be observed at large distances. The above techniques for age computation and for measuring metal abundance are now being used extensively to investigate the present composition of galaxies.

Figure 1 shows a plot of the age versus relative metal abundance for a number of star clusters in the Galaxy for which data are presently available. The first important point is demonstrated

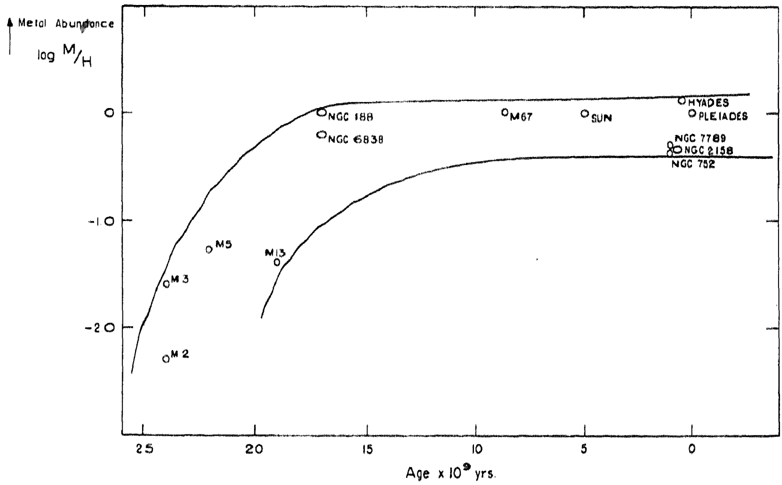

Figure 1. Age versus relative metal abundance for twelve clusters.

by the general relation between age and chemical composition. An envelope is sketched around the points to show the best general run of the correlation as it appears at present.

It is immediately apparent that the correlation between age and chemical composition is only approximate, and that there exist clusters which have formed at the same time but have different metal contents. This is first apparent in the globular clusters. Apart from the question of finding the age of a star of one chemical content relative to another, that is, regardless of whether the globular cluster ages are correct relative to the galactic cluster ages, the globular clusters should be dated correctly with respect to each other within the low metal content population. Figure 1 shows that M 13, which is unmistakably younger than M 5, has a lower metal content. Figure 1 also shows that M 3 and M 2 have about the same age but quite different metal content. We must conclude that, if the galaxy started from pure hydrogen, metal enrichment proceeded much faster in some parts of the galaxy than in others. Presumably, the metal enrichment proceeded fastest where star formation was fastest, that is, where the density was the highest.

Not much can be proved about the relative densities at the place of formation for specific globular clusters. It is, however, of interest to investigate still further the curve in Figure 1. Because of the difficulty of photometry in crowded star fields, the age shown for the nuclear globular cluster, NGC 6838, is only an upper limit [7]. The accuracy of the place of NGC 6838 in Figure 1 is sufficient, however, to show that the cluster is as young as, or younger than, the cluster NGC 188. Yet NGC 6838 has a definitely lower metal content than NGC 188. If NGC 6838 formed near the galactic nucleus and NGC 188 was formed in the disk near the sun, it is still difficult to say which was the most dense, or which material had suffered the most nuclear processing through stellar interiors.

In our Galaxy at present, young clusters are formed mostly near spiral arms. Therefore, they lie close to the central plane and are hard to observe because of reddening. Most of the well-investigated young clusters lie close to the solar neighborhood. Recently, however, the 200-inch and 100-inch telescopes have been used to investigate clusters increasingly remote from the solar neighborhood. Differences emerge. The results for two such clusters, NGC 2158 and NGC 7789, are shown in Figure 1. A third cluster, anomalous for the solar neighborhood, NGC 752, is also plotted because of its similarity. Because this is the first time metal-abundance differences have appeared in such young clusters, it is interesting to look more closely at the results.

Accurate photoelectric photometry of field stars and the occurrence of a distant Cepheid near NGC 2158, have fixed the reddening very closely. It appears that the cluster has a small but definite ultraviolet excess. The clusters NGC 7789 and NGC 752 have such similar color-magnitude diagrams (see Figure 2) that they were investigated again in order to see if they also showed evidence of metal poorness. Re-evaluation of the NGC 7789 observations shows that it, too, has an ultraviolet excess. NGC 752 not only has an ultraviolet excess of $(U - B) = -0\overset{m}{.}09 \pm 0\overset{m}{.}01$ but its K0 giants show CN weakening, and its main sequence stars are members of N. Roman's weak line F star group. All three clusters are about 10^9 years old.

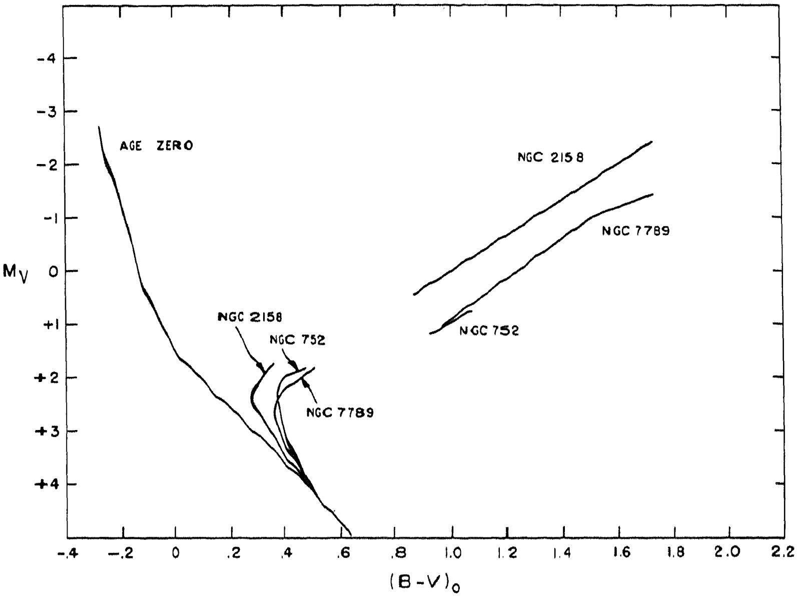

Figure 2. Color-magnitude diagrams for three clusters.

NGC 7789 is about 2000 pc beyond the sun near the Perseus arm. NGC 2158 is 5000 pc from the sun in the anticenter direction. NGC 2158 resembles, in structure and in richness, a globular cluster more than it does a galactic cluster. With respect to age, metal richness, and structure, NGC 2158 represents the long-sought-for transition system between a galactic and a globular cluster. It is more important to stress, however, that these three clusters appear to belong to regions beyond the sun, where the density is probably smaller. For the first time we are beginning to get observed correlations between low density regions and low metal content as predicted by the theory of element building and star formation.

It is significant that the only other example of recently formed clusters which are rich and globularlike is found in the Magellanic Clouds. There the metal content and interstellar density are also both down. In the region just discussed, beyond the sun in the Galaxy, Kraft has found some examples of Cepheids with amplitude behavior more like those in the SMC and with metal-line weakening in their spectra. A tentative picture emerges that the Magellanic Clouds are comparable in stellar population to the outer portions of our Galaxy. Indeed, this is common sense if one considers either visual analogy or the probable similarity of physical conditions and evolutionary stages.

Cepheids are useful in tracing population characteristics throughout a galaxy or into extra-galactic systems, and so are other kinds of variables. Very little photometry has been done on red, long-period variables, for example. It is well known, however, that, in the general field, the short-period variables, with periods around 150 days, are quite distant from the central plane, while variables with periods from 400 to 500 days lie relatively close to the plane. Obviously, the period is a rough population characteristic for the red variables. This is borne out because most globular clusters contain red variables with periods in the 100 to 150 days range. The old [8] and very metal-rich [9] cluster 47 Tuc contains red variables with periods of about 200 days. In the dwarf elliptical member of the local group of galaxies, NGC 185, W. Baade found among the stars which are just resolved, red variables with periods of 133, 162, and 243 days [10]. This is the first time we have obtained the slightest direct evidence, from individual stars themselves, of the kind of stellar populations that occur in elliptical galaxies.

A great deal more observational material is needed, however, before we can disentangle the two parameters of age and chemical composition by looking at variable stars alone. The period of a red variable is probably determined primarily by the depth into the zone of pulsational instability to which it is carried by its evolutionary track. This evolutionary track is determined by both its age and chemical composition.

Recent results on the red variables in 47 Tuc [11] illustrate this point. At the exact end of the 47 Tuc giant branch, three stars pass over into the condition of being variables. Their amplitudes are small and their periods irregular. But they definitely exhibit cycles of about 50-60 days. The implication is that this is the characteristic period at this stage. At an apparently later evolutionary stage, there are two red variables which are semiregular at about 150-160 days. Finally, there are the three variables with periods of about 200 days having amplitudes of 4^m or 5^m in both colors. Analysis of this behavior should yield valuable information on physical conditions during this stage. But it is clear that stars of the same age and chemical composition (members of the same cluster) can go through several stages of development and have more than one period. In addition, there is no guarantee that a star of a different age and different chemical composition could not pass through the same point in the instability zone and mimic stars of a different population found there.

The study of red variables such as those in clusters, in the nucleus of our Galaxy and in other galaxies will be very useful in outlining physical processes and population characteristics.

The cataclysmic variable U Gem stars and the novae are clearly not halo population stars nor are they young spiral arm stars. By their velocity distribution, the U Gem stars clearly belong to the disk population [12]. They are probably normal or slightly deficient in metals, and of intermediate or fairly old age. The novae, from their distribution in the Andromeda Nebula [13], appear to be the same kind of star. Kraft suggests that the binary nature of the cataclysmic variables may cause their outbursts. The present population assignment agrees in removing them from the old low-metal content stars which have so few binary stars.

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DISCUSSION

Feast: Are regular variables in 47 Tuc bluer than irregular ones (they are later in spectral type)?

Arp: The mean color index of the 200-day variables in 47 Tuc is bluer than that of the irregular variables and bluer than the end of the giant branch. This is probably due to Titanium Oxide absorption causing an artificially blue color index by chopping out light in the *V* wave lengths.

Christy: From the presence of young metal deficient clusters in the outer reaches of the galaxy, would you conclude that new material has recently been added to the galaxy?

Arp: I can only conclude that the material out of which these stars were formed has undergone less processing through stellar interiors. This is presumably due to the lower density in this region of the galaxy, hence, to the lower rate of star formation. Large-scale mixing or circulation through the galaxy in periods less than 10^9 years would tend to prevent such inhomogeneities, however.

De Vaucouleurs: There are 3 or 4 small, very bright compact dwarf galaxies (prototypes M 32, NGC 4486 - A, etc.) which seem to have a $0^{m}.1$ ultraviolet excess compared with the normal $U - B$, $B - V$ relation for ellipticals. Could this be an index of differences in chemical composition?

Arp: It could be due either to a lowered metal content or — in the case of a composite energy curve — it could arise from a larger proportion of blue stars being present in the dwarfs than in normal ellipticals.

Feast: What is the evidence for a low stellar and interstellar density in the SMC?

Arp: The average surface brightness of the SMC is much less than that of a normal galaxy. In general, it is less than the surface brightness in a spiral arm. This implies that where the density is down, the rate of star formation is down and metal enrichment is down. I am only pointing out that the structure of young metal-poor clusters which are also rich and spherical may be permitted by the lower density regions of formation.

As for the gas density, Frank Kerr has said that the SMC could have a density ranging from that of the solar neighborhood to much less.

Kerr: The total quantity of hydrogen in the SMC is quite large. Space density depends on the extent of the system in depth. If the Cloud is well flattened, the mean density might approach one atom/cc, but if it is more spherical, the density would be much less.

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Special Problems in the Magellanic Clouds

The importance of the Magellanic Clouds to astronomy arises partly from their proximity and partly from the number and variety of objects which can be studied in them. It is the latter point which distinguishes them from other nearby galaxies such as those in Sculptor, Fornax, and Draco, which are essentially population II systems. The Clouds have an apparent distance modulus of 18.5 or 19, and are at a distance of 50 or 60 kpc. This means that with the biggest telescopes available, the 74-inch reflectors at Pretoria and Mount Stromlo, stars can be measured in them to, say, $M = +1$ ($m = 20$), and stellar spectra obtained to, perhaps, $M = -6$ ($m = 13$). It means also that the clusters in the Clouds, of which there are hundreds, are near enough to be resolvable, so that in many cases one can construct color-magnitude diagrams for them. And the scale at which the Clouds are seen, about 15 pc per minute of arc, is great enough to allow detailed studies to be made of the surface distribution of their radio-frequency radiation, especially with the high-resolution systems at present under construction in Australia. A similarly detailed picture can be built up of the kinematic structure of the Clouds; this is discussed by Thackeray [1].

Table 1 summarizes the most important data about the Clouds; almost every figure in it is open to argument. The fourth row refers

TABLE 1

Basic Data on the Magellanic Clouds

Distance from the sun:	50 - 60 kpc	
Apparent distance modulus:	18½ - 19	
Angular extent:	LMC, 12°;	SMC, 4°
Neutral hydrogen envelope:	angular dimensions – 15° × 30° linear dimensions – 15 × 30 kpc	
Mean velocity relative to the sun:	LMC, +290 km/sec;	SMC, +160 km/sec
Mean velocity relative to the galactic centre:	LMC, +40 km/sec;	SMC, −15 km/sec
Luminosity:	$M_B = -18$ (LMC);	$M_B = -16$ (SMC)
Mass:	$10^{10} M_\odot$ (LMC);	? (SMC)

to the work of Kerr and his associates [2], who have shown that the Clouds are enclosed in an envelope of neutral hydrogen, sharply bounded on the preceding edge of the SMC and the following edge of the LMC. Clearly, the Clouds are physically related, and may well be separate condensations of the same mass of prestellar matter. The fact that they form a dynamical system, akin to a pair of double stars, of known separation and relative radial velocity, enables a lower limit of $10^{10} M_\odot$ to be set on their masses, and their low radial velocities suggest that they may form a triple system with the Galaxy itself.

One of the basic problems is to determine the distance modulus of the Clouds. This can be done, in principle at least, from luminosity classifications of known stellar members (usually supergiants or B stars), using perhaps narrow-band photoelectric techniques; from the luminosities of Cepheids, short-period variables, novae, or other variables; from the positions of the giant and horizontal branches of globular clusters; possibly from the color-magnitude diagrams of young clusters containing red giants; and so on. The objects have to be found and their membership established. Often this is obvious; when not, a radial velocity is usually the best criterion.

Attempts using most of these approaches have been made. They all run into a fundamental difficulty: what exactly is the relation between Cloud objects and galactic objects of generally similar type? We can illustrate this from globular clusters, or globularlike clusters, for five of which we now have color-magnitude diagrams. These fall into two groups, conveniently illustrated by the colors and magnitudes of the stars at the upper ends of their giant branches. For comparison, the giant branches of galactic halo clusters terminate at colors ranging from 1.2 to 1.6. Two results which are both well established observationally emerge: the giant branch of NGC 121 is $0^m\!.5$, or more, fainter than those of the other clusters in Table 2; except for NGC 121, the stars in this table are redder than the corresponding stars in any halo cluster. Either this difference is intrinsic, or NGC 121 is 20 kpc further away than the other clusters. Its radial velocity establishes it as a member of the SMC.

TABLE 2

Giant Branches in Globular Clusters

	V	$B-V$	Reference
NGC 121	16.7	1.6	Tifft [3]
NGC 361	16.2	1.7	Arp
NGC 419	16.1	1.8	Arp
NGC 1783	15.8 16.0	2.0 1.8	Eggen and Sandage Gascoigne [4]
NGC 1846	15.8	1.6	Hodge
NGC 1978	15.9	1.9	Hodge

I think that the first alternative is the correct one, and further, that NGC 121 can be identified with the galactic halo clusters. The evidence for this is the position and color of its giant branch, and the fact that it has both a horizontal branch and cluster variables. This identification has extensive implications. First, it puts the modulus of the SMC at something in excess of 19. Secondly, it means that NGC 361, 419, 1783, and 1978 constitute a new type of cluster, unrepresented in the Galaxy, and distinguished by a giant branch which runs up to $M_v = -3$, $B - V = 1.8$. This is not so unlikely when we think that in NGC 1866 and 1831 we already have clusters of types not found in the Galaxy. One may even surmise that they represent successive stages in the evolution of a halo-type cluster, running in the direction: NGC 1866 to 1831 to (361, 419, 1783, 1978) to (121, 1466, 2257). NGC 1466 and 2257 have been added because they contain considerable numbers of cluster variables.

Finally, it is already known that the SMC Cepheids of shorter period are relatively more numerous, have greater amplitudes and are somewhat redder than those in the Galaxy. A modulus of 19+ would mean that they are also several tenths of a magnitude brighter, at any rate in the 3- to 10-day period range [5] (the slopes of the period-luminosity relations may be different). This result cannot but weaken the position of Cepheids as extra-galactic distance indicators.

The color-magnitude diagrams of the young Cloud clusters are easier both to observe and to interpret. But again we encounter phenomena, such as gaps near the upper ends of the vertical main sequence, and unexpectedly blue colors for the "evolved" stars not matched by those in any comparable galactic cluster. The galactic sample is, however, small, being limited to H and chi Persei, kappa Crucis, and M 11. Sufficient attention may not always have been paid to field stars, and some of the stars which are supposed to have evolved anomalously may in some cases actually belong to the field. The Clouds are objects of low surface brightness. De Vaucouleurs found that even in the densest part of the LMC less than half of the stars brighter than 14.3 were Cloud members; five degrees out, the proportion had fallen to less than a fifth [6]. Hodge has made the interesting suggestion that the number of young clusters in the LMC is great enough to justify the assumption of an enhanced rate of star formation there over the past 10^7 years [7].

The Clouds provide the best opportunity by far for studying abundances and abundance differences through a galaxy. This problem has become the subject of a lively controversy which originated in a suggestion of the Burbidges [8] that the apparently low dust-to-gas ratio in the SMC is an indication of metal deficiency. There is still no agreement as to the amount of dust in the SMC. Following Shapley, Wesselink [9] has made galaxy counts and attributes the low numbers he finds within the Cloud to obscuration, hence, to dust. Sandage remarks on a correspondence between dust and young stars in galaxies at large which is almost one-to-one [10]. Bright blue stars undoubtedly exist in the SMC. Do these stars necessarily imply dust? But it is difficult to find an object in the SMC reddened by as much as $0\overset{m}{.}1$, and this includes reddening within the Galaxy. And the dispersion of the period-luminosity relation found by Arp for his Cepheids seems much too small to allow any appreciable absorption. The period-luminosity dispersion is much greater for Cepheids in the LMC, where also dust lanes and patches are obvious in a way they certainly are not in the SMC. We are still not sure what dust particles are, let alone whether the presence of heavy elements is essential to their formation. Even the complete absence of dust may not provide an unexceptional case for low metal abundance [11].

Another argument is provided by Cepheids. Shapley in the Clouds [12], Baade and Swope in M 31 [13], and, more recently, Bahner, Hiltner, and Kraft in the Galaxy [14], together have shown fairly convincingly that the mean period of a group of Cepheids tends to decrease with the density of the region in which the Cepheids occur. The argument is that the circumstances of Cepheid variation depend on metal abundance, and that the metal content of a less dense region will be lower because, in it, the rate of star formation, hence, of the transmutation of hydrogen to metals is lower. This is probably true. But the regions of high stellar density in which the long-period Cepheids tend to occur are not necessarily regions of high gas density – one would have to consider as well, for instance, the mechanics of disk formation. Also, we cannot follow the argument through without a more satisfactory theory than exists of the pulsation and evolutionary status of Cepheids. This line of attack – using the properties of Cepheids to infer those of the medium from which they originated – seems potentially a most powerful one.

Arp [15] has just shown how to estimate the age and composition of a globular cluster. His methods are not directly applicable to such clusters in the Clouds (tentatively identified as NGC 121, 1466, and 2577) because we cannot observe the breakoff point, hence, cannot make a direct age determination; the *UV* measurements from which a *UV* excess, consequently, a metal abundance, could be estimated are also difficult for such faint stars. Even in the most favorable circumstances we cannot observe the Cloud globular clusters to much below the horizontal branch. Something, however, can be inferred from the known sensitivity to metal abundance of the slope and position of the giant branch, and this, together with information on variables, spectra, and perhaps multicolor photometry, may enable us to disentangle the effects of age and abundance. It is quite practicable to obtain spectra of the integrated light of these clusters, which could be analyzed on the lines of Mayall and Morgan.

Color-magnitude work on these objects is difficult and slow, and very dependent on seeing; it is much to be hoped that easier methods can be developed for distinguishing between, say, halo and disk-type clusters in the Clouds.

The direct way to determine abundances is, of course, from stellar spectra. We are hampered here because only supergiants ($M < -6$) are bright enough to make this practicable, and these stars are not well suited to abundance determination. Feast, Thackeray, and Wesselink, who have by far the most extensive data, find no general spectroscopic peculiarities which cannot be attributed to high luminosity and, in particular, no evidence for metal-poorness. Code and Houck find some anomalies for SMC B stars; Buscombe [16] does not. Spectra of gaseous nebulae may give the most reliable answers we have, though they give information on only a few elements. H. M. Johnson and the Pretoria group working with spectrographs, and Aller and Faulkner with a scanner, find a good match between the Orion Nebula and 30 Doradus (LMC), and between 30 Doradus and IC 1644 (SMC). No significant abundance differences are suggested. Aller and Faulkner find normal H/He ratios in 30 Doradus and in NGC 348 in the SMC.

In summary, then, the Magellanic Clouds enable us to calibrate the luminosities of many distance indicators and to extend our knowledge of evolutionary processes because they contain clusters of types not found in our Galaxy. We should be able to determine abundances for a variety of objects in them, and by combining this information with data on luminosity functions, kinetics, distributions of neutral hydrogen and so on, build up an evolutionary history of the Magellanic Clouds in a way not possible in any other galaxy, including our own.

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DISCUSSION

Tifft: I would estimate from the NGC 121 region, both cluster and field, the apparent distance modulus of the SMC is 19.2. As one approaches the SMC core there appears to be a transition in stellar type to that seen by Arp. The comparison with the Galaxy in the 121 region is very close although some *small* differences remain.

Arp: Tifft has obtained a surprisingly large number of *old* stars in the SMC. Either the young stars in the center and following wing of the SMC overwhelm the substratum of old stars or the loci of old stars and young stars are displaced from each other.

Bok: Might we hear the reaction of the Radcliffe astronomers to the suggested modulus of 19.0 or 19.2?

Thackeray: I am very glad that Dr. Tifft has made this photoelectric determination of the distance modulus of the SMC, so much needed since our Radcliffe photographic determination was made in the face of considerable difficulties. I look forward to seeing Dr. Tifft's detailed results in order to assess the need for revising the distance modulus.

Sandage: If the true modulus of the SMC is about 18.7 from Arp's Cepheid data using Kraft's calibration of the period-luminosity relation, if Tifft's data for NGC 121 place the unreddened horizontal branch at $V = 19.6$, and if $M_V = +0.4$ for the NGC 121 RR Lyrae stars suggested by Tifft, then we have a $0\overset{m}{.}5$ discrepancy in the absolute magnitude of the Cepheids in the SMC and in our Galaxy. Is this significant? If it is, then Cox's calculation of the theoretical period-luminosity relation probably requires that the H/He ratio differs in the SMC and our Galaxy.

Arp: Tifft has found the RR Lyrae stars and associated horizontal branch in the preceding wing of the SMC at $M_V = 19\overset{m}{.}6$.

If there is no absorption in the field, and if the absolute magnitude of the RR Lyrae stars is $M_V = +0\overset{m}{.}4$, then the SMC long-period Cepheids are $0\overset{m}{.}5$ brighter than the Cepheids in our Galaxy. These are Tifft's assumptions.

If the absolute magnitude of these RR Lyrae stars is $M_V = +0\overset{m}{.}7$ and the absorption is $A_B = 0\overset{m}{.}1$, then the Cepheids in the Galaxy and the SMC have the same absolute magnitude. These assumptions may be more realistic.

The answer to the question, then, depends primarily on what kind of RR Lyrae stars the SMC contains – whether they are of the presumably more common kind, as in M 3, M 5, and as is RR Lyrae itself, which have been most recently calibrated at about $M_V = +0.65$, or whether the SMC RR Lyrae stars are more like those in the younger globular cluster M 15 ($M_V = +0\overset{m}{.}3$).

Tifft: I should emphasize that the SMC modulus fit is based mostly on the halo field, not the cluster which may be closer or more distant. If anything, the field component is fainter than the cluster.

Thackeray: I would confirm that, according to our published results, some variables near NGC 121, provisionally classed as field RR Lyrae variables in the SMC, are slightly fainter than the RR Lyrae variables in NGC 121.

Kerr: Apart from the absolute value of the distance modulus of the Clouds, the relative distances of the two Clouds are of interest in working out the space distribution of the over-all hydrogen envelope. Radio people ask for assistance from optical astronomers on this point.

Gascoigne: The difficulty here is chiefly that of being sure we are dealing with strictly comparable objects. The indications are, from Cepheids and globular clusters, that the LMC is nearer by perhaps $0\overset{m}{.}1$, but this is by no means certain.

Whitrow: What is the density distribution in the envelope surrounding the Magellanic Clouds, and does it have anything like a well-defined boundary?

Gascoigne: It is very sharply defined on the preceding edge of the SMC and the following edge of the LMC.

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Interstellar Matter in Galaxies as Revealed by Observations in the Optical Spectral Regions

The presence of interstellar matter in galaxies is revealed in the first instance in local regions in which emission lines appear identical to those appearing in the Galaxy in emission nebulae surrounding O and early B stars. Indication of a more general gaseous substratum was given by Mayall's [1] discovery of the frequent occurrence, from the nuclear regions outwards, of the line λ 3727 [OII], predominantly in galaxies of late type. Systematic investigations as to the frequency in different types of galaxies have been made by Humason [2] and Mayall [3]. The line begins to appear already in the E type, increases in strength in the successive spiral types, and is almost always present in the irregular types. The appearance of the line is clearly intimately correlated to the development of the population type I in the systems. The line λ3727 is most often accompanied by other lines, especially Hα. Page [4], who has studied emission lines in double galaxies, found that the ratio of intensity between λ 3727 and Hα is subject to considerable variations.

A few objects show strong and broad emission lines in the nuclear regions. Seyfert [5] found that the width of the lines indicate very high turbulent velocities, up to 8500 km/sec in NGC 3516 and 7469. In addition to the Balmer lines of hydrogen, the emission lines of He I, He II, O I, O II, O III, N I, S II, Ne II, Ne III, A IV, Fe VII. occur. The emission lines contribute a considerable part of the light of the very bright nuclei in these systems.

An interesting feature in many of these galaxies is the apparent small size of the nuclei in which the broad emission features arise, often only a few seconds of arc. E. M. Burbidge, G. R. Burbidge, and K. H. Prendergast [6] have investigaged the nucleus of NGC 1068 and have concluded that gas is being ejected from the nucleus of this galaxy at high velocities. The effect may be analogous to the hydrogen emission from the galactic nucleus observed by van Woerden, Rougoor, and Oort [7]. Woltjer [8] concludes that the high internal velocities are probably due to fast motions, circular and at random, in the gravitational fields of the nuclei which are probably extremely massive. In most cases the gravitational field may be sufficient to prevent the mass to flow out, and there may be a principal difference against the case of the galactic nucleus where the flow may be determined by electromagnetic forces. G. Münch [9] has investigated the line pair λ 3726-29 in the nucleus of M 31 within about $1'$ from the center. He finds indications of widely turbulent motion and a systematic trend when the radial velocities are corrected for the rotation which means a general expansion, if the NP edge of the nebula is the near one. Osterbrock [10] has investigated the broad emission lines in the nuclei of certain elliptical nebulae. For NGC 4278, he derives mass and rotational and turbulent velocities of the gas and concludes that it is similar to very extended gas clouds in the Galaxy.

In peculiar cases, often multiple nebulae in more or less close contact, as depicted in the extensive catalogue prepared by Vorontsov-Velyaminov [11], intense emission objects may be formed in quite irregular formations, as exemplified by NGC 2444-5, studied by the Burbidges [12]. The nature of this object, and similar ones, is still a great puzzle from a cosmological point of view.

Interstellar lines in absorption similar to those observed in the Galaxy have been found by Feast, Thackeray, and Wesselink [13] in the Magellanic Clouds. The lines appear in spectra of stars involved in nebulosity. The interstellar absorption lines cannot be easily studied in the spectra of more distant galaxies. We have reason to assume, however, that the gas and solid grains form clouds of about the same range in size and with the same order of internal velocity dispersion as in the Galaxy. A comprehensive review of the properties of interstellar matter, with special reference to the formation of stars, has been given by Spitzer [14].

The extinction effects of dust clouds in the interstellar matter is evident in the distribution of intensity and color over the apparent disks of galaxies. The effects begin to be conspicuous in the late E types and the S0 types and proceed with increasing intensity in the spiral and irregular types. As is well known, the effects are most conspicuous when the line of sight forms a small angle with the equatorial plane of the system. Even when this angle is fairly large, however, especially in spiral systems of late type, the effects of the dark matter on the intensity and color may be very considerable in certain regions of the apparent disk of the system, which will often cause great asymmetries in intensity and color along lines parallel with the apparent minor axis, and, especially, along this axis itself [15].

Measurable effects of polarization were first detected by Öhman [16] for certain clouds in M 31. A. Elvius [17] has found polarization effects of NGC 4565, 5055, 7331. In the case of NGC 5055 the electric vector appears to occur along the spiral arms. This is partly the case also in the results for NGC 7331, which will be further mentioned below.

Hiltner [18] has observed 21 globular clusters in M 31 for polarization. The polarizations range from $0\overset{\mathrm{m}}{.}006$ to $0\overset{\mathrm{m}}{.}061$. The best determined cases show that the electric vector is approximately parallel with the apparent major axis, as should be the case if the fields are longitudinal and approximately along the arms.

The starting point for theoretical investigations on the nature of the interstellar extinction of light has, in general, been Mie's classical theory of diffraction by small spherical particles. On the other hand, the interstellar polarization is ascribed to the alignment by electromagnetic forces of particles deviating from the spherical form.

Schalén [19] applied Mie's theory with success to the problem of interstellar extinction in the Galaxy and derived the radius of the grains, assumed to be metallic, from the observed color effect. The light scattering by small particles has been treated most extensively by van de Hulst [20], with special consideration of the application to astrophysical problems. Van de Hulst estimates that the grains responsible for the extinction of starlight in our surroundings is of the order 1.4×10^{-26} gr/cm^3, which is about one per cent of the hydrogen density. The radii of the particles are on the average about 3×10^{-5} cm. It is possible that most of the atoms heavier than helium are in the grains.

According to Platt [21], the dust clouds may contain particles of quite different optical properties–solid particles of radius less than 10^{-7} cm. which have grown by random accretion from interstellar gas. High visible absorption coefficient per unit mass, high albedo, and considerable magnetic moment and ellipticity are to be expected for tiny particles (3×10^{-3} λ) of normal cosmic composition as soon as it is assumed that their electronic energy bands are usually unfilled. Optically, the best comparable terrestrial species will be radicals and unsaturated compounds. Platt suggests the following conclusions: The longest wave length, λ, of strong allowed absorption parallel to the largest diameter, l, of a particle will probably be of the order gl, where g is about 400. The absorption cross-section of the particles is likely to be roughly constant and equal to the geometrical cross-section down into the ultraviolet. The albedo in the visible region may be large, perhaps near unity. Most of the particles will be paramagnetic. The particles will not be perfectly spherical and should give several per-cent polarization if aligned with their long axes (or short axes) all parallel. Spitzer [14] has remarked that it is difficult to see how such small grains could avoid growing to larger sizes, as long as most of the heavier elements are still present in the gas. Moreover, observed evidence on the composition of the gas indicates that an appreciable fraction of beryllium and calcium atoms is, in fact, locked up in the grains. At the present time, however, the lower mean density for the dust cannot be excluded.

Öhman [22] suggests that the negative hydrogen ion H^- may be present in sufficient frequency in interstellar space to be of importance for the extinction of light. He refers to a recent investigation by W. H. McCrea and D. Nally [23], who found good evidence for a frequent formation of H_2 molecules by surface reactions on interstellar dust grains. A first study of such reactions has been made by Kahn. These results draw the attention to the possibility that H^- may be formed from interstellar H_2 molecules by capture of electrons in the formation of H_2^- ions with subsequent dissociation of H_2^- into H and H^-. It was tentatively suggested by Öhman that this reaction may take place not only in the solar envelope but also in interstellar space. I. Fischer-Hjalmars [24] has taken up this idea in a careful and critical study and indicated how different reactions of this type can be realized. It is possible that the presence of a magnetic field would favor the process. She discusses also some experimental results by V. I. Khvostenko and V. M. Dukelsky [25] which indicate that H_2^- ions are formed as an intermediate step from collisions between electrons and hydrogen molecules. The high abundance of H_2 molecules in interstellar space predicted by McCrea and Nally gives reason to assume that a considerable amount of H^- may be formed as a consequence of this reaction. If so, Öhman concludes that interstellar absorption may, to some extent, be due to this effect. In order to produce the observed absorption (λ^{-1} law), the average size of the interstellar dust grains would be smaller than is generally assumed. If this suggestion is supported by further theoretical and experimental studies, Öhman points out that it would be of interest to know if H^- can produce polarization effects as well, and, in this connection, it is tempting to consider the influence of weak interstellar magnetic fields of the order of 10^{-6} gauss. If polarization effects can appear in H^-, a maximum degree of polarization would be expected in the infrared. From what we know about "spectroscopic stability" [26], polarization effects would not be expected in the interstellar atomic absorption.

E. Holmberg [27] has investigated the theoretical asymmetry of light intensity and color in models of stellar systems where the luminosity is represented by a three-dimensional spheroidal Gaussian distribution of a certain axial ratio f, and the absorbing matter by a similar distribution with the same center and equatorial plane but with the axial ratio pf. Only pure absorption is considered. For $p < 1$, and when the inclination of the equatorial plane against the line of sight differs from zero and 90°, the intensity on the near side of the minor axis of the apparent distribution is relatively fainter and redder than for corresponding points on the farther side. The maximum color excess is about $0^{m}_{.}2$ and varies little when the maximum optical thickness of the absorbing disk increases from $0^{m}_{.}5$ to $2^{m}_{.}0$.

Application of Mie's theory to the observed distribution of light intensity, color and polarization in typical spiral galaxies of a certain advanced type has been performed by A. Elvius [28]. For the galaxy NGC 4216 the photometric analysis in blue and yellow was performed on plates taken by B. Lindblad with the 60-inch telescope at the Mount Wilson Observatory. The material was completed by plates in ultraviolet and red obtained by Baade with the 200-inch telescope at the Palomar Observatory. The isophotes for ultraviolet, blue, and red light were constructed, and the intensity distribution along ellipses which correspond nearly to circles in the equatorial plane was discussed. It was found that the run of the asymmetry in the light distribution could be most easily explained by assuming the presence of a fairly considerable scattering of light from the intense nuclear regions in neighboring clouds of dark matter. On the basis of van de Hulst's theoretical results, the question of diffraction effects is extensively worked out and the possibility that these effects may cause a considerable part of the observed apparent bulge is demonstrated. Plotting the difference in asymmetry between two wave lengths, for instance between blue and red against the asymmetry in the red, gives a characteristic diagram which appears to have quite different properties in the case of "real spindles" like NGC 4565 and 5746 against late-type systems of intermediate tilt like NGC 4216 and 7331 where the spiral structure and the dark clouds extend close to the nucleus. The hypothesis was made that

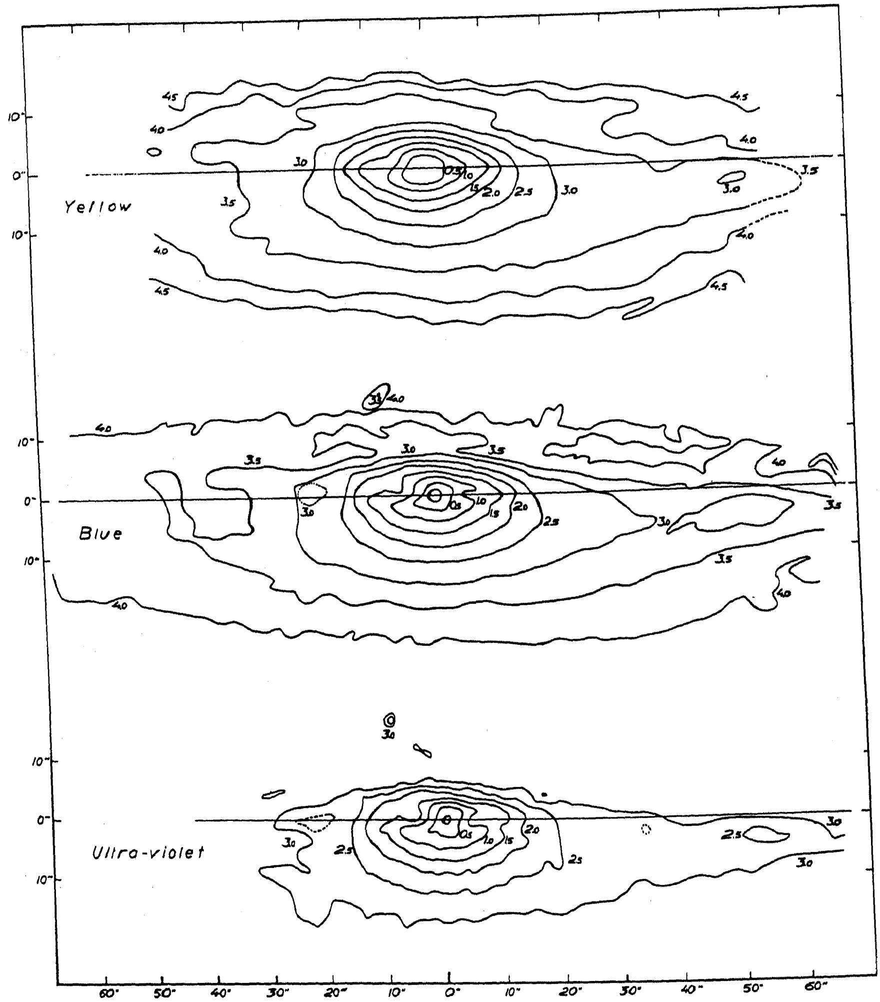

Figure 1. Isophotes in different colors for every half magnitude in NGC 4216, according to A. Elvius.

this difference may mean opposite rules regarding which side of the major axis shows the greatest absorption as a consequence of the tilt. For the galaxy NGC 7331, A. Elvius [29] has made a very elaborate study of the polarization, using Öhman's birefringent polarigraph, in which two images of narrow strips of the nebula of opposite polarization are photographed simultaneously side by side. On the brighter side, the electric vectors lie nearly parallel to the spiral arms with "positive" polarization, while, on the fainter side, a "negative" polarization appears with the electric vector nearly parallel to the apparent minor axis. The distribution of intensity in

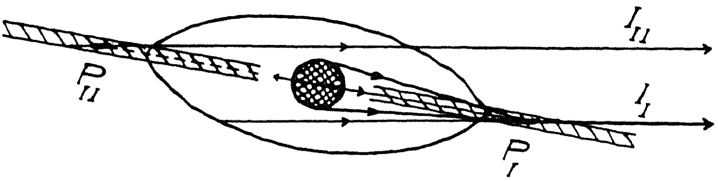

Figure 2. Contributions to the light intensity in points on the minor axis in a spiral galaxy model.

ultraviolet, blue, and red in NGC 7331 has been investigated by Lindblad [30], and in more detail by Jensen [31] on the basis of material obtained with the Crossley reflector at the Lick Observatory. Comparing the results of these authors, A. Elvius found the highest color index in the region where the strongest "positive" polarization has been discovered, whereas the regions with negative polarization are found to be considerably bluer. Therefore, it seems that the "positive" polarization is connected with extinction and that it is of the same nature as the interstellar polarization in our Galaxy. The "negative" polarization on the brightest side of NGC 7331 may be due to diffraction of light in the interstellar medium. It is important that the intensity of diffracted light must be expected to be very much stronger in the half of the spiral nearest to us than in the other half, because particles possessing the properties necessary to explain the observed extinction and reddening must be expected to scatter light much more forward than backward.

Van Houten [32] discusses the photometry of 20 galaxies based on plates taken by Oort and Oosterhoff with the 60- and 100-inch reflectors of the Mount Wilson Observatory and photoelectric observations by Oort, Hiltner, and van Houten with the 82-inch reflector of the McDonald Observatory. In order to get a starting point for an analysis of the influence of the absorbing material, a discussion of the optical properties of interstellar dust particles in the Galactic system is made on the basis of the measurements of diffuse galactic light by Elvey and Roach, Henyey and Greenstein, Elsässer and Haug. Two possible models were adopted, one with isotropically scattering grains with albedo 0.2, one with strongly forward-scattering grains with albedo 0.5. Of special interest is the extensive discussion of NGC 4594 which is dissected into two subsystems, the very wide bulge and the disk system. A remarkable difference between the distribution of luminous and dark matter in the disk is shown. The absorbing material is located in a thin ring, practically between 8 kpc and 33 kpc from the center, while the luminous part has maxima at 3 kpc and 10 kpc. (The distance of the system has been assumed to be 16.8 Mpc).

The amount of light from the bulge system scattered in the ring

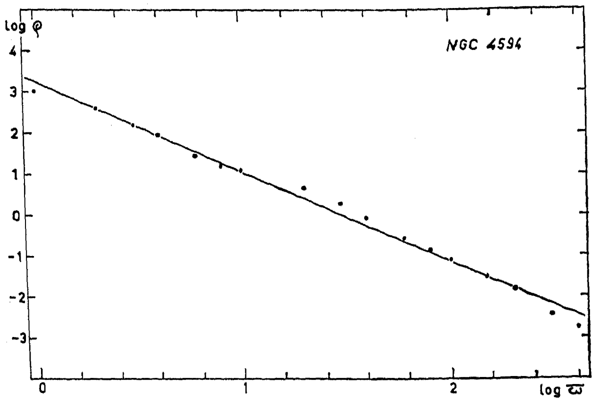

Figure 3. Density distribution of the bulge system in the equatorial plane of NGC 4594 according to van Houten.

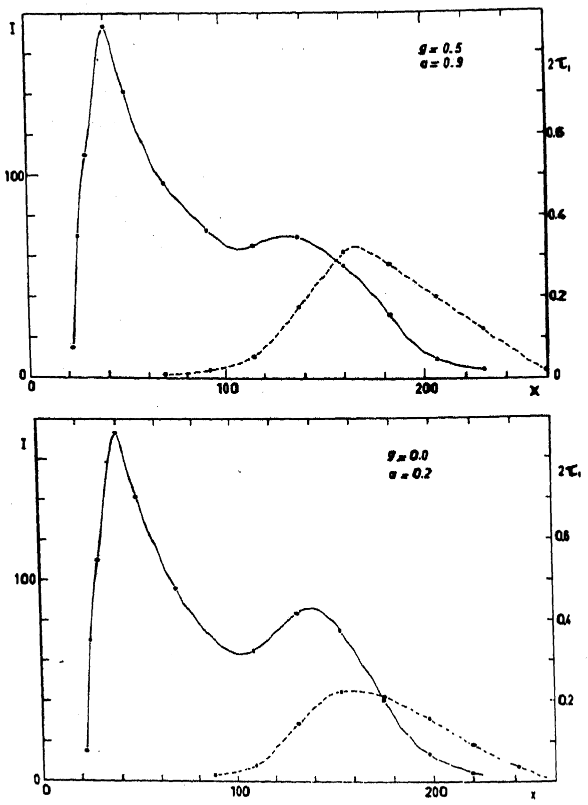

Figure 4. Distribution of integrated starlight and dark material in the disk system of NGC 4594 for high forward-throwing and high albedo (above), and for isotropic scattering and low albedo (below), according to van Houten.

is found to be comparatively small. Compared with the results derived by A. Elvius in the case of NGC 4216 and 7331 it may be pointed out that the effects must be expected to be considerably more developed and conspicuous in the last mentioned cases where fairly small very intense nuclei illuminate dust clouds in their immediate vicinity.

The low color reached on the minor axis in NGC 4594 as compared with that on the major axis is of interest. The possibility of an absorption outside the equatorial region is supported by the high color of the inner spiral arms observed by B. Lindblad [33] which correspond to the inner maximum in the disk system. The inner spiral structure shows plain traces of interstellar absorption and it is possible that van Houten has somewhat underestimated the presence of dust clouds in the inner regions.

A dissection of the systems NGC 4762, 7332, and 4526, similar to that of NGC 4594, shows that the luminous disk does not extend up to the nucleus and that the distance of the inner edge from the nucleus is correlated to the relative importance of the disk system.

The apparent impossibility of deciding between the two alternatives concerning albedo and degree of forward-throwing, indicates that the present material from our Galaxy and from external galaxies is not sufficient to determine the optical properties of the interstellar dust particles and calls for further observations.

Theories of the distribution of light over the apparent disks of galaxies are bound intimately to the question regarding the true tilt of the equatorial planes in space, and, thereby, with the direction of winding of the spiral arms with respect to the direction of rotation. The direction of tilt is obvious in cases when the absorbing layer in the equatorial plane is clearly projected on the bulge of the nuclear regions, but in these cases it is often very difficult to trace the direction of winding of the spiral arms. After examining the direction of asymmetry indicated by the strongest dark lanes in nebulae of intermediate tilt, and correlating it with the observed radial velocities due to rotation, Hubble [34] expressed the opinion that a unique relation appears to exist between the direction of rotation and the direction of winding of the arms. Assuming, by analogy with the nebulae seen nearly edge-on, namely, that the side showing the strongest absorption lanes is the near one, Hubble concludes that spiral arms are trailing in the rotation. Lindblad and Brahde maintained an opinion earlier expressed by Lindblad that, in nebulae of intermediate tilt, "secondary" lanes on the farther side may develop in greater intensity than the "primary" lanes on the near side which are strong in nebulae seen nearly edge-on. Other indications pointing towards "leading" spiral arms were mentioned. Criteria in favor of "trailing" spiral arms have been further developed by Irwin

[35] and, more recently, by de Vaucouleurs [36] on the basis of photographs of galaxies obtained by different observers with the 200-inch telescope of the Palomar Observatory.

In the case of NGC 4594 the direction of tilt is unambiguously determined by the heavy absorption lane along the near edge of the disk. The low color of the part of the minor axis which is turned towards the observer compared with the color on the major axis probably indicates the presence of a flattened subsystem of dark matter in the nuclear regions. From an ultraviolet photograph with the 200-inch telescope taken by Baade, the author [33] derived a spiral pattern which appeared to give "leading" arms fairly clearly. This conclusion has been doubted by de Vaucouleurs, who has tried to show that the observed pattern can be reconciled also with "trailing" arms. The principal features of the pattern are the "main ring," apparently of heavy H II regions, and the way in which the spiral arms appear to diverge from this ring. An ultra-violet picture of longer exposure taken in best seeing conditions would be very valuable.

The result of an analysis which is based on the geometry and intensity of the dark lanes, like that underlying the criteria formulated by de Vaucouleurs, is sometimes at variance with the results based on the observed intensity and color of the luminous spiral arms. For instance, in the case of NGC 4216 the interpretation given by A. Elvius of the distribution of intensity and color indicates a direction of tilt which will make the arms "leading," whereas criteria applied by de Vaucouleurs to the appearance of the dark lanes give the opposite tilt and "trailing" arms. It may perhaps be pointed out that the structure of this nebula when "rectified" in its own plane shows a strong asymmetry and that the main dark lane is in reality an extremely complex formation, probably also extending very widely in the radial direction in the plane.

As to NGC 7331, de Vaucouleurs claims to have found a conclusive criterion of tilt in the fact that the one-sided heavy absorption region in the plane of the nebula appears to cut over a faint apparent "bulge" which can be traced fairly widely along the minor axis in both directions. This apparent "bulge" was interpreted by B. Lindblad and A. Elvius as essentially a phenomenon in the spiral structure. This can clearly be argued because the main part of the light of the "bulge" on the photographs appears to come from luminous patches of spiral arms which are quite strong especially in light of short wave lengths and are mixed with dark matter up to a very small nucleus. These conditions are in marked contrast to the composition of the "bulge" of population II in the earlier spiral types. The spiral structure is very complex, which is best shown when trying to reconstruct it in its own plane. The outer branches of arms appear to assemble into a very wide, rather elongated, annular formation. Inner, steeply outgoing branches of arms appear to touch, and interfere with, the outer branches in the neighborhood of the minor axis, and it seems possible that the faint extensions of the apparent "bulge" may be due to this circumstance.

The interpretation of the observed quantities defining the distribution of light over the apparent disk of a galaxy involves the relative geometrical arrangements of bright and dark matter, as well as the optical properties of the absorbing dust. The present uncertainty on these points, no doubt, demands that conclusions be drawn with caution. Moreover, the great variations from one system to another, and, in many cases, lack of rotational symmetry, are likely to make more schematic, purely morphologic, criteria uncertain. Further investigations of the distribution of light intensity, color and polarization of the light in the galaxies are very desirable.

From the point of view of dynamical theory it appears that both "leading" and "trailing" arms may be possible. Though "trailing" patterns may well dominate, it is by no means certain that important cases of "leading" patterns will not occur.

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DISCUSSION

Biermann: What are some of the E galaxies that have interstellar gas?

Mayall: About 20 per cent have λ 3727 [O II], and NGC 1052 is one of those in which it is strongest. Many others are listed in the red-shift catalogue.*

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*M. L. Humason, N. U. Mayall, and A. R. Sandage, *A.J.* **61,** 97 (1956).

Harwit: I would like to hear some more discussion of Platt's particles. For them, it might not be necessary to invoke large magnetic fields to account for alignment. The argument against these particles attributed to Spitzer need not be valid. The particles appear to be strongly affected by light pressures, and they might well be pushed out of the galaxy before they had time to grow large.

Lindblad: It is likely that Platt's particles can be aligned by weak magnetic fields. The effect of light pressure should be further investigated.

E. M. Burbidge: Recent McDonald work on the variation of Hα/[N II] λ 6583 intensity ratio with position within many spiral galaxies shows that in spiral-arm regions the ratio is about 3, in agreement with the average ratio found by, for example, Courtés and H. M. Johnson in diffuse Struve-Elvey regions in our Galaxy. In some nuclei the same ratio occurs, and in these nuclei bright high-temperature stars seem to be present. In reddish nuclei, Hα/[N II] may drop as low as one tenth, and, if one does not wish to postulate differences in composition in interstellar gas in different locations within galaxies (H and N have rather close ionization potentials), a possible explanation may be a low radiation temperature and high electron temperature ($\sim$20,000°) so that excitation and ionization are done by electron collision. A low Hα/[N II] ratio can then be achieved.

Münch: In relation to the H II regions in NGC 4594 mentioned by B. Lindblad, I wish to refer to my unsuccessful attempt to detect them in spectral plates taken at Palomar last spring. Exposures up to four hours taken with the nebular spectrograph in the position of the supposed H II regions did not show detectable emission in λ 3726–29 of [O II] or Hα. A direct plate taken with a 1032-D emulsion and a Corning 3484 filter, a combination which isolates a spectral region free of emission lines, shows the same features which have been interpreted as H II regions. On this basis, it is suspected that such features derive their light predominantly from stars, perhaps illuminating the dust.

Zwicky: With regard to particles in interstellar space which are made up of free radicals, I think that the physical chemists working on high-energy propellants probably have the most useful information on *(a)* recombination of free radicals; *(b)* association of free radicals; *(c)* formation of free radicals in solid particles by "radiation damage."

Lindblad: The possibility of free radicals in interstellar space should be further investigated.

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A Summary of our Knowledge of the Neutral Hydrogen in Galaxies

The study of the 21-cm. line emitted by neutral hydrogen has opened a vast new field of galactic and extra-galactic research. It is generally assumed that hydrogen in its neutral atomic state is, by far, the most abundant element in the interstellar matter. The possibility of studying its distribution is of fundamental importance since this distribution is the basis of star formation and, thus, of galactic structure.

The 21-cm. line originates in a hyperfine transition in the ground state of neutral hydrogen. It is characterized by an excitation temperature which, in the normal conditions in interstellar space, is equal to the kinetic temperature of the gas. In inter-galactic space and in the neighbourhood of bright radio sources, the radiation field determines the value of the excitation temperature.

The 21-cm. line is observed by measuring the intensity as a function of frequency in all points of the sky. The antennas which are in use at present have resolving powers between 0°.2 and 3°. The simplest possible model which explains the spatial distribution of the gas is that of a large number of small clouds (2 to 10 pc in diameter) often concentrated in large cloud complexes. This is not necessarily a final model as it seems quite possible that what we call "clouds" are, again, an assembly of much finer substructures, perhaps like the wisps we see in reflection nebulae.

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The intensity received in the wide antenna beam is the sum of the intensities of a large number of clouds, both in the line of sight and perpendicular to it, each with its own density, peculiar radial velocity, kinetic temperature, and internal turbulent velocities. None of these is very well known. Disentangling a line profile is thus beset with many problems. We shall now survey some of these and try to assess the accuracy with which they can be solved.

The temperature of the gas, although not entering directly into the determination of the density if the optical depth is not too high, is so poorly known that density estimates might be wrong by factors of two on this account. It seems fairly well established that the harmonic mean of the temperatures of a large number of low-density clouds is roughly constant in an area of a few kpc around the sun and has a value of about 125°K. Only in cases where cool clouds appear in front of high-intensity regions is it possible to determine the temperature of individual clouds. Clouds with temperatures as low as 50°K have been found. If many of the neutral hydrogen clouds are in the process of cooling off after a collision with other clouds, then, depending on the speed with which they do so due to molecular (H_2) or electron-ion cooling, mean temperatures are calculated to be in the range 200 to 500°K. Since optical depths measured in absorption spectra are of the order of one in a large number of cases, we would expect to measure temperatures in this range at least in some places. However, the maximum temperatures found are about 120°K. It seems, therefore, that either the suggested heating mechanism is not very effective, or that the cooling is much more rapid than hitherto assumed, for example, through dissipation of heat in the small-scale turbulent motions.

So far, the only thing we can say about the temperature of the interstellar gas is that it varies from cloud to cloud and may have a considerable range.

The velocity distribution of the gas is also poorly known. Doppler broadening due to thermal motions only accounts for part of the width of profiles of individual clouds observed either in absorption or in emission at higher galactic latitudes. It cannot be decided from these observations whether the concept of a cloud as a real compact mass with some turbulence, or as a loose collection of thin wisps moving with respect to one another, is the right one.

Apart from partaking in the galactic rotation, every single cloud has its own peculiar motion, and groups of clouds, or even parts of spiral arms, may have velocities deviating from the rotational velocity by amounts of up to 20 km/sec.

The only motion which is clearly visible in the line profiles and which can be easily interpreted is the large-scale galactic rotation. But again, small systematic deviations from circular motion are difficult to discover. Such deviations are quite obvious in the centre and anticentre directions, but the question as to whether they are not just localized phenomena is hard to answer.

The rotational motion of the gas in the part of the Galaxy between the sun and the centre can be found from the 21-cm. line observations themselves. There is some discrepancy between the motions north and south of the line sun-centre. This could be attributed to a systematic outward motion of the local standard of rest of the order of 7 km/sec. As the local standard of rest is mainly determined by early-type stars, this is not impossible in principle.

In any case, the gas motion between 8 kpc and 3 kpc from the centre is known to within some 10 km/sec. In the central region, large deviations from circular motion occur. These have tentatively been explained as expansion; expansional velocities of 50 km/sec and higher are reached. Within 600 pc from the centre, the observations seem to indicate the presence of a small, rapidly rotating gas disk. This nuclear region will be discussed in a later paper.

The velocity law for regions further away from the centre than the sun is determined by optical means, or deduced from models of the mass distribution in the Galaxy. It is less well known; and its uncertainty leads to an uncertainty of up to 2 kpc in the distances from the centre. In the region inside the sun, such uncertainties are only of the order of 1 kpc. Uncertainties in smaller details of the order of 1 kpc, due to the random motions of groups of clouds or parts of spiral arms, also arise.

A determination of the density distribution of the neutral hydrogen thus involves a number of assumptions. The large-scale density distribution can be deduced almost immediately from the original line profiles; it is in the finer details that the assumptions begin to play a major part. The gross spiral structure seems well established, although it is difficult to connect parts of spiral arms to form continuous structures. The arms are not as tightly wound as in Sa galaxies and not as loose as in Sc's: the impression one gets from the data is that the type of our Galaxy is somewhere between Sb and Sc.

Another thing which is quite clear and does not involve any assumptions is the flatness of the layer of neutral hydrogen. The

deviations from a flat layer are no greater than some 50 pc out to about 8 kpc from the centre. Further out, the layer bends upward in the Northern Hemisphere and downward in the Southern Hemisphere, to distances of the order of 500 pc from the plane. It has been suggested that the reason for this curvature is the influence of the inter-galactic wind. The thickness of the layer is remarkably constant and amounts to about 200 to 250 pc, perhaps somewhat thicker in the outer parts.

The density of hydrogen as a function of the distance to the centre is also fairly well known. It rises from the centre, reaching a maximum value around 6 kpc, and then decreases to very low values beyond 12 kpc.

You may have noticed that I have carefully avoided giving accurate figures. I feel that our present knowledge enables us to give the rough picture of our Galaxy described above, but that in a more detailed description, density estimates might be uncertain by a factor of two and distances by one kpc or so. A concerted effort to tackle the basic uncertainties and to try to come to a more complete and detailed knowledge is most necessary. Such detailed studies are now well under way.

The correlation between gas and dust has been definitely established, and I have the feeling that in regions where such correlation is as yet doubtful, perhaps the observations, optical or radio, or their interpretation, are in error rather than the Galaxy. The ratio of the densities of gas and dust seems to decrease from low-density regions to high-density regions, a decrease which might be due to the formation of H_2 molecules in the high-density regions.

In the study of young associations, the 21-cm. line research provides an important tool. Expanding motion of the hydrogen has been found in several associations, and the size of the hydrogen regions seems to be systematically larger than that of the associations. It remains very difficult to separate hydrogen belonging to an association from foreground and background gas. For the study of more distant associations higher resolving power is clearly necessary.

The distribution of the gas close to the sun is studied by observing line profiles at high galactic latitudes. Gas at the galactic poles is moving in towards the plane, whilst there seems to be a connection between the intensity distribution and the position of Gould's Belt.

Quite unexpectedly, in a few recent surveys, hydrogen has been found in fairly concentrated regions high above the galactic plane. It is suggested that such a concentration, observed south of the double cluster in Perseus, consists of gas which was blown away by the expanding association. The outermost arm of the Galaxy has a very asymmetric cross-section perpendicular to the plane; gas which clearly belongs to it is found more than 3 kpc above the plane, in the same direction as the tilt of the gas layer in the plane in that region.

The question of the connection between the gas in the plane and possible neutral hydrogen in the galactic halo is as yet unsolved. One would expect some sort of a circulation pattern, since gas can quite clearly both evaporate from the plane and condense in it. Recent observations of the region between the sun and the centre indicate that at least in these regions the transition between halo and disk occurs rather smoothly and that the spiral arms are only condensations in the central 100 pc of the disk. Whether the halo contains a smoothly distributed ionized or neutral gas, or neutral gas concentrated in clouds, is unknown. One can only say that dense extended clouds would have been observed long ago, and that a cloud must have a low internal velocity dispension not to evaporate immediately once it gets into the halo.

The general picture of the gas content of our Galaxy arising from the present 21-cm. line data, is, as far as the large-scale features are concerned, rather clear: the galaxy type is between Sb and Sc, the thickness of the gas layer is well known, the layer is distorted at the edges, and the density distribution is rather irregular. Rotational motions on a large scale are fairly well known, but deviations from circular motion, whether systematic or random, are hardly known at all. Therefore, detailed distance estimates might be in error by one kpc or so. Absolute values of the density are still uncertain, but the integrated hydrogen mass must be of the order of 0.03 of the total mass of the Galaxy.

The nearest galaxies have now all been studied for 21-cm. line radiation. Of these, the Magellanic Clouds, the Andromeda nebula, M 33, and M 101 are the only ones for which more than just the total amount of hydrogen and the velocity spread could be found. The limited resolving power of the instruments used so far, however, has made it impossible to study detailed structure. The radial velocities through these nebulae and the density as a function of distance to the centre have been deduced from the observations, and a very rough estimate of the density distribution has been made. From the measured radial velocity, given the tilt of the nebula to the line of sight, the rotational velocities could be determined, and from these it was possible to make an estimate of the total mass. In the case of the Magellanic Clouds there is some doubt about this tilt, so that mass estimates are still uncertain. For the more distant nebulae, the rotational velocities can be estimated from the velocity spread of the line profile; this gives a very uncertain value of the total mass. The hydrogen mass, of course, is much better known, since it follows from an integration of the line profiles. Only in spirals and irregulars has neutral hydrogen been found; it has escaped detection in ellipticals so far.

From the data on some ten nebulae available at present we find that the ratio of hydrogen mass to total mass varies from somewhere between 0.1 and 0.3 for irregulars, through 0.06 to 0.01 for Sc and Sb nebulae, to a value smaller than 0.04 for the one elliptical observed. The Galaxy fits very well into this sequence.

It is clear that before long the list of observed and observable extra-galactic nebulae will be considerably extended due to the use of more sensitive receivers and narrow-beam antennas.

I hope I have made it clear how much we already know, and also, how much there is yet to be done to obtain a detailed picture of the relation between galactic structure and neutral hydrogen gas. There is a wealth of information waiting to be obtained.

Let me finish by mentioning briefly a few of the problems which seem to be outstanding at present: the structure, density, and temperature of individual hydrogen clouds, to be studied by observing both emission and absorption profiles; the motion of the gas in the galactic plane, in particular as regards the deviation from circular motion – a very extensive program indeed, incorporating distance comparison between hydrogen clouds and optical objects; the connection between the gas in the disk and that in the halo, and the study of the circulation pattern of this gas – probably intimately connected with this is a detailed study of the gas in the neighbourhood of the galactic centre; further study of associations with narrow-beam, and narrow-band, instruments; and last, but not least, a study of extra-galactic nebulae with high-resolution antennas and high-sensitivity instruments.

DISCUSSION

Gold: The "general assumption" that most gas is in atomic form is not likely to be correct. The ratio of molecular to atomic hydrogen is given by the balance between association on dust and photodissociation. Estimates of the rate of photodissociation have gone down recently (work of R. Gould) and the best estimates now look as if a large fraction is in molecular form. The survey at 21-cm. may refer merely to the loci of recent dissociation, i.e., the vicinity of blue stars. It is important to keep this in mind in the discussion of dynamics of gas, positions, and all the physical characteristics like the temperature, rate of grain growth, and so on.

Münch: The hypothetical existence of interstellar H_2 molecules introduces serious difficulties in the thermal balance of the interstellar H I medium, because it is such an effective radiator in the $J = 2 \longrightarrow J = 0$ quadrupole transition of parahydrogen. Without taking into account the cooling effect of H_2, we hardly can maintain a temperature of the order of 100°K, because of the increased values for the collisional cross-section for the excitation of the low metastable levels of C and Si found recently by Seaton. This is so provided that the energy input to the medium takes place only through photoionization. The possibility of heating by intercloud collisions is, in my opinion, very questionable, as it would lead to a rapid dissipation of all interstellar structures. If we are willing to increase the energy input expected from collisions with low energy cosmic rays, then we might have H_2. But the cosmic-ray physicists, by increasing the energy density of the cosmic radiation from the value accepted today, would have to face the difficult problem of the origin of such energy.

Gold: We cannot contemplate that so little H_2 exists that the present picture of the thermal balance would be upset. I suppose we see H_I only in regions where dissociation and heating by blue stars has taken place recently. The molecular rich stuff elsewhere is probably very cold.

Westerhout: The presence of a large amount of molecular hydrogen would cause the gas to cool off very rapidly. Thus, unless we had a very efficient heating mechanism, we could not explain the high temperatures of 125°K observed. Such a mechanism is, however, not a priori impossible, but in discussing H_2 the temperature question should be considered carefully.

Zwicky: To my knowledge great relative abundance of H_2 molecules in interstellar and inter-galactic space may be expected, because of the following considerations: (*a*) comparison (or prediction) of abundance of H_2 with (or from) abundance of CH, CN, NH; (*b*) absolute equilibrium calculations (thermodynamics); (*c*) kinetic considerations of recombination possibilities. I agree with Dr. Gold's outlook rather than with Münch's.

Pishmish: You made the assertion that gas was falling into the galactic plane. At which distance from the sun is this taking place? Is it in the region of the galactic nucleus or near the sun?

Westerhout: This is gas close to the galactic poles, not in the direction of the nucleus. Therefore, it must be near $R = 8$ kpc and, thus be local. Gas falling into the nucleus has not (yet?) been observed.

De Vaucouleurs: How do you know that the negative velocities at high galactic latitudes refer to nearby hydrogen and not to the halo?

Westerhout: The velocities are small (10 km/sec or so); in halo-gas I would expect much higher velocities.

Münch: By means of the interstellar absorption optical lines I have detected clouds up to 1 kpc from the plane. On what basis, then, is it believed that the gas detected in 21-cm., to which Westerhout referred, is within 200 pc? I may add that the observation of low velocities does not provide evidence in this respect, because the distribution of velocity components in the z-direction should be independent of height above the plane.

Westerhout: You have detected absorption lines in stars up to 1 kpc above the plane. But could not the clouds be anywhere between us and the star? The low velocities are perhaps not a good argument, but the large extent is: almost all the gas with $b > 40°$ has negative radial velocities. The distance of the gas cannot be derived from the data, but I believe the gas is local and belongs to the plane. The figure of 200 pc is only an order-of-magnitude estimate.

Pawsey: In addition to the reason given by Westerhout (the low velocities), the distribution of intensity with galactic latitudes indicates a broad stratification parallel to the galactic plane which we assume to be the continuation of the roughly 200 pc thick disk known at greater distances.

Roberts: An extensive program of hydrogen line studies of extra-galactic nebulae is being conducted with a 21-cm. maser located at the focus of the Harvard 60-foot radio telescope. Over two dozen galaxies have been observed, the majority by Mr. E. Epstein. Most of these galaxies are smaller than the beam width of the telescope. About three-fourths of the galaxies studied have a measurable signal. The radiation from these galaxies has been detected as (*a*) H I only, (*b*) continuum only, and (*c*) a combination of H I and continuum.

Since this is the first time a 21-cm. maser has been used for such work, as many check procedures as possible are being employed in the present phase of the program. These checks have involved (1) repeating observations separated by as long a time period as possible; (2) independent observations of the same object by two observers; (3) different observing techniques, i.e., drift curves and declination scans; (4) comparison with observations made at other observatories of objects in common. In all instances, these checks have been very satisfactory.

Some of the results indicate that NGC 253 and 2903 are continuum sources only, while systems such as NGC 628, 2403 and IC 10 show H I radiation with no detectable continuum. In the latter cases, velocity profiles have been obtained and, therefore, a measure of the total amount of neutral atomic hydrogen.

Zwicky: A triangular patch on the outskirts of IC 10 shows thousands of remarkably blue stars, indicating remarkably little absorption (or reddening) in spite of the location of IC 10 in the Milky Way plane. This agrees with Roberts' finding that the center found from 21-cm. observations checks with the optical center.

Bok: Has Dr. Roberts any evidence with regard to the ratio of total mass of stars to neutral atomic hydrogen for the galaxies studied at Harvard?

Roberts: If we assume that the mass to luminosity ratio for Sc galaxies is 5, then a combination of the optical magnitude and the 21-cm. data yields the following: the three Sc galaxies NGC 628, 2403, and 4244 have about 10 per cent of their total mass in the form of neutral atomic hydrogen. This calculation assumes that the opacity is small.

Tifft: NGC 2903 has a very peculiar *blue* nucleus setting it apart from normal Sc systems. NGC 3351 (SBb) also has a peculiar blue nucleus. In the outer parts the galaxies have normal colors. In NGC 2903, at least, the blue center is due to continuous radiation, not an emission-line effect.

Roberts: It would be interesting to get the color variation in NGC 253 to see if it is similar to NGC 2903. Both galaxies are continuum radio sources which are morphologically similar.

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The Radio Emission from Normal Galaxies

INTRODUCTION

An attempt will be made in this paper to summarize briefly some of the principal facts about the radio emission from normal galaxies. In the present context, a "normal" galaxy means a typical or average galaxy of the population which is observed optically; roughly speaking, the data on *normal* galaxies has been obtained from radio surveys of galaxies which are bright and well-known optically, while the data on *radio* galaxies or *abnormal* galaxies have been obtained by optical surveys of bright and well-known radio sources.

GALAXIES OF TYPE Sb AND Sc

Observations of normal spirals have been published by several authors [1,2,3] and their results are in reasonable agreement. The largest set of homogenous observations is that published by R. Hanbury Brown and C. Hazard [1] and their results are as follows:

Twenty bright spirals of type Sb and Sc were surveyed at 158 Mc/s and their radio magnitude m_r was measured. The radio magnitude was defined by the equation:

$$m_r = -53.45 - 2.5 \log S_{158} \tag{1}$$

where S_{158} $\mathrm{wm}^{-2}(\mathrm{c/s})^{-1}$ is the flux density at 158 Mc/s. The measured values of m_r were then combined with the photographic magnitudes m_{pg} to give a measure of the ratio of the total radio emission to the total light. This measure was called the *radio index R* where,

$$R = m_r - m_{pg} \tag{2}$$

The results of this survey show that the mean value of R for type Sb is +1.4, and for type Sc is +1.2; the associated root-mean-square deviations are +0.8 and +0.7, respectively. The probable error in measurement of the mean values of R was about ±0.1; therefore, the observed deviations represent a true dispersion in the properties of galaxies.

The results also indicated that the radio index of a galaxy depends upon its inclination to the line of sight and on its galactic latitude. The magnitude of both these effects can be explained satisfactorily in terms of the absorption of light in the observed galaxy and in our Galaxy.

In addition to the measurement of radio magnitude, detailed surveys have been made of M 31 at 400 Mc/s [4,5] and at 237 Mc/s and 158 Mc/s [6]. The isophotes can be interpreted in terms of two distributions: a *disc* distribution which is roughly coextensive with the visible body of the galaxy, and an extended *halo* which is considerably larger; the disc component contributes about 10 per cent of the total radiation at 158 Mc/s and the halo about 90 per cent. The halo appears to have an elliptical outline with an apparent diameter between 1 per cent contour lines of 10° × 7°. If we assume that the halo is an oblate spheroid whose principal axes coincide with those of the visible galaxy, then the true axial ratio is 0.6 and the diameter between contours of 1 per cent emissivity is about 120 × 70 pc. The emissivity in the central regions of the halo is of the order of $10^7 \mathrm{w(c/s)^{-1}ster^{-1}pc^{-3}}$ at 158 Mc/s.

The observation of an extended halo around M 31 raises the question as to whether this is a general property of spirals of the same type. The apparent angular sizes of NGC 598, 3031, 4736, and 5194 have been measured at 158 Mc/s [7] and these results, together with those obtained by P. R. R. Leslie at 178 Mc/s [2], support the suggestion that Sb and Sc galaxies also have extended halos; furthermore, there is evidence for a halo around our Galaxy [8]. We may, therefore, conclude tentatively that normal Sb and Sc galaxies have extended halos which, at frequencies of the order of 100 Mc/s, are responsible for the majority of their radio emission.

EARLY-TYPE GALAXIES

Very little is known about the radio emission from a normal early-type galaxy. A search of the literature shows that 18 bright galaxies of types E, S0, and Sa have been surveyed for radio emission. In only one case [2], NGC 4526, was a radio source associated with one of these galaxies, and it cannot be excluded that this single case represents a chance coincidence.

An analysis of the upper limits set to the radio emission from these galaxies shows that their radio index must be significantly greater than that of Sb and Sc spirals. For example, if we assume that the dispersion in the radio index is similar to that found for Sb and Sc spirals (± $0^{m}.7$), then the limits are consistent with a 50 per cent probability that the average radio index of an early-type galaxy is greater than +2.5. It also follows that the absolute radio magnitude of these galaxies must be significantly less than that of the spirals.

MAGELLANIC IRREGULARS

The results of surveying 7 Magellanic irregulars are shown in Table I. One galaxy, IC 1613, appears to be markedly different from the others; however, an examination of the radio magnitude obtained for this galaxy at different frequencies suggests that the results are affected by confusion between sources and that the association of IC 1613 with a relatively intense radio source is questionable.

If we exclude IC 1613, then the data in Table I show that the average radio index of Magellanic irregulars is significantly greater than that for Sb and Sc spirals. Tentatively, we may ascribe to it a value of +3.

TABLE I

Observations of Single Magellanic Irregular Galaxies

GALAXY	m_{pg}	m_r (158 Mc/s)	R (158 Mc/s)
LMC	0.6	3.4 (M)*	+2.8
SMC	2.5	5.5 (M)	+3.0
NGC 55	7.8	10.9 (M)	+3.1
NGC 4449	9.9	> 10.6 (B.H)†	>+0.7
NGC 4214	10.1	> 11.0 (B.H)	>+0.9
NGC 6822	9.2	> 10.7 (B.H)	>+1.5
IC 1613	8.9	9.1 (B.H)	+0.2

*(M): Mills [3] at 85 Mc/s.

†(B.H): Hanbury Brown and Hazard [7] at 158 Mc/s.

DISCUSSION

Figure 1 summarizes the results obtained on the radio index of galaxies. It shows that the maximum ratio of radio emission to light is found in spirals of types Sb and Sc. This conclusion was first put forward by Mills [9].

The results have also been examined for correlations between the absolute radio magnitude and the absolute photographic magnitude, and also between the radio index and the mass, linear size,

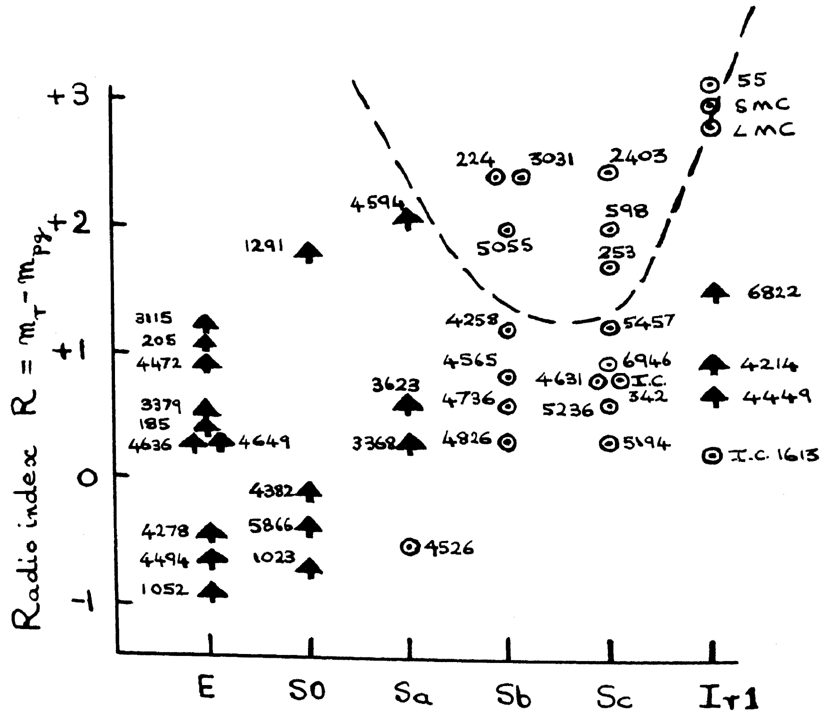

Figure 1. The variation of radio index with type of galaxy at metre wave lengths. The arrows represent lower limits to the radio index; the circles represent measurements. The NGC number of each galaxy is given on the diagram. The broken line is intended to suggest how the radio index may vary with type of galaxy.

and colour index. Unfortunately, the data are too few to establish any correlations satisfactorily, and more observations are needed.

An interesting conclusion from the present scanty evidence is that Magellanic irregulars do not have extensive halos. The absence of a halo around an irregular is suggested by the radio isophotes of the Magellanic Clouds [3], but their large angular size and proximity to the Galaxy makes this conclusion uncertain. However, if we assume that NGC 55 and the other four irregulars in

Table I are similar to the Magellanic Clouds, then all the measurements taken together show that this type of galaxy cannot have an extensive and powerful halo similar to that found in types Sb and Sc. Any halo which may exist in these irregulars must emit less than the main body of the visible galaxy.

Another interesting point is that the radio index, that is to say, the ratio of radio emission to light, of the Magellanic Clouds is similar to that found for the disc component of M 31. This result suggests that the same mechanism for producing radio emission operates in the bodies of spirals and irregulars, and that spirals may be associated with large-scale magnetic fields, which are, in turn, responsible for confining radiating particles in an extended halo.

As far as early-type galaxies are concerned, all we can say is that they are relatively weak radio emitters in comparison with Sb and Sc spirals, but we cannot say if they are weaker than Magellanic irregulars. Thus, while the present data exclude the possibility that they have extended halos of comparable emissivity with those found for type Sb and Sc, they do not exclude the possibility that the visible body of an early-type galaxy radiates with the same radio index as that found for the visible regions of Sb and Sc spirals and also Magellanic irregulars.

Finally, one cannot draw the conclusion that radio emission from a normal galaxy necessarily depends upon the abundant existence of population I. The failure to detect normal early-type galaxies, and also some large globular clusters [9], suggests that this may be so; nevertheless, the limits of observation of radio emission from early-type galaxies must be significantly lowered before any conclusion can be reached about this particular question.

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DISCUSSION

Baldwin: It appears on the present evidence that it is not clear that elliptical galaxies are poor radio emitters relative to the Sb and Sc types. The lower limits to $(m_v - m_{pg})$ for the elliptical galaxies quoted are all less than +1.4, which is the mean of the values for the spirals. Would you agree with this?

Palmer: The limits of the indices for early-type galaxies are about equal to the mean indices for Sb and Sc. So, if the dispersion is similar, the mean value of R for early-type galaxies is nearly 1^m greater.

King: The dispersion of radio indices for Sb and Sc spirals does not necessarily indicate a true dispersion. The optical magnitudes often have uncertainties of this size, especially when one considers that at least two of the galaxies included are obscured ones in low latitude.

Palmer: The dispersion amongst the indices for Sb and Sc galaxies may be affected as you say, but the mean values of the index for each type show a significant variation with type.

Seeger: Radio-telescope resolution is not yet good enough to derive statistics of external galaxies of total radio intensity to 10 per cent. Halo-to-disk separations are similarly crude. To develop good statistics we need much better resolution. Witness the change in the halo picture of M 31 derived by 2° and 3/4° beams at Leiden and Manchester.

Palmer: All radio astronomers are in favor of increased resolution, provided sensitivity can be maintained.

Kerr: The paper pointed out that spirals have halos and irregulars do not when observed in the continuum. On the other hand, for neutral hydrogen, the Magellanic Clouds do show a halo, whereas the Galaxy does not.

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The Measurement of Rotation in Spiral, Irregular and SO Galaxies

I. INTRODUCTION

Rotations in galaxies were first detected by V. M. Slipher [1] from the inclination of the spectral lines. A rotation was first measured by Pease in the case of M 31 [2]; Öpik [3] saw that this determination could be used to obtain an estimate of the mass of M 31 interior to the point to which the measures extended (which was only 2½′ from the nucleus). It was not until H. W. Babcock's work twenty years later that a rotation curve for the major part of a galaxy – M 31 – was obtained from painstaking observations and used in a determination of the mass distribution of M 31 [4].

Since the main purpose in obtaining the rotational velocity of a galaxy as a function of distance from its center is to determine from this the mass distribution and total mass of the galaxy, this paper is closely connected with that by Prendergast on mass determinations (see P. 126). In this paper, therefore, I shall not discuss the methods of analyzing rotation curves to obtain masses, but shall consider the methods of observation used in obtaining rotation curves and shall discuss some intrinsic problems involved in

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the interpretation of the measures. The problem of the rotation of the Galaxy will not be discussed, as this is a special case, and the purpose here is to discuss external galaxies. Finally, all available data will be collected.

II. METHODS OF OBSERVATION

Three methods of measuring velocity from the wave lengths of spectral features are now available. First, emission lines in the optical spectral region can be measured – these give velocities of ionized gas in the galaxy. Second, the 21-cm. line can be measured for the nearer galaxies and, as radio telescopes with greater resolution become available, it will be possible to extend this to more distant galaxies. These measures give velocities of neutral atomic hydrogen in the galaxy. Third, absorption lines arising in the stars of the galaxy can be measured – these give the velocities of the stellar component. Before discussing these three types of observation further, let us consider the selection of orientation of galaxies whose rotation curves we wish to measure.

In order to obtain the rotational velocity as a function of distance from the center of a galaxy, it is necessary to select, for observation, galaxies whose equatorial planes are inclined to the tangent plane of the sky at angles not less than about 45°. If a galaxy is viewed too nearly face-on, the factor by which the observed velocity has to be corrected for projection becomes large and errors are magnified. However, if a galaxy is viewed edge-on, so that its equatorial plane is perpendicular to the plane of the sky, and if such a galaxy contains much obscuring dust, or if observations are made by means of emission lines arising in the gas of the galaxy where an optical depth of unity is reached for only a small linear penetration into the galaxy, then we shall be observing only the outermost rim of the galaxy – like the rim of a wheel. In this case it is easily seen that the line-of-sight component of velocity, when plotted against distance from the center, will give a straight line and will give no information on the variation of rotational velocity with distance from the center. Such measures can give the total mass of a galaxy, but not its mass distribution.

On the other hand, if a galaxy that is viewed edge-on contains little or no gas or dust, and the velocity is measured by means of absorption lines arising in the stars of which the galaxy is composed, then this restriction does not occur. For the optical depth in stars, even along a line passing through the nucleus of a galaxy, is small. Thus, if one observes velocity along the major axis of an edge-on SO galaxy, for example, one is measuring the integrated velocity along the whole line of sight throughout the galaxy. Because of the impossibility of determining the orientation of the equatorial plane of an elliptical galaxy that is not viewed edge-on, it is desirable in studies of rotation curves of such galaxies to select only the most elongated objects, where one can reasonably assume that the line of sight lies in the equatorial plane.

(a) OPTICAL METHODS OF OBSERVATION

It has always been emphasized by Mayall, one of the pioneers in the study of rotations of galaxies, that one should, wherever possible, make use of emission lines rather than absorption lines – first, because greater accuracy is possible in measuring them on the spectra and, second, because emission lines can be registered on the photographic plate in objects of much lower surface brightness than is the case for absorption lines (see e.g., [5]). However, in the case of SO galaxies, the nuclear bulges of Sb and Sa spirals, and even in the spiral arms of some Sb and Sa galaxies, emission lines may be absent or weak because there is little uncondensed gas. For example, NGC 4826 is an Sb galaxy in which the inner spiral arms have a dusty, knotty structure and the emission lines are quite strong, but the outer arms are smooth and seem to be mainly stellar – at any rate, emission lines have not been detected in the outer parts of this galaxy.

For a nearby galaxy in which great resolution is possible, it is clearly desirable to select H II regions which have been detected from survey work on direct photographs with filter-plate combinations selecting the wave lengths around Hα. This is the best way to pick out such H II regions, and then individual velocities of these regions can be measured. M 31 and M 33 are the two obvious cases for which this method is indicated, and it has been applied to both [4,5,6]. A further great advantage is that H II regions in our Galaxy are known to lie close to the equatorial plane, so that it can be hoped that in other spirals this will also be the case and the measured velocities and positions of the regions will have to be corrected only for projection in the equatorial plane, not in the z-direction.

The emission lines that usually appear are, in decreasing order of strengths of the various elements, Hα and sometimes Hβ; [O II] $\lambda\lambda$ 3729, 3726; [N II] $\lambda\lambda$ 6583, 6548; [S II] $\lambda\lambda$ 6731, 6717; and, less often, [O III] $\lambda\lambda$ 5007, 4959. Those who work in the photographic region of the spectrum make use mainly of [O II]; this has the slight disadvantage that, unless moderate dispersion can be used, the two lines are blended; further, the relative intensities of the pair vary as a function of electron density in the gas. If one can work in the red spectral range, Hα and [N II] can be used and it has been the general experience of both Page and ourselves, working at McDonald, that Hα is a more widespread and generally stronger feature in the spectra of spiral-arm regions of external galaxies than is [O II].

Ideally, a network of velocities across the whole galaxy should be measured. If this were done, an attempt could be made to distinguish two separate components of velocity that might be present: (1) Rotational (circular) velocities; (2) Radial motions in the equatorial plane. In general, since workers have been primarily interested in obtaining the rotation, emission patches that lie mainly near the major axis have been chosen, with few patches near the minor axis. If radial motions of any size are present, unless observations are made exactly on the major axis, the rotational velocity deduced from them will be in error. In the analysis of both M 31 and M 33, it has been assumed that radial motions are negligible; this is probably justified in the outer parts although their existence in the inner region of M 31 has been shown by Münch [7]. In other spiral galaxies, as we shall show later, the neglect of radial motions may not be justified.

The two galaxies which are near enough for very detailed investigation to be made are the Large Magellanic Cloud and M 31. Somewhat further away, but still close enough for separate emission patches to be studied, are M 33, M 81, and M 82, and a few bright southern galaxies such as NGC 55 and NGC 253. For galaxies more distant than these, the best technique is to use a spectrograph with a long slit which can be set along the major axis (and also in one or two other position angles in order to provide a check on the orientation of the major axis and on the possible existence of radial motions). This technique was developed at Lick Observatory, where the prime focus spectrograph on the Crossley telescope has a slit 6ʹ.2 long. This was used by Babcock [4] for the inner parts of M 31, and by Mayall for the study of rotation periods of many galaxies by measuring the angle of inclination of the spectral lines [8].

The same technique has been used at McDonald Observatory (e.g., by Burbidge and Burbidge [9]), where the B-spectrograph, designed at Yerkes by Babcock and first used for work on galaxies by Page, has been used at the prime focus of the 82-inch telescope. This spectrograph has a grating blazed in the Hα region, where the dispersion is 330 A/mm. Although the blaze is in the red, the sensitivity of the 103a-F film used is such that the spectra may extend right down to λ3900, or even to λ3727, according to the length of the exposure. A second grating, blazed at λ 4000 A, which is much more efficient for measuring the H and K absorption lines of Ca II, has just been put into operation as a supplement to the Hα grating.

The full length of the slit in the B-spectrograph is 290″; this means that, for galaxies at distances around 5×10^6 pc, the whole major axis can usually be covered by two spectra taken with the nucleus set at opposite ends of the slit. Thus, rotation curves can be obtained for suitably oriented galaxies that have plenty of H II regions throughout them with a relatively short amount of observing time compared with the one or two whole observing seasons necessary to collect information on individual H II regions. It has, therefore, been possible to collect data on rotations of a fair number of galaxies. The spectra have to be measured in a two-coordinate measuring machine, so that the location in the galaxy of each setting for velocity is known, and it is preferable to make settings with the intersection of two cross wires set at 45° to the horizontal so that the appearance of the spectrum line immediately above and below the place where the setting is being made does not influence the setting.

This method does place an observational selection upon the masses that become available. Irregular and Sc galaxies, with their more widespread and strongly emitting H II regions, are much more easy to observe than Sb galaxies which, in turn, are easier than Sa spirals. In the latter, the meager data from emission lines have to be supplemented by data from absorption lines which are much harder to measure. Further, one obviously cannot obtain such detailed rotation curves as could in principle be obtained for the nearby galaxies, just because of the lack of scale. This becomes particularly important in and near the nucleus, as will be seen in the next section where M 31 is discussed in detail.

In the measurement of the rotation of S0 galaxies, where one has to use absorption lines, there is an additional problem involving the very steep gradient in intensity as one goes away from the center of the galaxy. The full force of the luminosity gradient in the galaxy comes into play when one is observing absorption lines, while, in spirals, the intensity of emission lines is often weaker in the nuclear region than in the outer parts and, in any case, the emission-line intensity does not usually show a very large range across the galaxy. In measuring S0 galaxies, there will be a danger, if a long slit is used and is set along the major axis, of light from the nuclear region diffusing out and becoming superimposed on light from an adjacent region further from the nucleus. For this reason, observing has to be restricted to the best conditions, i.e., the seeing has to be very good; in addition, it is necessary often to use a slit set at right angles to the major axis and crossing it at various points rather than the technique of a long slit set along the major axis. Consequently, in the table of collected results given at the end of this paper, it is not surprising that only two S0 galaxies are listed – NGC 3115 and NGC 4111 (from the work of Humason, Minkowski, and Oort).

A final point worth emphasizing is that, if it is hoped to obtain a reliable measure of the total mass of a galaxy, observations must be made right out to a distance from the center that is exterior to the bulk of the mass. This appears to be easier to do with 21-cm. observations than with optical ones [10,11]. In M 31, the furthest-out emission patches for which velocities were obtained from optical spectra lie beyond the main, easily visible body of the galaxy; the radio measures extend still further. Whether the measures for more distant galaxies reach out sufficiently far or not can be determined only after the rotation curve has been plotted; if it extends well past the turnover point, where maximum velocity relative to the center is reached, then probably most of the mass lies interior to this. A fragmentary rotation curve is of little value unless it can be established that the turnover point has been passed. However, one measure of velocity in a distant H II region on or near the major axis is of more value in determining the total mass of a galaxy than a wealth of information in the interior regions. In general, it is easier in Sc than in Sb galaxies to measure past the turnover point, because of the generally greater extent and emitting power of their H II regions.

(b) RADIO-ASTRONOMICAL METHODS OF OBSERVATION

In a galaxy of large angular size, the rotation curve can be determined with good precision by means of the 21-cm. hydrogen line. This has been done for M 31 [10], M 33, and M 101 [11] with the Dwingeloo radio telescope. The velocity corresponding to a given setting of the telescope can be measured with high accuracy by recording the intensity at different frequencies and building up the profile; the center of such a profile is well determined. The telescope can be set accurately to about 0°02 in general; the main difficulty is that the beam width of 0°6 is large enough that a considerable area of the galaxy is integrated into each setting, so that an integral equation must be solved to yield velocities at given points along the major axis. Solution of such an equation demands the assumption that there is circular symmetry in each half of the galaxy, and that the motions are purely circular. These assumptions are likely to be valid in the cases so far considered.

In the next six sections, we shall discuss data obtained by the methods described above. This material is all contained in the literature except for some recent observations by the Yerkes-McDonald group which are not yet published. In section III, M 31 will be considered separately because this galaxy has the greatest amount of information available from observations by various techniques In section IV, data for irregulars and spirals of little structure are discussed, while in section V are collected data for a number of regular Sc and Sb galaxies. Next, a group of Sb spirals and one Sa spiral, the rotation curves of which pose some problems, are considered, after which section VII is devoted to the barred spirals and their particular problems. Finally, two groups of peculiar galaxies are discussed.

III. M 31

In Figure 1, we reproduce the rotation curve published in [10], in which Mayall's measures from optical spectra of H II regions are superimposed on the 21-cm. curve. The accuracy of the points forming the latter is well within 10 km/sec. Probable errors of velocities from the optical spectra are ±26 km/sec [5], so that errors of 50-60 km/sec could easily occur; thus, the discrepancy between optical and radio measures on the NE side of the galaxy, though uncomfortably large, could be the result of errors. These observations all refer to the gas, ionized in one case and neutral in the other.

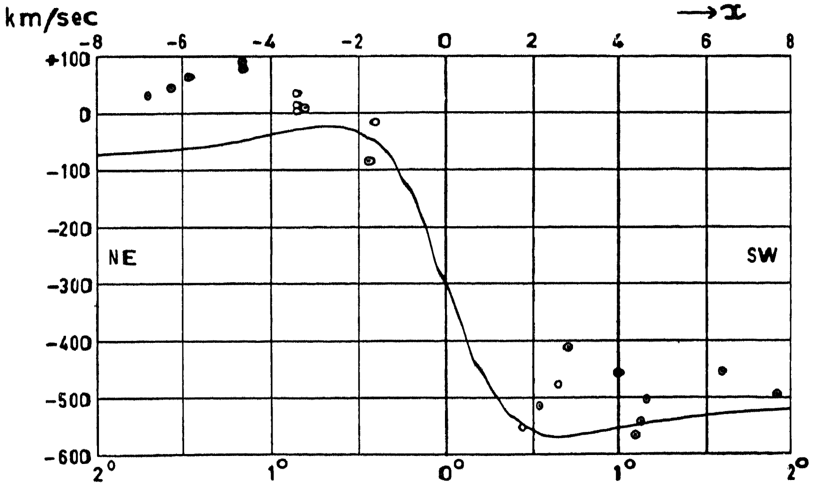

Figure 1. 21-cm. rotation curve of M 31 with optical measures of H II regions superposed (Courtesy *B.A.N.* [10]).

In Figure 2, we reproduce the rotation curve for the innermost few parsecs from observations with a large scale at the coudé focus of the 120-inch Lick telescope [12]. These observations are from absorption lines and refer to the stars in the nucleus. There is a sharp rise to 87 km/sec at a distance of 2″.2 from the center, just at the "edge" of the bright nucleus that looks stellar with smaller telescopes, followed by a sharp drop.

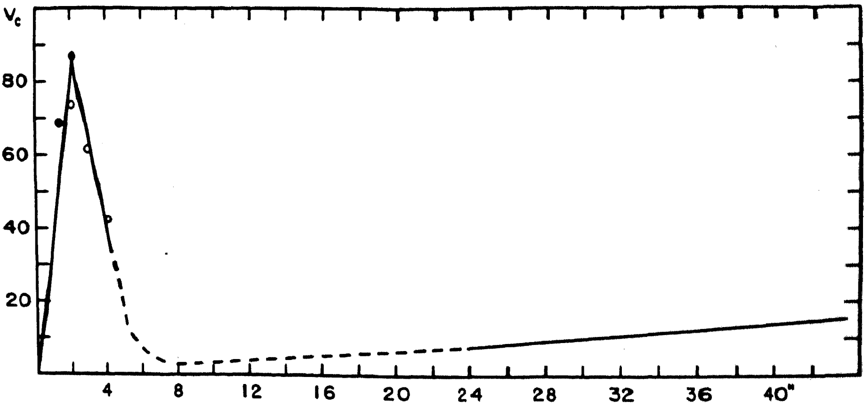

Figure 2. Absorption-line velocity measures in innermost few parsecs of M 31 from 120-inch coudé measures (Courtesy *P.A.S.P.* [12]).

Between these measures, ending 3″ from the center, and the first points giving the curve in Figure 1, lies the region covered by Babcock's observations [4]. From the center to 30′ out, he used the long-slit technique to measure the velocity from the absorption lines produced by stars in the amorphous nuclear region. Observations on both sides of the nucleus, made with both the Crossley telescope and a small telescope on the Crossley mounting for obtaining a more compressed scale, agreed in giving a rise in velocity from zero at the center to 100 km/sec at a distance of 3′ from the center, followed by a drop to near zero at about 10′ from the center, followed by a second increase in which the curve joined fairly smoothly onto that given further out by the H II regions.

That the peak at 2″.2 observed by Lallemand, Duchesne, and Walker should not have appeared in the Crossley observations is easily understood, because the nuclear rotation would only be detected with great scale and very good seeing. A peak at 3′ would presumably not show in the 21-cm. measures because the settings were spaced at 0°.25 intervals, but the lack of agreement between 3′ and 30′ is hard to understand. Here it must be remembered that Babcock's observations in Figure 3 refer to the *stellar* component and the observations in Figure 1 to the *gaseous* component. In our work with the B-spectrograph on Sb galaxies, we have sometimes suspected a discrepancy in the bulge or disk regions between the measures from absorption lines and those from emission lines. The Sb galaxy NGC 5005 [13] is an example. However, the probable errors of our absorption-line measures in regions outside the nucleus are large and we have never believed the discrepancy to be significant.

In the case of M 31, it seems desirable to remeasure the rotation of the stellar component within 30′ of the nucleus with moderate dispersion for a close check with velocities of the gaseous component. Because the stellar motions reflect conditions at a presumably early stage of evolution of the galaxy, and some stellar orbits will have considerable inclinations to the equatorial plane, the average rotation curve need not agree with the gaseous rotation. One may put the problem physically by saying that the mass of the interior region may be supported by a pressure term in addition to centrifugal force.

IV. THE LARGE MAGELLANIC CLOUD, M 82, AND NGC 3556

Because of the size and degree of resolution of the Large Magellanic Cloud, optical observations must be made by measuring individual H II regions. A rotation curve has been derived by de Vaucouleurs [14] by this means; a rotation curve from 21-cm. measures has also been determined [15]. Two major difficulties are encountered here: first, the neutral atomic hydrogen appears to have a rather widespread distribution instead of being confined to the equatorial plane; this is presumably connected with the irregular nature of the object which shows only a rather primitive spiral structure. Second, the object appears to be viewed rather face-on and its orientation to the plane of the sky is hard to determine with any precision. With these uncertainties in mind, the agreement between the two rotation curves is fair, but this is an object in which a network of velocities from optical spectra is required.

M 82 and NGC 3556 are both irregular systems with signs of rudimentary spiral structure; NGC 3556 is classified Sc in the *Hubble Atlas of Galaxies*. Both are viewed nearly edge-on so that one is presumably getting a "rim-of-the-wheel" view of the rotational velocity since both contain a great deal of dust. The rotation curve of M 82 was measured by Mayall [16] and that of NGC 3556 by Burbidge, Burbidge, and Prendergast [17], and both are approximately linear, as would be expected if the galaxies are viewed nearly edge-on. Mayall's curve of M 82 is shown in Figure 3. Such

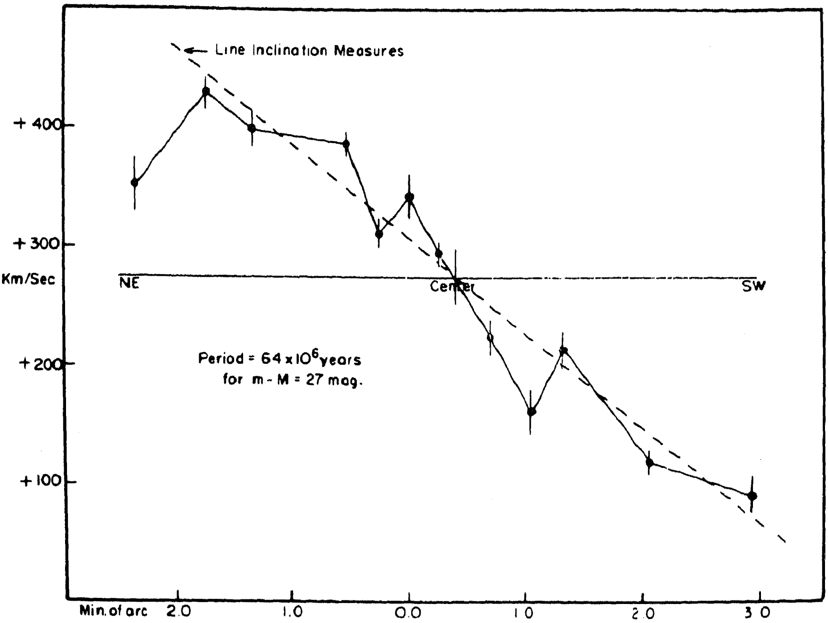

Figure 3. Rotation curve of M 82 from Mayall's measures (Courtesy *Ann. d'Ap.* [16]).

curves give information only on the total masses of the galaxies, if the measures extend far enough. This is probably the case in NGC 3556.

V. REGULAR Sc AND Sb GALAXIES

In this section, we shall discuss only galaxies with a fairly regular spiral structure, whose major axes are easy to determine from direct photographs and which exhibit none of the problems that sometimes arise in the form of possible departures from coplanarity, or steeply inclined spiral arms, the outline of which is far from circular in the equatorial plane. Examples of these will be considered in the next section. When optical spectra are obtained by the long-slit method, with the slit set in more than one position angle, there is no disagreement between the direction of the major axis thus determined and the position angle obtained from direct photographs.

Sc galaxies in this category for which rotation curves have been determined are M 33, M 101, and NGC 157. The first two are of large angular size and have had curves determined by the 21-cm. line [11], while velocities of individual H II regions have also been measured in M 33 [6]. The scatter in the latter is bad, and a program for the measurement of velocities in a large number of H II regions in M 33 is under way by Brandt at Mount Wilson. The biggest departure between the optical and radio measures occurs at the north side of the galaxy (M 33), where the optical measures have a scatter of about 100 km/sec. The H II regions are $0^{\mathrm{m}}.5$ to 1^{m} brighter in M 33 than in M 31 [5]. Surveys of a considerable number of galaxies show that this is a characteristic difference between Sc and Sb galaxies [18].

NGC 157 has been measured by the long-slit technique by Burbidge, Burbidge, and Prendergast [19]. Ionized gas is widespread through the whole galaxy out to a point where the luminosity on direct photographs falls off rather sharply; the rotation curve reaches a distinct turnover point just before this is reached. The rotation curve from measures of the Hα and [N II] emission lines with the slit set along the major axis is shown in Figure 4.

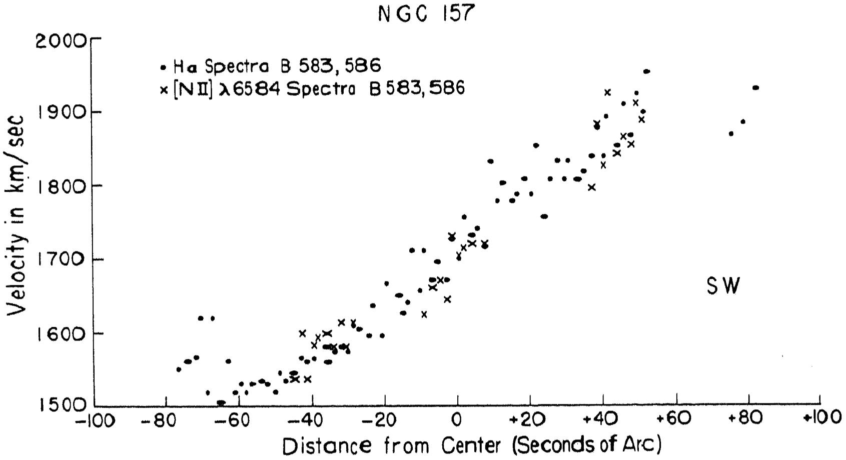

Figure 4. Rotation curve of NGC 157 (Sc) from long-slit McDonald spectra.

NGC 5055 is an Sbc galaxy with a well-determined rotation curve obtained from optical spectra with the long-slit method [20]. Here again, emission regions are widespread through the whole galaxy and give a rotation curve extending past the turnover point. This curve is reproduced in Figure 5.

M 81 is an Sb galaxy in which Münch [21] has obtained a rotation curve from optical measures of individual H II regions. This curve extends very well past the turnover point; indeed, one H II region was measured at about twice the radius of the main luminous body of the galaxy. If random motions of H II regions are small, this should determine the total mass very well.

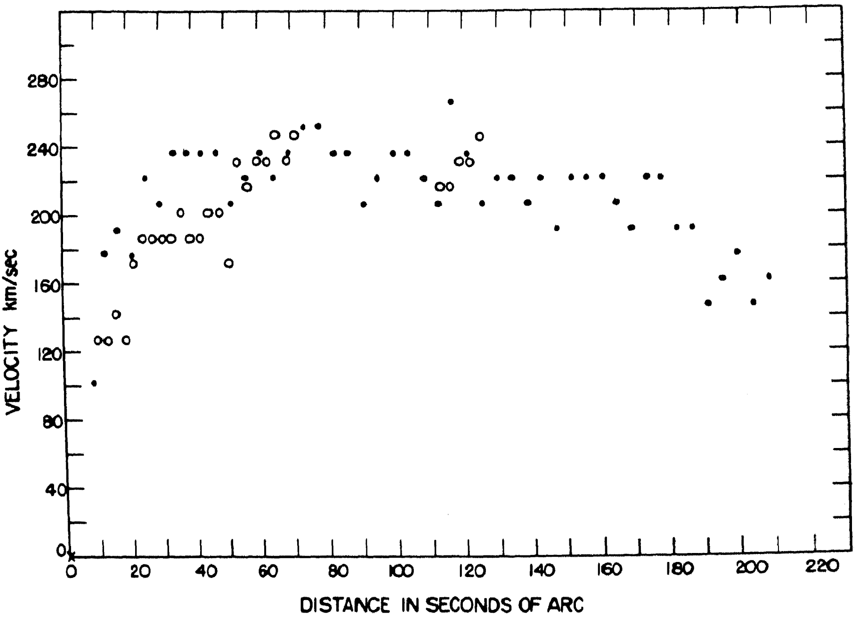

Figure 5. Rotation curve of NGC 5055 (Sbc) from long-slit McDonald spectra.

VI. SPIRALS WHOSE ROTATION CURVES PRESENT SOME PROBLEMS

Here are discussed a group of spirals whose rotation curves have been obtained by the long-slit method with optical spectra by Burbidge, Burbidge, and Prendergast. In all of these cases, the observations have presented some anomalies.

In NGC 2903 [22], the major axis, as defined by the inner bright arms (11°), differs from that defined by the faint outer arms (24°). Spectra set in these two position angles give rotation curves which can only be made to coincide if the true position angle is 33°, i.e., 22° different from that given by the bright inner main body of the galaxy. This discrepancy cannot be explained by the outer arms being steeply inclined, making a considerable angle with a circle in the equatorial plane, because it is in the opposite sense; the departure from circular outline must be greater for the inner bright portion. Further, the rotation curve is not smooth and symmetrical on the two halves; it exhibits some bumps which are probably real because they are measured on more than one spectrum. Possibly the measured velocities here refer to matter lying out of the equatorial plane in this region.

It may be that there is more than one plane of symmetry in this galaxy. NGC 2685 [23] is a fairly obvious case of this, and there may be others. NGC 3504 [24] is a transition between a barred and a normal Sb spiral; a long slit set along the apparent major axis yields a rotation curve that rises very sharply to a maximum of 140 km/sec and then falls off; if the slit is set along the minor axis zero rotation is measured. But the galaxy has outer faint arms which are practically circular in the plane of the sky, suggesting that they do indeed lie approximately in the plane of the sky and not in the equatorial plane of the bright inner part.

NGC 3623 [25] is an Sa spiral in which the direction of the major axis changes quite markedly from a set of inner dust arms, through an outer luminous arm, to the outermost faint arms, suggesting non-coplanarity among these different parts. In this case, the rotation curve, as measured from long-slit optical spectra, is less informative because, as would be expected in an Sa galaxy, emission lines are generally weak since there is probably not much ionized gas. Observations were made only in a range of ±5° in position angle, with settings along the best guess at the major axis and 5° each side of it. The observed rotation curves are in good agreement, with the exception of one discrepant point from the only bright H II region in the entire galaxy, located just off the major axis in the bright arm at the south end of the galaxy. We have concluded that the observed velocity here probably contains a rather large noncircular component.

NGC 5005 is an Sb galaxy fairly similar to NGC 3623. The rotation curve obtained by means of the long-slit technique [13] is not very smooth; it has a break at a position on the major axis where the setting of the spectrograph slit crosses a spiral arm that fans out like a plume from a point on the outer edge of the highly luminous inner part of the galaxy. Possibly the measured velocities here come from matter lying above the equatorial plane; a prominent dust arm lies parallel and just exterior to the bright arm.

NGC 4736 is an Sb galaxy with a large central nuclear region. There are some H II regions in an outer ring and, although the galaxy is viewed rather face-on, we had hoped to obtain a rotation curve from measures with the long-slit technique in these H II knots and in a bright ring which lies in the peripheral region of the central bulge. However, the measures could not be fitted together in any reasonable way, and we have concluded that the geometry of the galaxy is such that in the outer parts there is a tilt with respect to the rotational plane defined by the inner parts. Photographs of NGC 4736 [23] show that the outer ring has a major axis which is oriented at nearly 90° to the major axis of the inner parts.

VII. BARRED SPIRALS

The barred spirals present special problems. First, they have bilateral symmetry instead of approximate circular symmetry which makes it much more difficult to estimate the orientation in space and the position angle of the major axis. Second, in two out of the few barred spirals for which we have any data, noncircular motions are present.

At present, a full rotation curve has been obtained only for NGC 7479, an SBb spiral that has H II regions right along the bar as well as in the spiral arms [27]. The rotation curve is approximately linear in the bar, and falls off sharply outside. Such a pattern of velocities can be explained in terms of a rotating fluid in the potential field of a long prolate ellipsoid of uniform density (the bar); Prendergast [28] has shown that the fall-off in velocity outside the bar is in agreement with the calculated flow pattern.

NGC 1365 is one of a group of barred spirals with fast-rotating nuclei in which there is considerable structure. It has dust lanes emerging from opposite sides of the nucleus; these stretch out on either side to form the bar. This type of structure is characteristic of one class of barred spirals. From a preliminary study of measures of emission lines coming from the ionized gas [29], the nuclear region appeared to be rotating rapidly with a linear relation between velocity and projected distance from the center. Recently, it has been found that this plot of velocity against distance from the center begins to curve at the edge of the nuclear region, and measures of velocity made in five position angles cannot be explained in terms of rotation alone. Some kind of flow in an inward or outward direction, a flow which does not exhibit circular symmetry, must occur.

NGC 1097 is a barred spiral that is rather similar to NGC 1365. Its nuclear region consists of a small ring or spiral of ionized gas and a central amorphous nucleus [30]. This ring is rotating very rapidly [29]. Radial motions have not been found in this case.

VIII. PECULIAR GALAXIES

In this section, we shall discuss some galaxies which are peculiar in various ways. One, NGC 3646, has a rotation curve which shows that the galaxy is not dynamically stable. Two are radio sources: the first is NGC 5128, the strong Centaurus A source; and the second is NGC 1068, which also belongs to the small class of galaxies studied by Seyfert [31] whose nuclei have emission lines of high excitation with large Doppler broadening. Finally, we mention a few peculiar objects discovered by Vorontsov-Velyaminov

[32] although these are too distant for rotation curves to be obtained and only the inclination of the spectral lines can be measured for an estimate of rotational velocity.

(a) NGC 3646

NGC 3646 is a very large, bright galaxy which possesses rather a peculiar structure. It has been studied for rotation by the long-slit technique [33], the velocities being measured from emission lines everywhere except in the nucleus where absorption lines were measured. Figure 6 reproduces a sketch of the galaxy together with a few of the velocities that have been measured. These velocities and other measures along the straight arm at the west side of the galaxy cannot be reconciled with circular motion in a stable rotating galaxy. We have concluded that the pattern of velocities measured in the outer ring suggests that this ring is in a state of rapid dynamical evolution, i.e., the galaxy is not stable in its present form. In this case, there is no reason to believe that the outer structure, which has a diameter of 41 kpc, should lie in the same plane as that defined by the more regular inner structure.

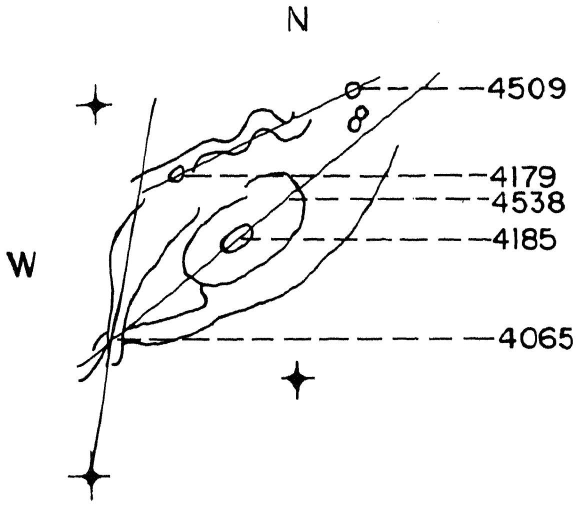

Figure 6. Sketch of peculiar Sc galaxy, NGC 3646, with velocities measured at various points.

(b) NGC 5128

The nearby southern galaxy NGC 5128 has an enormous central stellar component crossed by a wide, dark lane with considerable filamentary structure. Its rotation has been studied by the long-slit method [34]. Ionized gas, in the dark lane, and immediately above and below the dark lane and parallel to it, is rotating about an axis approximately perpendicular to the dark lane. No significant rotation of the stellar component about an axis perpendicular to the axis of rotation of the gaseous component has been detected [35]. Studies of the rotation curve of this object are particularly interesting because it is a strong radio source.

(c) NGC 1068

The rotation curve of the inner regions of this galaxy, studied by the long-slit technique by means of emission lines [36], is, as far as we can determine it, linear in form, indicating little concentration of mass towards the center. The great widths of the emission lines in the nucleus, if interpreted as random Doppler broadening, indicate velocities in excess of the escape velocity. More detailed study of the rotation curve with greater scale than that available at the prime focus of the 82-inch McDonald telescope should be made. Rotation has also been measured in NGC 2782; the rotation is large in the inner region of diameter about 25″ [37].

(d) VORONTSOV-VELYAMINOV OBJECTS

A number of the peculiar galaxies discovered by Vorontsov-Velyaminov have rapid rotations shown by the inclinations of the emission lines in their spectra. The double galaxy NGC 4676 [38], which appears to belong to the Coma Cluster, has fast rotation of both its components amounting to a difference of 400 km/sec across the diameter of each. NGC 6621-2 (V-V 247 in Vorontsov-Velyaminov's catalogue), V-V 122, and V-V 261 are all rotating very fast as shown by the considerable inclinations of the strong emission lines in the spectra of all three. These objects, from their irregular form and content of ionized gas with the high-temperature exciting stars that must be present, appear to be in an early stage of evolution and we have suggested that they may be newly-formed galaxies. Their fast rotations should lead to the formation of some symmetry about the axis of rotation in a time of the order of a few rotation periods.

IX. CONCLUSION

The discussion in sections III-VIII has substantiated the intuitively obvious fact that, the more symmetrical and regular a galaxy appears to be from direct photographs, the more regular will its

TABLE 1

Galaxies Whose Rotation Curves Have Been Determined

Galaxy	*Type*	*Obs.*	*Reference*
NGC 55	Irr	H II*	[26]
NGC 157	Sc	l.s.†	[19]
M 31	Sb	l.s., abs., H II, 21	[4, 5, 10, 12]
M 33	Sc	H II, 21	[6, 11]
LMC	Irr-S	H II, 21	[14, 15]
NGC 1068	Sbp	l.s.	[36]
NGC 1365	SBb	l.s.	
NGC 2146	Sbp	l.s.	
NGC 2782	Irr	l.s.	[38]
NGC 2903	Sb	l.s.	[22]
M 81	Sb	H II	[21]
M 82	Irr	l.s.	[16]
NGC 3115	S0	abs.	
NGC 3504	SBb	l.s.	[24]
NGC 3556	Irr-Sc	l.s.	[17]
NGC 3623	Sa	l.s.	[25]
NGC 3646	Sc	l.s.	[33]
NGC 4111	S0	abs.	
M 101	Sc	21	[11]
NGC 4736	Sb	l.s.	[23]
NGC 5005	Sb	l.s.	[13]
NGC 5055	Sbc	l.s.	[20]
NGC 5128	Ep	l.s.	[34, 35]
NGC 7479	SBb	l.s.	[27]

*H II refers to measurement of individual H II regions, 21 refers to measurement by 21-cm. radio observations.

†l.s. refers to measurement by long-slit technique.

abs. refers to velocities from absorption lines.

Observations of inclinations of spectral lines which do not give rotation curves although they do give the average rotation period from which estimates of the total mass can be obtained, are not included.

rotation curve be and, consequently, the better-determined will be its mass distribution and total mass. For the collection of data on masses, the most regular objects with favorable orientations are the best to observe. However, even the most regular of galaxies, e.g., M 31, when examined in sufficient detail, show features such as radial motions in the central part and peaks on the rotation curve indicating a discontinuous mass concentration at the center. Clearly, it is in the nuclear regions of galaxies where large-scale observations should be made to search for: (*a*) structure of this sort, (*b*) possible radial motions, and (*c*) possible differences between the rotation curve of the gas and that of the stellar component.

The existence of quite large noncircular motions in barred spirals has been shown, but further observations are needed before a theory of barred spiral structure can be developed.

Finally, rotation curves obtained by the long-slit method with the slit set in several orientations across a galaxy may yield information as to whether some galaxies possess more than one plane of symmetry.‡

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DISCUSSION

Roberts: Have you observed any galaxies which have been described by Dr. Morgan as having "hot spots" in their nuclei? These "hot spots" are presumably giant H II regions located right in the nucleus!

E. M. Burbidge: Yes, we have observed a few and they tend to show rapid rotation across the nucleus. Emission lines are strong in the nuclei, with intensity ratios like H II regions in the arms, so there must be high-luminosity, high-temperature stars there to give the excitation.

Fehrenbach: Mme. Duflot-Augarde observed the rotation of NGC 2782 (Electronic Camera). She obtained for this Seyfert Object a rapid rotation. Its mass was $15 \times 10^{10} M_{\odot}$ and its mass/light ratio, $M/L = 7.6$.

Minkowski: It seems to be typical for highly flattened S0 galaxies, such as NGC 3115, 4111, 4762, that the nucleus shows very high rotation with an abrupt change to slower rotation in the outer parts.

Whitrow: To what extent is it known that the bars in barred spirals rotate as rigid bars?

E. M. Burbidge: Observations have been made in one barred spiral, NGC 7479, which has emission regions along the whole length of the bar and, in this, the rotation curve was linear along the bar indicating rigid-body rotation.

Poveda: I would like to know if there is any correlation between the maximum velocity of rotation and the morphological type of the galaxy considered.

E. M. Burbidge: There is a general tendency for the mass of galaxies to increase on going from Irr to Sc to Sb to Sa types, so, in this sense, the maximum velocity of rotation does increase with this sequence. However, we have too small a sample available at present, and it is too heavily weighted by observational selection towards the Sc types, to study this question in detail yet.

Fricke: I would like to remark that the measured rotational velocities in regions near the center of extra-galactic nebulae will generally not give information on the circular velocities in these regions (because of the prevailing and unknown velocity dispersion), and that, therefore, scarcely any conclusions can be drawn from it for the distribution of mass. I am wondering whether you would agree to this.

E. M. Burbidge: We have not measured the rotations of any elliptical galaxies; in our work we mainly use emission lines where the problem does not arise. I agree that in the nuclear regions of galaxies, if absorption lines are measured, these give velocities which, if analyzed as a rotation curve, would give too small a mass because the noncircular components of the orbital velocities of stars about the center act like a pressure which partly supports the mass.

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Motions Within the Magellanic Clouds

This contribution will be mainly concerned with the Large Magellanic Cloud in which we have 61 stars with usable radial velocities published by the Radcliffe Observatory [1]. These 61 optical velocities have been combined with the 17 Lick nebular velocities in an analysis of the velocity gradient across the Large Cloud [2]. After correcting for the solar component of galactic rotation, we find that the radial velocity gradient is closely in a north-south direction (position angle 171°) and amounts to 14.2 km/sec per degree.

The Radcliffe stars cover a considerably larger area of the Cloud than the nebulae with known radial velocities, although they do not extend so far out as the hydrogen 21-cm. observations of the Sydney team [3].

The optical velocities are satisfied fairly closely (though not perfectly) by solid-body rotation. This means that we could, if we chose, attribute the gradient of velocities to linear translation across the sky (as Hertzsprung did) and not to rotation. However, we have three very good reasons for believing that the velocity gradient is mostly due to rotation, and not to translation. They are: (1) there is some evidence of differential rotation, particularly from the radio observations; (2) the translational velocity required to account for the whole velocity gradient is 813 ± 65 km/sec and this is too large for a member of the local group of galaxies; (3) the

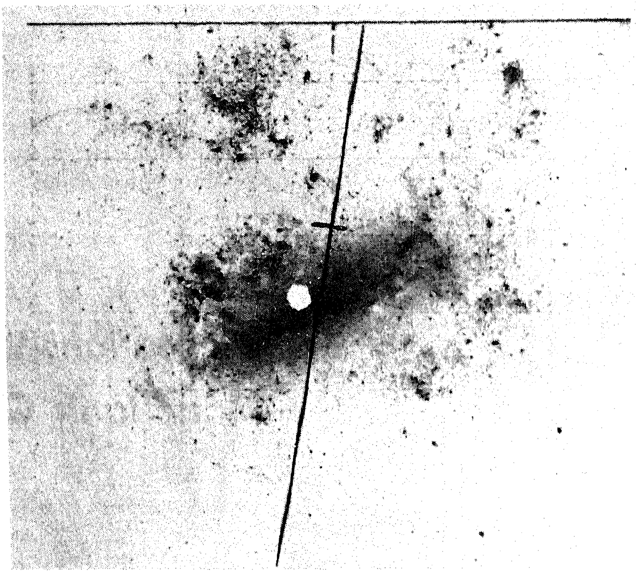

Figure 1. The Large Magellanic Cloud, showing direction of maximum gradient of radial velocities (continuous line, crossed by short line indicating the ratio center). The white spot indicates de Vaucouleurs' optical center.

position angle of our maximum velocity gradient agrees well with the major axis of de Vaucouleurs' faint outer isophotes.

Despite these considerations, it must be stressed that our unavoidable ignorance of the tangential velocities of the Magellanic Clouds sets a limit on what we can say about their rotation, masses, and so on. For instance, if the Large Cloud has a tangential velocity of 100 km/sec along our velocity gradient, then this would contribute 12 per cent of the observed gradient. The true velocity gradient due to rotation must be subject to an uncertainty of order 12 per cent. This ambiguity is an important consideration only in studying the Magellanic Clouds because, as external systems, they are so near–yet not near enough to measure proper motions. As soon as one reaches the distance of M 31, the ambiguity becomes trivial, provided the tangential velocity does not increase with distance, and we have no reason to suppose that it does.

In Figure 2 the individual observations of corrected radial velocity are plotted against the angular distance along the "major axis" of the Large Cloud. Crosses refer to nebulae; open circles, to observations of low quality or some peculiar emission objects. Some curvature is shown at the two ends of the axis. This curvature is shown more clearly when the observations are grouped into means of equal weight, as plotted in Figure 3.

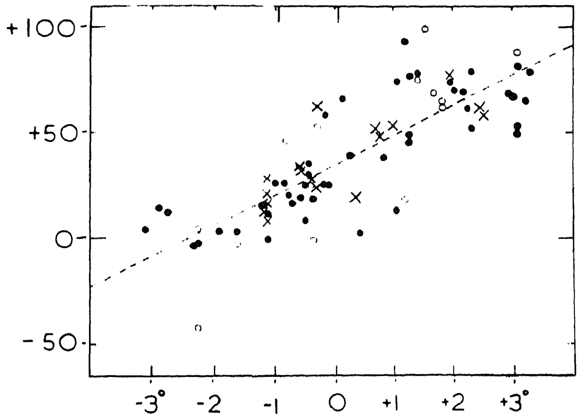

Figure 2. Radial velocities in LMC (corrected for solar motion and galactic rotation) plotted against angular distance along p.a. 171° measured from radio center.

The curvature at the two ends is significant in view of the indicated size of the errors. The centre of symmetry defined by the curvature is much closer to the radio centre than to the optical centre, despite the fact that the radio centre is displaced a full degree from the main axial bar of the Large Cloud (Figure 1). We have, therefore, folded our optical rotation curve over about this radio centre. The curve defined by the *peaks* of the 21-cm. profiles is shown for comparison; the 21-cm. profiles are asymmetrical, and if the medians had been used instead of the peaks, the agreement with the optical curve would be very poor.

We have also analysed the Large-Cloud velocities by the method used by de Vaucouleurs [4] with much less material, assuming a plane system inclined by 27° to the plane of the sky. Figure 4

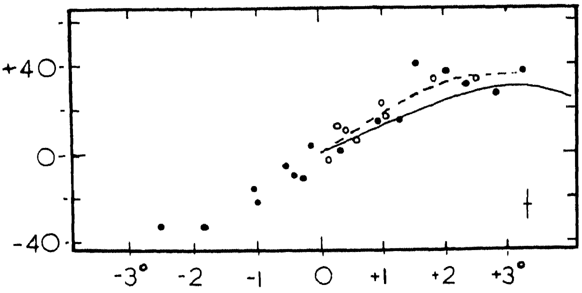

Figure 3. Figure 2 repeated, with measures grouped into means of equal weight. Open circles: left-hand points reflected in radio center. Dashed curve: drawn free-hand through open and closed circles. Full curve: 21-cm. curve from corrected peak velocities.

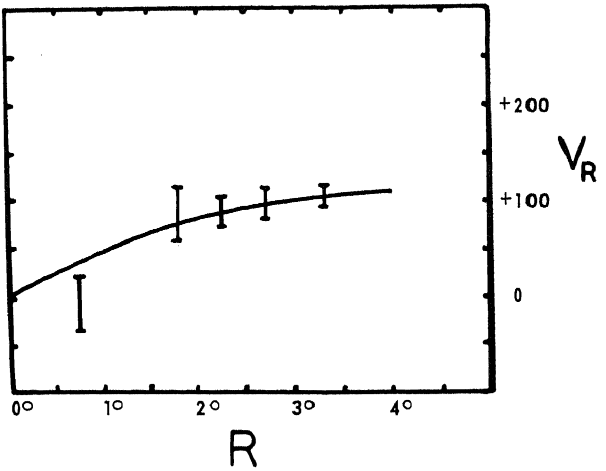

Figure 4. Rotation curve about radio center; LMC inclined-plane model. The coordinates are distances R from the center and velocities V_R in the plane of the system, assuming an inclination of 27° to the plane of the sky. Observations grouped into five mean points with indicated errors.

shows the rotation curve obtained, based on five mean points with indicated errors. The coordinates are now velocities in the assumed plane, and calculated distances from the centre are also in the plane. We cannot distinguish certainly between a model of a sphere and an inclined plane system.

There is, however, one consideration which favours de Vaucouleurs' inclined-plane model. The dispersion in radial velocities, after allowing for rotation and observational error, is 14 ± 2 km/sec, which is much less than the range in velocities due to rotation. If the Large Cloud is inclined at a small angle to the plane of the sky, as in de Vaucouleurs' model, then the observed velocity dispersion is essentially the dispersion in the z-direction. The smaller the inclination, the greater the rotational velocities in the plane will become, implying a greater degree of concentration to the plane.

There are indications that the velocity dispersion is significantly larger for *(a)* the inner regions compared with the outer; *(b)* brighter stars compared with fainter stars. Table I shows numerical results for these two comparisons. σ_1 and σ_2 refer to two different methods of calculating dispersions from observations of different weight [2].

A large velocity dispersion in the central regions might arise in various ways. In particular, in de Vaucouleurs' inclined-plane system, it would be consistent with the presence of a central bulge as found in ordinary spirals.

TABLE 1

Corrected Velocity Dispersions in Large Magellanic Cloud (relative to curve of Figure 3.)

	σ_1 (km/sec)		σ_2 (km/sec)		*Number of Stars*
Inner zone	18.3±	3.3	18.9±	3.1	29
Outer zone	6.9	3.0	11.0	3.2	29
Inner-Outer	+11.4	4.5	+ 7.9	4.5	
Brighter	15.5	3.0	18.2	2.5	26
Fainter	12.4	3.0	12.8	2.7	32
Brighter-Fainter	+ 3.1	4.2	5.4	3.7	
All stars	13.4	2.1	15.1	1.9	61

Before correction for observational scatter, the computed dispersion for all stars is 17.5 km/sec.

With regard to the larger velocity dispersion of the brighter stars, there is perhaps a similar phenomenon in the Galaxy in the high frequency of 0 stars with velocities exceeding 50 km/sec, discussed by Oort and Blaauw. The largest residual found in either Magellanic Cloud is +81 km/sec shown by one of the brightest (B type) stars in the Small Cloud. But this probably does not exceed the velocity of escape from the system.

In the Small Cloud, 40 stellar velocities have been analysed, but since they are confined within 2° along the major axis no significant conclusions regarding rotation can be drawn. However, the velocity gradient derived, admittedly with great uncertainty, is only one third that of the 21-cm. observations. This is just the opposite to the situation in the Large Cloud. It would be important to determine whether the 21-cm. profiles are as asymmetrical in the Small, as in the Large, Cloud.

The corrected velocity dispersion in the Small Cloud is 15.4 ± 3 km/sec, and this is very similar to that found for the Large Cloud. The velocity dispersion in the hydrogen gas appears to be at least 20 km/sec from the 21-cm. profiles–significantly larger than our figure of 15 km/sec for supergiant stars, which, again, is significantly larger than the velocity dispersion of main sequence B stars in the Galaxy. Feast finds a velocity dispersion 11.3 ± 3 km/sec for the nebulosities within 30 Doradus, while de Vaucouleurs gives 7 km/sec for the general velocity dispersion of Large Cloud nebulae.

The large velocity dispersion from the 21-cm. profiles, if real, must be due to the greater extension of the hydrogen gas than of stars in depth. The asymmetry of the profiles indicates differential rotation; the outer parts rotate more slowly. We can study the outer regions best through the 21-cm. profiles because the hydrogen gas extends further out than the supergiant stars. Even if the stars are confined strongly to a plane, the hydrogen gas must be regarded as extending well out from that plane.

We can say nothing at present about the velocities of main sequence stars in the Magellanic Clouds, but there is no reason to suppose that they differ from those in the Galaxy. But it does look as though the birth of excessively luminous stars in the Clouds may be accompanied by some accelerating mechanism such as has, for instance, been suggested by Oort and Spitzer.

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DISCUSSION

Kerr: Some new observations of the neutral hydrogen have been obtained by Hindman, McGee, *et al.*, of our 21-cm. group in Sydney. These show a bridge of gas between the two Clouds, and a large gaseous envelope surrounding the whole composite body. Line profiles are available for the whole area. Most of these are about 50-60 km/sec in width, between half-intensity points, and 100–110 km/sec overall.

The Large Cloud shows a clear rotation curve, going well past the velocity peak into the "planetary" region. However, rotation is less clear in the Small Cloud, but is probably present. There is also a general velocity gradient across the whole hydrogen body. If the velocities of individual peaks are plotted out for the Small Cloud, two features can be seen, with a roughly constant difference in radial velocity of 28 km/sec over a large area of the SMC. These might represent matter falling in from both sides towards the equatorial plane.

Future high-resolution observations will be important for sorting out the various motions that are present.*

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* A simple three-dimensional model was put on display to demonstrate the space relationships between the Galaxy and the Magellanic Clouds.-Ed.

Feast: It is of interest to note that the optical results give a mass of about 10^{10} solar masses for the LMC, somewhat smaller than that given by de Vaucouleurs.

Gascoigne: Kerr has mentioned that the SMC velocities suggest an inflow of gas. Is there any reason to prefer an inflow to an outflow? And how would such an outflowing mass compare with the expanding ring around the center of the Galaxy?

Kerr: An inflow is possible, but a closed velocity ellipse would be required for an expanding ring, and the full ellipse cannot yet be delineated.

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Internal Dispersion of Velocities in Other Galaxies

It has long been known that the absorption lines in the spectra of galaxies are broadened by internal motions. The quantitative determination of the velocity dispersion is difficult. Even in spectra with moderate broadening most lines are blends. When the dispersion becomes large, no individual lines remain whose contours can be determined with sufficient accuracy. Under these circumstances, the only way to determine the velocity dispersion is to compare the spectrum of a galaxy with spectra whose absorption lines are artificially broadened in a controlled way.

To carry out this procedure, it is necessary to start with the spectrum of a star of suitable spectral type and to impose on it a series of synthetic velocity dispersions. Once the velocity dispersion of a galaxy has been determined, the spectrum of that galaxy may be used as standard and synthetic velocity dispersions may be imposed on it. The spectra may be compared by juxtaposition or with the aid of tracings or plots of the intensities. Tests showed that both methods lead to the same results. The use of tracings is possible only if the noise of the tracings can be suppressed by widening the spectra. If low intensity prohibits adequate widening or if the mixing of different parts of a galaxy is to be avoided, direct comparison of the unwidened spectrograms is better than the use of noisy tracings.

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The use of the same spectrograph for the calibration spectra and the spectrum of a galaxy eliminates the necessity to make corrections for the effects of the instrumental contour of a sharp line. But the width of the instrumental contour, expressed in km/sec, should not be much larger than the smallest velocity dispersion to be measured. It is, therefore, desirable to use a dispersion of at least about 80 A/mm; for this dispersion, the width of the instrumental contour is 140 km/sec if the monochromatic image of the slit has a width of 25μ. It is necessary to emphasize this point because it has sometimes been overlooked and claims have been made which are not justified, such as estimates of a velocity dispersion of 100 km/sec from spectrograms with 400 A/mm.

To obtain synthetic spectra, a model for the distribution of radial velocities has to be selected. The choice of the model is not limited by any technical difficulty of synthesizing a spectrum. Any desired model could be used. A Gaussian distribution has been chosen; this is not meant to imply that the real distribution is Gaussian. The fraction Φ of stars with radial velocities between v_r and $v_r + dv_r$ is then assumed to be

$$\Phi\,(v_r)\,dv_r = \frac{1}{(2\pi)^{1/2}\sigma}\,e^{-v_r^2/2\sigma_r^2}\,dv_r \tag{1}$$

where σ_r is the velocity dispersion of the radial velocities. If $I\,(\lambda)$ is the spectral intensity of the spectrum on which the velocity distribution (1) is to be imposed, the resulting spectral distribution $I\,(\lambda_0)$ is, with $v = c\,(\lambda - \lambda_0)\,/\,\lambda_0$;

$$I(\lambda_0) = \frac{c}{(2\pi)^{1/2}\lambda_0\,\sigma}\int_{-\infty}^{+\infty} I(\lambda)\,e^{-\frac{c^2(\lambda-\lambda_0)^2}{2\lambda_0^2\,\sigma^2}}\,d\lambda. \tag{2}$$

If $I\,(\lambda)$ represents a spectrum with the velocity dispersion $\sigma_1, I\,(\lambda_0)$ will have the dispersion $\sigma_0 = (\sigma^2 + \sigma_1{}^2)^{1/2}$.

Two methods have been used for the evaluation of the integral.

The first *(Method A)* uses the coudé spectrograph of the 200-inch telescope as analog computer [1]. The regular slit is removed and replaced by glass plates with a nonuniform deposit of aluminum made in such a way that the transmission in the direction of dispersion varies as a Gauss function. A spectrum recorded with such a slit then shows the spectral intensity $I\ (\lambda_0)$. The slits were prepared by evaporating aluminum from a linear source onto glass plates with a narrow shielding strip parallel to the source in front of the glass. By adjusting the distance of the shielding strip from the glass, transmission curves could be obtained that followed a Gaussian distribution with deviations of less than 2 to 3 per cent of the central transmission; the width of the shielding strips determines the dispersion. With the aid of such slits, series of spectra of standard stars of various types were obtained which show the appearance of the spectra with different values of the velocity dispersion. By comparison of the spectrum of a galaxy with these calibration spectra, spectral type and velocity dispersion could be determined.

The second method requires measurement of the spectral distribution $I\ (\lambda)$ and numerical computation of the integral in (2). This procedure has been described in detail by the **Burbidges and Fish** [2]; the quantity designated as dispersion in that discussion is actually $2^{1/2}\sigma$ [3]. They used widened spectra and compared intensity tracings with plots of the computed intensity distributions *(Method B1)*. To make computed distributions usable for unwidened spectra, synthetic spectra were reconstituted from the computed intensity distributions *(Method B2)*.

For this purpose, the area underneath a plot of $I\ (\lambda_0)$ was blackened. A negative was made on which the height of the transparent area is proportional now to $I\ (\lambda_0)$. A uniformly illuminated slit was then photographed while the negative was moved uniformly across the slit in the direction normal to the wave length. The exposure at each wave length thus was made proportional to $I\ (\lambda_0)$; exposure times were of an order in which reciprocity failure is not important. Care was taken to obtain as nearly as possible the same contrast as that of the spectra which were to be compared with the synthetic spectra.

Method A was used with a dispersion of 40 A/mm for NGC 221 and NGC 224 [1]. To obtain widened spectra of the nuclei, a window of a width of 2″ was trailed along the slit at the same rate as the nucleus. The slit was always in the direction of the minor axis. No observable difference was found between the spectra of the nucleus and areas 5″ to 10″ distant from the nucleus. The spectra covered the range from λ3900 to λ4500; the region λ4200 to λ4400 was found to be most suitable for the determination of the velocity distribution. The best fit of spectral type was obtained with spectra of type G8 III to K0 III. This spectral type later was also found by Morgan and Mayall [4]; it agrees with measurements of color and spectral intensity distribution [2].

The long exposure times required by the use of a dispersion of

40 A/mm, the smallest dispersion of the coudé spectrograph of the 200-inch telescope, made the investigation of other galaxies with Method A unattractive.

Method B1 was used by the Burbidges and Fish[2] to investigate NGC 221 with δ Tauri, K0 III, as standard star, and NGC 3379 with NGC 221 as standard [5]. A dispersion of 75 A/mm at the Cassegrain focus of the 82-inch telescope at the McDonald Observatory was used. The region λ 4200 to λ 4400 was used for the evaluation.

Method B2 was used for observations of a number of galaxies at the prime focus of the 200-inch telescope with a dispersion of 85 A/mm. NGC 221 was used as standard. The spectra were not widened and had to be compared therefore with reconstituted spectra. To find out what accuracy could be achieved by this method, independent estimates of σ were made by the author, by E. Flather, and by J. S. Mathis. The average deviation of the three estimates ranged from ±15 km/sec to ±60 km/sec and was essentially determined by the quality of the plate; individual values are given in the seventh column of Table 1.

TABLE 1

Velocity Dispersion in Galaxies

			Method					
Object	*Type*	M_p	*A*	*B1*	*B2*	*AD*	*Region*	*Note*
NGC 221	E2	–15.2	100	96	--	--	A:nucl. to 10″; B1:wid.	
Anon	EOp	–15.2	--	--	370	40	Widened	(1),(2)
NGC 4111	SO_2	–18.4	--	--	215	38	Centr. region to 5″	
NGC 3115	$E7/SO_1$	–19:	--	--	205	23	5″ to 15″	
NGC 3379	EO	–19.6	--	187	--	--	Widened	
NGC 4621	E5	–19.6	--	--	450	64	Nucleus	(1)
NGC 4762	SO_1	–19.6	--	--	195	24	4″ to 18″	(1)
NGC 224	Sb	–19.9	225	–	225	14	A:nucl. to 10″; B2:10″	
NGC 4697	E5	–20.2	--	--	310	63	Nucl. to 5″	(1)
NGC 6703	SO	–20.2	—	--	270:	--	Nucleus	
NGC 4406	E3	–20.3	—	--	415	29	Widened	(1)
NGC 7626	E1	–20.7	--	--	480:	--	Nucleus	
NGC 4486	EOp	–20.8	—	–	490	55	3″ to 20″	(1)
NGC 4472	E1	–21.3	--	--	365	21	6″ to 20″	(1)

(1) Virgo cluster. (2) 7.′2N prec. NGC 4486.

All results are collected in Table 1 and plotted in Figure 1 against M_p. The region of the galaxies to which the velocity dispersions refer are given in the eighth column of Table 1. Correlation between velocity dispersion and M_p exists, but it is poor; this can be clearly seen without quantitative evaluation by inspection of the

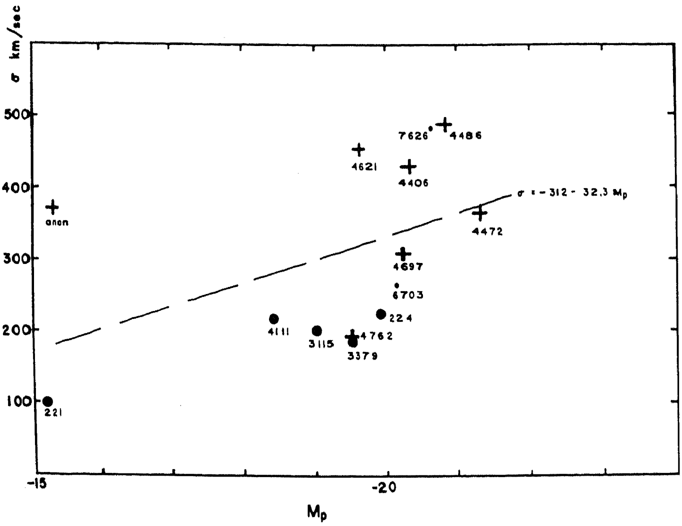

Figure 1. Velocity dispersion σ plotted against absolute photographic magnitude M_p.

plates [6]. Some of the objects show rotation (NGC 224, NGC 3115, NGC 4111, NGC 4621, NGC 4762), but their velocity dispersions are not conspicuously out of line. The velocity dispersion does not seem to be a good criterion for absolute magnitude, a possibility suggested by Mayall and Morgan [4].

Six of the seven members of the Virgo cluster have relatively high velocity dispersion. The seventh, NGC 4762, is extremely flattened. The velocity dispersion applies to the flat disk; the strongly exposed nuclear region does not show appreciably higher dispersion. The prevalence of high dispersions for objects in the Virgo cluster is odd, but the total number of objects is too small to rule out the possibility of a statistical accident. It seems important to extend the observations to more objects, especially at low and medium absolute magnitudes.

Particularly remarkable is the high velocity dispersion in the anonymous dwarf galaxy 7.′2 north preceding NGC 4486. Color [6] and spectrum of this object are quite normal. The intensity distribution in the object shows much less central condensation than is usual in elliptical galaxies. This was noticed by Baade who drew attention to the object. It was recognized as peculiar only because it is close enough to NGC 4486 to be in the area of good definition on plates of that object taken with the Ross corrector. It cannot be recognized as definitely unusual on the 48-inch Schmidt plates. A search for similar objects is therefore difficult. The rarity of objects of this type is an open question.

The author is greatly indebted to Dr. R. A. Fish, who took care of the numerical integrations, and to William C. Miller, who carried out the difficult photographic work of preparing the reconstituted spectra. Thanks for their help in evaluation are due to Miss E. Flather and Dr. J. S. Mathis of the Washburn Observatory, where part of this work was done during the author's stay. Drs. E. M. and G. R. Burbidge deserve thanks for helpful discussions and for making results available prior to publication.

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DISCUSSION

Neyman: Because of the smallness of the number of observations, the regression coefficient of velocity dispersion in magnitude may not be significantly different from zero. Was this question studied?

Minkowski: The material is too small for elaborate statistical studies. The regression coefficient, however, is significantly different from zero.

Thackeray: It is perhaps worth pointing out that the stellar velocity dispersions in the Magellanic Clouds previously quoted are very much smaller than those plotted in your diagram. Were there any Irregulars in your list?

Minkowski: No.

McVittie: Would you explain why you expect to find a linear relationship between velocity dispersion and absolute magnitude?

Minkowski: I do not expect a linear relationship. The straight line in Figure 1 is meant only as an indication of the order of magnitude of the dependence on absolute magnitude.

King: A significant correlation of velocity dispersion with absolute magnitude should include the radius of the galaxy in question. This makes the Virgo object especially peculiar. The only way to reconcile its high velocity dispersion with its faint absolute magnitude is to assign it a small radius. Quite to the contrary, the radius of its central core seems to be rather large.

Minkowski: The radius should indeed be taken into account. The dependence on absolute magnitude alone was considered because the use of the velocity dispersion as criterion for absolute magnitude has been suggested.

The Virgo object cannot be compared directly with other known dwarf ellipticals. It may have a higher mass-luminosity ratio than normal dwarf ellipticals.

De Vaucouleurs: I am rather surprised that you describe NGC 4486-B (the small anonymous elliptical near M 87) as a nearly uniform disk. Is it not possible that its peculiar appearance is caused by a high surface brightness combined with a very steep luminosity gradient and insufficient resolution compared with the small diameter of this object? The abnormal $U - B$, $B - V$ relation of this object (together with NGC 4008, 4478, 4158, and 5846) is discussed elsewhere.*

Minkowski: The object was noticed by Baade because of its unusual appearance. It is well enough resolved to see that, unlike normal elliptical galaxies, it has very little central condensation.

E. M. Burbidge: However, this anonymous object in Virgo is large enough that you would easily detect a bright nucleus or bright nuclear concentration with the 200-inch, would you not?

Minkowski: Yes.

Hoyle: Can you give us the angular diameter of this dwarf elliptical?

Minkowski: About 10″.

Vorontsov-Velyaminov: Is the velocity dispersion deduced from observations due to the peculiar velocities of stars or does it include the result of integration of the radial component of the rotational velocity along our line of sight? In that case, the velocity dispersion must depend on many circumstances and not only on the luminosity of galaxies.

Minkowski: The velocity dispersion includes the result of integration along the line of sight. It does not depend only on the luminosity of the galaxies. The dependence on absolute magnitude alone was considered here because of its possible use for the determination of absolute magnitudes. Obviously, other factors play great roles also.

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*See *Ap.J. Suppl.* 5, 233 (1961).

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Motions of the Interstellar Gas in the Central Regions of Galaxies

I

The observation at the 21-cm. wave length of emission and absorption features in directions close to that of the galactic center has provided a view of the space distribution and state of motion of the neutral interstellar gas near the galactic nucleus. According to Rougoor and Oort [1], at a distance of 3 kpc from the nucleus there is a well-defined, rather dense spiral arm rotating around the center with a velocity around 200 km/sec. The unexpected feature in the motion of this arm is that it also has an outward component of velocity amounting to 53 km/sec, which is detected as an absorption feature against the radio source Sagittarius A [2]. From the study of the 21-cm. line profiles obtained within 4° from the galactic center, the Leiden astronomers [1] have suggested a model for the hydrogen distribution at closer distances from the nucleus also. Between 500 and 600 pc, there appears to be a ring, with density around 1 atom per cm^3, in a state of pure rotation, with velocity close to 265 km/sec. Still closer to the nucleus, there is a practically empty region, the density becoming appreciable again only at $R = 300$ or 350 pc. It then graduallyrisesto about 3 atoms per cm^3 at $R = 100$ pc, and, further in, it increases sharply right to the boundary of the Sagittarius A radio source. The rotational velocities in the inner disk amount to 220 km/sec at $R = 260$ pc, 190 km/sec at $R = 140$ pc, and 180 km/sec at $R = 70$ pc. On the assumption that the motions of the gas in the ring and disk are governed by gravitational forces only, the concentration of mass density in the nuclear region of the system has been derived by the Dutch astronomers.

II

A further discovery apparently related to the phenomena revealed by the 21-cm. work has been achieved by Courtès and Cruvellier at the Haute Provence Observatory [3]. Photographing the region of the galactic center, through a Fabry-Perot interferometer, these authors have detected Hα emission displaced by about -190 km/sec at an angle of 1° from the center. The emitting mass can not thus be closer than 140 pc from the nucleus. The close agreement of the rotational velocity of neutral hydrogen at this distance with the observed radial velocity of Hα provides strong evidence that the emitting gas is, indeed, located in the nuclear region. On the basis of this remarkable observation of the French astronomers, it is suggestive to think that the various regions more or less sharply outlined by emission or absorption of neutral hydrogen may be imbedded and mutually separated by gas in the ionized state.

III

Following the discovery of the nonrotational motions in the Galaxy, it became apparent that the study of similar phenomena in other systems had to be attempted by optical means because of the limitations in resolving power of existing radio telescopes. The occurrence of optical emission in the nuclei of galaxies is quite general [4]. In spheroidal systems, the emission lines are so broad that their measurement for accurate radial velocities is not feasible [5]. In the Sb system M 81, contrariwise, sharp emission lines have been observed right at the nucleus [6], but they do not extend beyond 60 pc. Near the nucleus of M 31, the system is generally believed to resemble our Galaxy closely. The presence of the [O II] doublet at $\lambda\lambda$ 3726-29 in emission was suspected by Babcock [7], but it never was studied in detail. Accordingly, in the fall of 1959, an attempt was made to detect and measure the [O II] emission doublet in M 31. Somewhat unexpectedly, it was found [8] that, in spectra taken at a dispersion of 67 A/mm with the nebular spectrograph of the Hale reflector, the [O II] lines appeared in appreciable strength well beyond the nucleus proper, with exposure times of the order of four hours. While the study in detail of the gas motions in the nuclear region of M 31 is not yet completed, in view of the present interest in the phenomena concerned, it is appropriate to present here a brief report on the present status of the program.

In Figure 1, a photograph appears of the nuclear region of M 31 taken with a blue sensitive plate. The directions of the apparent major and minor axes are indicated, as well as the length of the slit used in the spectroscopic observations. Attention is called to the dark markings seen projected against the stellar background, to their spiral arrangement with respect to the nucleus, and also to the asymmetry shown by their distribution in the NE and SW quadrants. In the lower part of Figure 1, representative spectra of the [O II] doublet, taken at various position angles, are reproduced. The results of the radial velocity measurements are given graphically in Figure 2 for the various directions across the nucleus as functions of distance in the symmetry plane of the system. The radial velocities are accurate to within 10 km/sec, an amount derived from the agreement between three overlapping, well-exposed plates taken along the NE major semiaxis. At points where the intensity of the emission lines is low, the accuracy may be considerably lower. On the basis of the material so far obtained, we can summarize our present knowledge about the distribution of interstellar gas in the central parts of M 31 as follows.

(*a*) Within 15″ (about 45 pc) from the nucleus along the major axis, the [O II] lines are so broad that no accurate radial velocity measurements may be made. At greater distances from the nucleus, the [O II] doublet appears well resolved. In consideration of the resolving power of the spectra, it may be concluded that small-scale motions on the line of sight do not exceed 75 km/sec and probably are less. This, in turn, implies that the motion of the gas takes place very nearly in the symmetry plane.

(*b*) The intensity and velocity of the [O II] lines are not smoothly varying functions of position, but show sudden breaks in a manner suggesting the existence of discontinuities in the flow. There is evidence that these discontinuities have some relation to the dust lanes referred to above. In general, the lines are stronger in the NE quadrant where the dark markings are more prominent.

(*c*) The existence of motions without rotational symmetry is shown by the differences in velocity at either side of the major axis in a fixed direction. The radial velocity is not constant and equal to that of the nucleus, particularly along the apparent minor axis, where the components of rotational motions vanish. A point at 200 pc from the nucleus on the SE side (the far side) has a velocity of +100 km/sec with respect to the nucleus, while the

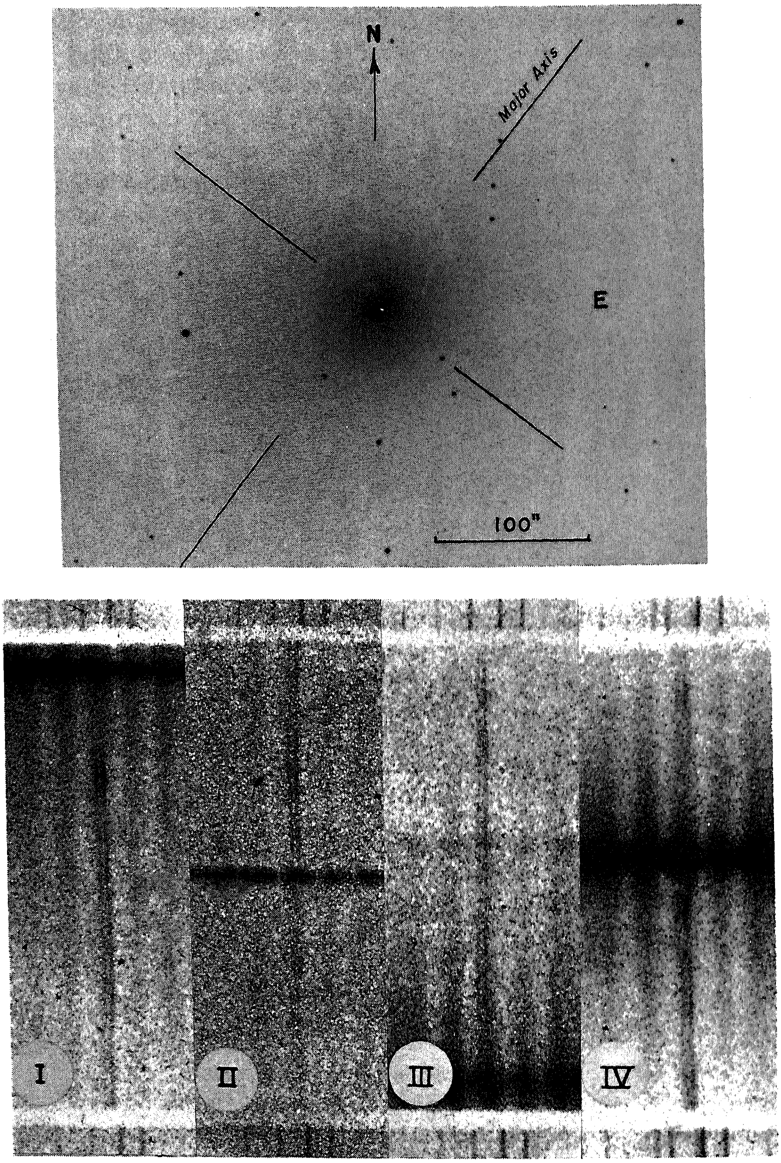

Figure 1. (*Top*). The nuclear region of M 31, from an original obtained by W. Baade with the 100-inch reflector on a 103a-0 emulsion through a UG2 filter. (*Bottom*). The [O II] doublet at λλ 3726–28 in the central region of M 31, from spectra obtained at 67 A/mm dispersion. The length of the slit subtends an angle of 100 seconds of arc. Spectra I and II were taken along the Nf half of the major axis, spectrogram II being contiguous to I. Spectrum III is on the Sp side of the major axis, while spectrum IV is across the nucleus in the NS direction, with N at the bottom.

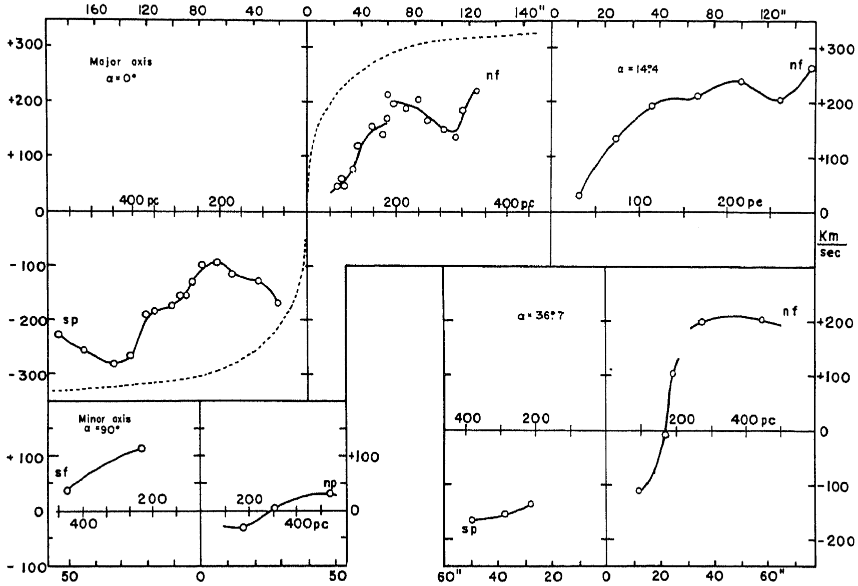

Figure 2. Radial velocities of the [O II] lines in the nuclear region of M 31, measured with respect to the centroid determined by 21-cm. line observations and projected on the symmetry plane. The abscissae are expressed both in seconds of arc in the plane of the sky and in parsecs measured in the symmetry plane, for the various directions α with respect to the major axis.

opposite symmetric point has a velocity of −40 km/sec. Thus, there appears to be an outflow of gas from the nucleus.

An estimate of one solar mass per year for the quantity of matter Q streaming out of the nucleus was given in a preliminary announcement [8]. This value followed from the electron density inferred from the intensity ratio between the two components of the [O II] doublet. This intensity ratio is so near the limiting value for low densities, that the resulting value of Q is, thus, only a very rough upper limit. A more realistic estimate for the electron density N_e may be obtained from the surface brightness in the [O II] lines. From the six-color photometry for M 31 [9], the brightness distribution in M 31 [10], and the measured contrast between the [O II] lines and the mean neighboring continuum, it is estimated that the surface brightness S of M 31 in the [O II] lines, at a distance of 50 pc from the nucleus, is

$$S = 10^{-5}\ \mathrm{erg/cm^2/sec}.$$

The emissivity of a low density gas containing oxygen, at an electron temperature T, is known to be [11]

$$1.6 \times 10^{-17}\, T^{-1/2}\, e^{-38600/T}\, N_e\, N\,[\mathrm{O\ II}],$$

in erg/cm^3/sec. Assuming that the emitting medium in M 31 has thickness h (in parsecs) and that the abundance of oxygen with respect to hydrogen has the cosmical value, for a temperature of 10000°K we find

$$N_e^2 \, h = 1 .$$

For $h = 100$ pc, then, $N_e = 0.1 \text{ cm}^{-3}$, a value 500 times less than the earlier estimate. Thus, the value of Q given previously is an overestimate by two orders of magnitude.

IV

The kinematical properties of the interstellar gas in the nuclear regions of the Galaxy and of M 31 have revealed that very likely we are observing manifestations of the same basic phenomenon. The side-by-side presence of neutral and ionized hydrogen in the Galaxy would suggest that M 31, when observed in 21-cm. with sufficiently high angular resolving power, would show also large amounts of neutral hydrogen. An independent suggestion for the presence of cool matter near the nucleus of M 31 is provided by the dust observed in the dark spiral-like lanes. The importance of the [O II] observations in M 31 is that the motions of the gas can be compared with those that would be expected from gravitational forces alone. In Figure 2 we see that the radial components of the motion along the major axis are considerably lower than the circular velocities inferred from the light distribution. The differences provide a measure for the importance of hydromagnetic forces in determining the motion and, probably, the physical state of the gas also.

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DISCUSSION

Oort: I wonder if there is good reason to suppose that part of the gas in the nuclear region of M 31 is outside the equatorial plane. In the nuclear region of our Galaxy, all the neutral hydrogen, at least, is very close to the galactic plane.

Dr. Münch has mentioned that the rotational velocities of the gas appeared lower than the circular velocities inferred from the star density. Exactly this same phenomenon is found in the Galaxy if we look at regions outside the small, rapidly rotating disk of radius 600 pc. For distances larger than 600 pc, the rotational velocity of the gas appears to fall far below the circular velocities corresponding to the gravitational field.

Münch: I agree with Professor Oort. The velocities of the gas could lie in the equatorial plane and be less than the stellar circular velocities if nongravitational forces (magnetic) were operating.

Mayall: Have you made any similar observation of the nuclear regions of M 33? Have you observed NGC 205 in a similar manner?

Münch: I have not observed either M 33 or NGC 205.

Courtès: M 33 has some very strong Hα emission close to the center, but the lines are asymmetrical. There are two H II regions, a very bright one near the center and a fainter one which seems to be connected with the nucleus itself.

Kinman: What dispersion was used?

Münch: The spectrograms of $\lambda\lambda$ 3726-28 were obtained with a 3-inch camera and a grating with 750 grooves/mm in the fifth order. They have, thus, a dispersion of 67 A/mm or 5.5 km/sec/μ.

Woltjer: Do you still believe that the gas in the nuclear region of M 31 does expand?

Münch: Yes. There is no question about it.

De Vaucouleurs: Has Dr. Münch tried to derive the velocity of the nucleus of M 31 instead of assuming it from radio data?

Münch: The motions revealed by [O II] lines are obviously not symmetric on either side of the major axis. The determination of a centroid by taking some sort of a mean would not be meaningful.

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The Determination of the Masses of Galaxies and Their Internal Mass Distributions

INTRODUCTION

In this review we shall consider only those methods of estimating the mass of a galaxy which depend in some way on dynamical considerations. The general procedure is to map the gravitational field of the galaxy by observing the acceleration of some test body. If we use another galaxy as the test body, we can obtain only total masses and, in practice, only statistical information about the average mass of a certain class of galaxies. The results for double galaxies have been summarized by Page [1], and are discussed elsewhere in this volume. If we wish to know something about the density distribution, the gravitational field must be explored in considerable detail, and it is necessary to use a subsystem of the galaxy itself as the test object. An equation of motion, such as the Boltzmann equation, or the hydrodynamic equations, must be used to infer the gravitational field from the accelerations. But accelerations can never be measured directly in a galaxy: they must be deduced from observations of the velocity field at what is effectively a single instant. Since this is the case, we are compelled to assume that the velocities are independent of time. Furthermore, the specification of the velocity field requires a vector function of a vector position $u(r)$, and this is observationally impossible except for a very limited part of the Galaxy. In all other cases, we can observe only the radial velocity, and even that is averaged in some fashion along the line of sight. Instead of three functions of three variables, then, we can measure at most one function of the two variables which specify position on the sky. Clearly, we are not only restricted to those galaxies which we believe to be in a reasonably steady state, but we must rely heavily on symmetry considerations to extend our knowledge of the velocity field. Fortunately, there are good reasons for supposing that steady motions and a high degree of symmetry are usually to be found together, but this means that irregular galaxies present a particularly difficult problem.

If we had completely self-consistent theoretical models of galaxies (that is, simultaneous solutions of Poisson's equation and the equations of motion) the observational data could be used to choose the particular model, and the mass would follow immediately. At present, this program is feasible only for spherical or slightly flattened ellipticals. In all other cases, we choose for the density some plausible expression containing several adjustable parameters, compute the gravitational field, and determine the parameters from the observations. The details of this procedure depend on whether the galaxy is an ordinary spiral, a barred spiral, or an elliptical.

ORDINARY SPIRALS

Except for the Galaxy, which is a special case, the material usually available for analysis consists of direct photographs (perhaps in several colors, or with isophotometry) and a collection of radial velocities obtained from 21-cm. line profiles or from emission lines in the optical spectrum. It is sometimes possible to obtain a short segment of the rotation curve in the center of the galaxy from absorption lines, and the widths of the lines occasionally give useful information about velocity dispersions in the stars or the gas. A much wider variety of material is available for the Galaxy, but this advantage is partially offset because relative distances are quite difficult to determine with any accuracy.

For an external galaxy, the first step in the reduction is to determine the orientation. Although these objects present a somewhat mottled appearance on direct photographs, there is some reason to hope that the underlying mass distribution is essentially axisymmetric, and, if this is true, the orientation is specified by the angle ξ between the axis of symmetry and the line of sight, and the position angle η of the line of nodes. These angles can be fixed either by photometric or spectroscopic methods. If there is an elliptical isophote in the galaxy, the position angle of the major axis is η, and, if the emitting region is sufficiently thin, the axis ratio is $\cos \xi$. This simple method of measuring ξ and η often encounters difficulties in practice. The mass distribution may be nearly axisymmetric, but the light distribution exhibits the spiral pattern, and this may make it difficult to find useful elliptical isophotes. (Danver [2] has determined the orientation of a large number of galaxies by matching the visible arms to logarithmic spirals.) The isophotes are often not symmetric with respect to the nucleus, and it is frequently observed that the position angles of the major axes change progressively as one goes out from the nucleus. Even if these complications are absent, the axis ratio should be corrected in principle for the thickness of the emitting region, but if we can rely on analogy with the Galaxy and with edge-on systems in general, the outermost parts of most spirals are so thin that this correction can often be neglected.

The orientation can also be determined from radial velocities if we assume that the lines are formed in a thin plane layer, and that only circular velocities are present. Let (ϖ, ϕ) be a polar coordinate system in the plane of a galaxy, and (s, α) be a polar coordinate system in the plane of the sky, with common origin at the nucleus and with the angles measured from the line of nodes. Then, if the circular velocity is $u_\phi(\varpi)$, the radial velocity V is given by

$$V(s, \alpha) = \sin \xi \cos \phi \, u_\phi(\varpi), \tag{1}$$

where

$$\tan \phi = \sec \xi \tan (\alpha - \eta), \tag{2}$$

and

$$\varpi^2 = s^2 [\cos^2 (\alpha - \eta) + \sec^2 \xi \sin^2 (\alpha - \eta)]. \tag{3}$$

If a sufficient number of radial velocities are available, it is possible to determine η from these equations, and, with considerably greater uncertainty, ξ as well. These estimates may or may not agree with the geometrical evidence. When a discrepancy arises, as it does in NGC 2903 [3], for example, it is seldom possible to resolve the difficulty in a unique way.

In the remainder of this section, it will be assumed that we are satisfied with the values of the projection angles, and have, therefore, found the best values of u_ϕ from the observations. Ignoring for the moment all forces except gravitation, we would put

$$-\frac{u_\phi^2}{\varpi} = \frac{\partial\psi}{\partial\varpi}, \tag{4}$$

and

$$\nabla^2\psi = -4\pi G\rho, \tag{5}$$

where ψ is the gravitational work function and ρ is the total density. Since the first of these equations says nothing about the z-dependence of ψ, the observations are not sufficient to determine ρ unless we make an additional assumption about the mass distribution. The following methods have been suggested by various authors:

A. Assume that the galaxy can be represented by a plane sheet of variable surface density [4].

B. Assume that the equidensity surfaces are oblate spheroids of constant eccentricity [5].

C. Assume that the galaxy can be represented by the superposition of several oblate spheroids of density $\rho = p + q/\varpi$ [6].

The choice of method depends to a certain extent on how much information is available in addition to the velocities. Method A uses no data except the rotation curve, but is somewhat troublesome to apply since the integrals which arise in computing the potential cannot always be evaluated in closed form. We will describe the second method in greater detail shortly, but we note here that the eccentricity of the equidensity surfaces cannot be extracted from the spectroscopic data. The third method needs much more information than the radial velocities to give a unique result, and is particularly suitable for application to the galactic system, since the force in the z-direction or the flattening of a particular subsystem of stars can be embodied in the model.

Method B leads to the integral equation

$$u_\phi^2(\varpi) = 4\pi G\sqrt{1-k^2}\int_0^{\varpi} \frac{\rho(a)\,a^2\,da}{\sqrt{\varpi^2 - k^2a^2}} \tag{6}$$

for the density $\rho(a)$, and is solved by substituting the expansions

$$u_\phi^2(\varpi) = \varpi^2 \sum_{l=0}^{\infty} v_l\,\varpi^l, \tag{7}$$

and

$$\rho(a) = \sum_{n=0}^{\infty} \rho_n a^n, \tag{8}$$

in Eq. (6) and equating the coefficients of a^n. It turns out that each v_n gives rise to one and only one ρ_n, with

$$\rho_n = \frac{k^{n+2}}{4\pi G \sqrt{1-k^2}\; A_{n+2}(k)} v_n, \tag{9}$$

where

$$A_m(k) = \int_0^{\sin^{-1} k} \sin^m \theta \, d\theta . \tag{10}$$

(The explicit values of these integrals are given in [5].) In practice, of course, the power series for u_ϕ^2 is replaced by a polynomial and the coefficients are determined by least squares directly from the radial velocities. Once ρ has been found, it is a simple matter to compute the total mass within a given distance of the center, or to find the mass per unit area for comparison with the light distribution.

The density obtained by this method depends on the parameter k, which is very difficult to estimate unless the galaxy is nearly edge-on, and if this is the case, the apparent rotational velocity may be falsified by absorption within the galaxy. The relevant eccentricity is that which best characterizes the mass distribution, and may be quite different from the one indicated by the isophotes. However, the total mass and even the mass per unit area seem to be remarkably insensitive to the value of k: Fish [7] has examined the effects of changing k in the reductions of NGC 5055 [8] and finds that the surface density is nowhere altered by more than 13 per cent, and the total mass increases only 11 per cent when the axis ratio of the equidensity surfaces is increased from 1/15 to 1/5.

The method just outlined has certain drawbacks. If there are gaps in the observed rotation curve, the least-square procedure may produce artificial humps: this situation can be handled best by drawing in a plausible curve and fitting the curve by the usual methods. The density distribution may not be too well represented by spheroids of constant eccentricity (and there are no theoretical models to indicate that it should be), but this is at least a reasonable model which can be adopted if no other information is available. It should also be remembered that Eq. (6) makes no allowance for forces other than gravitation, e.g., pressure gradients and magnetic fields. It is sometimes possible to estimate an effective pressure gradient from the widths of the emission lines (cf. NGC 2146 [5]) and the analysis can be modified in a rough way to take this into account.

No reduction procedure can give satisfactory information unless one starts with an abundance of radial velocities; even if these are available it may be impossible to obtain a reasonable result by any method. In certain galaxies, for example NGC 2146 [5] or NGC 2903 [3], there appear to be gross local departures from simple circular motion. Such anomalies are undoubtedly real and no simple explanation has been found to account for them. However, the greatest single uncertainty affecting the determination of a total mass is usually the distance scale. Galaxies which are suitable for extensive analysis usually lie in the range where the red-shift may be an unreliable distance indicator. The mass is proportional to the assumed distance scale, except for the Galaxy, for which Brandt [9] has shown that the mass is roughly proportional to R_0^3.

BARRED SPIRALS

Barred spirals are clearly objects of a lower order of symmetry than ordinary spirals. We may still assume that there is a plane of symmetry, but instead of axisymmetry, we suppose only that

$$\rho(\mathbf{r}) = \rho(-\mathbf{r}), \tag{11}$$

and

$$\mathbf{u}(\mathbf{r}) = -\mathbf{u}(-\mathbf{r}). \tag{12}$$

Three angles are now needed to fix the orientation of the galaxy: the angles ξ and η defined earlier, and the angle ζ between the line of nodes and the axis of the bar. It is almost impossible to determine these angles from an inspection of photographs, as there are usually no clearly defined elliptical isophotes. Furthermore, the assumption of a steady state must be relaxed somewhat, and we now suppose only that the velocity with respect to a coordinate system rotating with the bar is independent of time. It is quite clear that if the mass distribution is not axisymmetric, then the velocity field cannot be axisymmetric, and we have to consider a radial component u_{ϖ} of the velocity as well as the rotational velocity u_{ϕ}; both of these velocity components will depend on ϕ as well as ϖ. In this case the spectroscopic data no longer determine unique values for the projection angles.

The theory of possible gas motions in barred spirals is in a rudimentary state, and the observational data presently available are meager. However, it has been observed in NGC 7479 [10] that the bar rotates as a rigid body, and there is a sudden drop in the velocity just beyond the end of the bar. It can be shown that this last result is reasonable if the mass distribution is highly elongated [11]. It is possible to estimate the mass of the bar if we take as a crude model either a prolate spheroid or a general ellipsoid of uniform density; with this configuration, it is possible at least to balance the centrifugal and gravitational forces along the axis of the bar. Aarseth [12] has considered an assembly of spheres and cylinders of variable density with the same property. Once the model has been chosen, the total mass can be computed from the angular velocity of the bar.

It might seem at first sight that the nuclear regions of barred spirals can be considered axisymmetric, but radial velocity observations of NGC 1365 indicate that this may not always be true [13]. Six rotation curves have been obtained in five different position angles, and all are apparently linear within 20″ of the center, but the slopes of these curves are not compatible with an axisymmetric velocity field. Indeed, a general linear rotation, expansion and shear cannot be distinguished from simple rotation unless one knows the projection angles, and is equally incompatible with the observations. It appears that ordinary and barred spirals present problems of a different order of complexity.

ELLIPTICAL AND S0 GALAXIES

Most information about motions in spiral galaxies comes from radio or optical measurements of emission lines. Some ellipticals show λ 3727 in emission in the nucleus [14], but mass estimates must rest mainly on velocity dispersions obtained from the widths of absorption features, and on accurate photometry. Although a number of ellipticals show evidence of rotation in the nucleus, it is only in quite exceptional cases, such as NGC 3115 [15], that an extensive rotation curve can be obtained.

Again, the equations of motion provide the link between the observed velocities and the gravitational field. For an axisymmetric system, under fairly general conditions, these equations are

$$-\frac{u_\phi^2}{\varpi} = \frac{\partial\psi}{\partial\varpi} - \frac{1}{\nu}\frac{\partial}{\partial\varpi}(\nu < v_\varpi^2 >) - \frac{1}{\varpi} < v_\varpi^2 > + \frac{1}{\varpi} < v_\phi^2 >, \quad (13)$$

and

$$0 = \frac{\partial\psi}{\partial z} - \frac{1}{\nu}\frac{\partial}{\partial z}(\nu < v_z^2 >), \quad (14)$$

where ν is the density of a certain dynamical subsystem of stars, and $<v_{\varpi}^2>$, $<v_{\phi}^2>$, and $<v_z^2>$ are the mean square random velocity components of these stars. In writing these equations, we have already assumed that the system is in a steady state and that the cross-correlations of the random velocities (for example, $<v_{\varpi}\, v_{\phi}>$) are everywhere zero. It is evident from these equations that if we are to determine ψ, hence the total density ρ, we must know the behavior of the pressure tensor $<v_i v_j>$, the relative density ν and the rotational velocity as functions of ϖ and z. (Since ellipticals are not so flat as spirals, it is not allowable even in the first approximation to neglect the random motions entirely.) From the widths of the absorption lines we can estimate one component of the pressure tensor averaged along the line of sight, and from the tilt of the spectral lines we can get the apparent angular velocity of rotation; however, the light is so strongly concentrated towards the center in these systems that we can seldom observe these quantities outside the nucleus. This does not take us very far towards determining the gravitational field, and we have to rely heavily on accurate photometric results and on theoretical models for additional information. The fact that it is almost impossible to determine the orientation in most cases is another complication, although there is some evidence [16] that spherical systems are distinguishable from highly flattened systems seen face-on.

The simplest objects of the present class are the spherical galaxies, for which the equation of motion is

$$0 = \frac{\partial \psi}{\partial r} - \frac{1}{\nu}\frac{\partial}{\partial r}(\nu <v_r^2>) - \frac{2}{r}<v_r^2> + \frac{1}{r}<v_\theta^2> + \frac{1}{r}<v_\phi^2>. \qquad (15)$$

This is the only kind of galaxy for which completely self-consistent models are available – that is, solutions of the combined Boltzmann and Poisson equations. (Most of the investigations in the literature were motivated by a desire to understand the structure of globular clusters, but the results can frequently be applied to ellipticals.) Besides the usual isothermal and polytropic gas spheres, models have been constructed with a velocity cutoff [17], with a mass function and equipartition of energy [18], with primarily radial motions [19] and recently an evolutionary model has appeared which takes stellar collisions into account [20]. These models naturally have different gravitational potentials and density distributions. The most that can be done with the measured velocity dispersion is to evaluate one of the parameters needed to fit a theoretical model to an actual galaxy, and at least one other number is required to fix the scale of length. Photometric results must be used to determine this additional constant and to narrow the range of choice among the various models. Several purely photographic investigations are to be found in the literature [21] and a few photoelectric studies have been made, notably of M 87. Also, Hubble [22] and de Vaucouleurs [21] have proposed empirical formulae for representing the light distribution along the major axes of ellipticals. Such data can be made to yield information about the relative density distribution by a variety of treatments. For example, if the color of the galaxy is constant or nearly so, we may try the hypothesis that the mass-to-light ratio is constant throughout the galaxy. The emissivity per unit volume J can be recovered from the observed intensity I by solving Abel's equation

$$I(\varpi, z) = 2 \int_{\varpi}^{\infty} \frac{J(x, z)\, x dx}{\sqrt{x^2 - \varpi^2}}, \tag{16}$$

and, under the present hypothesis, J is proportional to ρ.

Even after making the above assumption, we still do not have enough information to make use of Eq. (15), since we do not know the dependence of the random velocities on the distance from the center. As it does not appear that line widths can be measured outside the nucleus by current techniques, we must fall back on theoretical models, which, however, allow a very wide range of possibilities. The virial theorem can be used if we are interested only in the total mass; but even then we cannot avoid adopting some sort of model, since neither the kinetic nor potential energies can be estimated from the observations without one.

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DISCUSSION

McVittie: If you alter your theoretical assumptions, do you change the total mass by a factor of a few per cent, by one of less than 10, or by one of the order of 100?

Prendergast: The mass within the *observed* part of the galaxy is almost independent of the theoretical assumptions; for example, if the assumed axis ratio of the spheroidal equidensity surfaces is changed from 1/15 to 1/5 in NGC 5055, the mass out to the last measured point changes by less than 10 per cent, and the projected density by less than 13 per cent. The masses which we quote nearly always refer to the mass out to the last measured point.

B. Lindblad: In barred spirals there are often indications that the bar is made up by a central mass and symmetrical condensations at the side. These may indicate whorl motions which would be of great interest for investigation, if possible, by determining the variation of motion at right angles to the bar.

Prendergast: It would, indeed, be useful to know more about the velocity field in barred spirals, but this information is very difficult to obtain.

Page: By what factor can the total mass of a galaxy be changed when observations farther from the nucleus are added to the velocity curve?

Prendergast: This depends on the galaxy; in the best cases, I should think, by considerably less than a factor of 2. For some galaxies we have estimated this effect by computing the distance required to double the mass if the density remained constant beyond the last observed point. If the velocity curve reaches a distinct turnover, this distance is usually well beyond the point where the galaxy fades out on long-exposure direct plates.

Thackeray: Could the width of the slit seriously affect the derived rotational velocities at the critical turnover point when the long slit lay along a rather narrow major axis?

E. M. Burbidge: I do not think it likely that the observations could be seriously in error.

Ambartsumian: What are the nature and size of the inconsistency you have obtained in the case of NGC 1365?

Prendergast: Six rotation curves have been obtained in five different position angles, and all are apparently linear within 20″ of the center. However, the slopes of these curves are not at all compatible with the hypothesis that the velocity field is axisymmetric.

Schmidt: Concerning the questions by McVittie and Page about the accuracy of mass determinations, I would like to say that, for M 31, a model of infinite mass could be constructed which would fit all the available 21-cm. data on rotation.

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Dynamics of Regular Galaxies

The perfect symmetry and regularity of elliptical galaxies suggest that these systems have reached some kind of steady state. In general, the steady state in an elliptical galaxy will differ from that which we find in a mass of gas. In the latter case, the velocity distribution of the particles is determined by collisions, and will be Maxwellian. Usually, in a galaxy the effect of encounters between individual stars cannot have been important. Thus, the velocity distribution of the stars will be largely determined by the initial conditions, that is, by where and with what initial velocities the stars were formed.

We may ask how elliptical galaxies as well as central bulges and halos of some spiral galaxies have acquired their regularity. As one possibility, we might imagine that they obtained it when they were still in a gaseous state before the formation of stars had begun [1]. In that case, the original mass of gas must have had a very high temperature, at least several million degrees for the giant ellipticals. It is uncertain whether star formation could have occurred in a medium of such high temperature. The other, perhaps more probable, alternative is that, at the time the stars were formed, no equilibrium distribution had yet been reached by the gas. The present regularity would then have been brought about by subsequent mixing of the stars.

Whatever the *cause* of the regularity, the *present* configuration is certainly kept up by the motion of *stars*. The velocity distribution of the stars will not ordinarily resemble the Maxwellian distribution of the gas particles. I do not think that it can be inferred from theoretical considerations; it has to be found from observation.

Before continuing this discussion, we should outline the purpose of our investigation. This may be set forth by the following questions:

(*a*) Why are elliptical galaxies built the way they are? Why is the pattern of the light-distribution approximately the same in all these nebulae, and why does it follow the law $I = I_0 (1 + r/a)^{-2}$ first given by Hubble (or the alternative formula,

$$I = I_0 \, e^{-\sqrt[4]{r/a}},$$

proposed by de Vaucouleurs)?

(*b*) How can we determine the distribution of mass density within an elliptical galaxy and in the amorphous parts of spirals?

The first problem has not been solved. It is probably intimately related to the origin of the galaxies. I shall not discuss it here, but will instead concentrate on problem (*b*). We shall deal separately with spherical and flattened systems.

SPHERICAL SYSTEMS

If u, v are the radial and transverse components of the velocity, $\phi(r, u, v)$ is the normalized velocity distribution at a distance r, and $\nu(r)$ the density, we have in a steady state the following relation between ϕ and ν at a distance r_2 and the same functions at r_1

$$\nu(r_2)\, \phi(r_2, u_2, v_2) = \frac{r_1}{r_2}\, \nu(r_1)\, \phi(r_1, u_1, v_1). \qquad (1)$$

The relation holds only for stars whose orbits reach both distances considered. The variables, r_2, u_2, v_2, are connected with r_1, u_1, v_1 by the following relations

$$v_1 = v_2\, r_2/r_1, \qquad (2)$$

$$u_1^2 = 2\,\{\Phi(r_2) - \Phi(r_1)\} + u_2^2 + v_2^2\,(1 - r_2^2/r_1^2), \qquad (3)$$

where Φ is the potential.

In general, we do not know the relation between the potential Φ and the density ν. If $\phi(u, v)$ were accurately known for various r, we might obtain a solution for Φ from these data. But, with present observational limitations, we must, in most cases, content ourselves with a very rough knowledge of the radial-velocity distribution in a small cylinder passing through the centre.

If we wish to make an estimate of the mass density from existing data, we have to make some assumption concerning the density distribution. The simplest working hypothesis is to assume that bright and faint stars have identical density distributions and that the interstellar gas can be neglected. In this case, the distribution of mass density is the same as that of the light. The potential can then be determined from

$$\Phi(r) = -4\pi G \frac{M}{L} \int_r^\infty r^{-2} \int_0^r r^2 \nu(r)\, dr, \tag{4}$$

where M/L is the mass-to-light ratio.

Because of the lack of sufficient observational data, we also have to make an assumption concerning the velocity distribution. In particular, it will be important to know whether this is nearly isotropic, or whether there is a great preponderance of radial motions, such as would occur if the stars had condensed from a regular gas sphere. We can do no better at present than to try both extremes.

From a computational point of view it is practical to represent the velocity distribution in any one coordinate as a sum of Gaussian distributions. For the radial components, u, we would thus have for a given value of r, say r_1:

$$\frac{1}{\sqrt{\pi}} \sum_i \theta_i h_i e^{-h_i^2 u^2}, \tag{5}$$

where $\sum_i \theta_i = 1$.

For the distribution of transverse velocities, v, we would similarly have

$$2 \sum_i \theta_i k_i^2 v e^{-k_1^2 v^2} \tag{6}$$

If we now consider another distance r_2, each Gaussian component will contribute the following amount to the density at that distance

$$\frac{\theta_i \nu(r_1) \{k_i(r_1)/h_i\}^2}{1 + [\{k_i(r_1)/h_i\}^2 - 1]\,(r_2/r_1)^2}\; e^{-2h_i^2\{\Phi(r_2) - \Phi(r_1)\}}\, \frac{c_i(r_2)}{c_i(r_1)}. \tag{7}$$

In this expression $c_i(r)$ is the factor by which the density is reduced due to the fact that the velocity distribution is cut off at the velocity of escape. The ratio $k_i(r_1)/h_i$ is a parameter which determines the elongation of the velocity ellipsoid in the direction of the radius vector.

If there is no preference for radial motions, the expressions can be written as

$$\theta_i \nu(o)\, e^{-2h_i^2\{\Phi(r)-\Phi(o)\}} \cdot \frac{c_i(r)}{c_i(o)}. \tag{8}$$

In case there is one Gaussian component only, and if the cutoff factors are neglected, this gives the density distribution in an isothermal sphere.

We can now use the following procedure to determine the value of M/L and to find out whether the structure of the galaxy is compatible with the various assumptions made.

We first try an isotropic velocity distribution and choose values of θ_i, h_i and M/L such that the observed density distribution can be reproduced as the sum of the expressions (8) for all values of the index i, and such that, at the same time, we obtain the observed value for the average velocity dispersion in a cylinder passing through the centre. As starting point for the computations we may take the centre of the cluster; the assumed values for θ_i should then refer to an element of volume at the centre.

We should subsequently make a similar analysis with elongated velocity distributions. In practice, it may be sufficient to choose one or two values for the parameter $k(r)/h$ at a given distance from the centre and take the same ratio for all values of the index i. The value for M/L will evidently come out smaller than in the case of an isotropic velocity distribution.

So far, no complete computations have been made for a spherical galaxy.

One difficulty in applying the above formulas is that we do not know whether a system which is observed to be circularly symmetrical is spherical; it might be a flattened system observed face-on. Statistically, however, it has been found that a sensible fraction of the round systems must have practically spherical symmetry. Knowledge of velocity dispersions in a number of round galaxies might, therefore, give us at least an approximate determination of M/L and at the same time give an indication about the general form of the velocity distribution.

From the theoretical side, we might also try to find the most general type of velocity distributions for spherical systems which are in dynamical equilibrium and, at the same time, satisfy Poisson's law. Analyses of this kind have been made by Camm [2] and Kurth [3].

FLATTENED SYSTEMS

We shall assume that the flattened systems have rotational symmetry and are symmetrical with respect to their equatorial planes. We are again faced by the difficulty that we have no a priori knowledge of the inclination of the nebular equatorial plane on the plane of the sky. However, for true elliptical galaxies, there is a lower limit to the axial ratio, which is about 0.3 according to Hubble. The lower limit depends somewhat on the exact definition of an elliptical galaxy. The elliptical galaxies which appear flattest are presumably galaxies seen practically edge-on. Assuming this to be so, we can derive the space distribution of the light. In addition, we can determine the apparent rotational velocity at various distances from the centre and the velocity dispersion for a cylinder passing through the centre. If observations of the rotation are available over a sufficiently large interval of r, we can use them in conjunction with the data on the space distribution of light to obtain the true average rotation as a function of the actual distance from the centre.

The rotational velocities Θ_o differ from the circular velocities Θ_c corresponding to the gravitational field because of the dynamical pressure gradient. The difference is obtained from the following relation

$$\Theta_c^2 - \Theta_o^2 = \overline{\Pi^2}\left[-\frac{\partial \log \nu}{\partial \log \varpi} - \frac{\partial \log \overline{\Pi^2}}{\partial \log \varpi} - \left\{1 - \overline{(\Theta - \Theta_o)^2}/\overline{\Pi^2}\right\}\right]. \quad (9)$$

Here, ϖ is the distance from the axis of symmetry, Π is the velocity in a direction perpendicular to this axis and to Θ, Θ is the velocity in the direction of rotation. Bars indicate averages.

Throughout a large part of an elliptical galaxy, ν varies approximately as ϖ^{-3}, so that $\partial \log \nu/\partial \log \varpi$ is about -3. If the distribution of Π were Gaussian, $\overline{\Pi^2}$ would be independent of ϖ, and the second term is zero. The actual value of the term is hard to predict, but it is likely to be much smaller in absolute value than the first term. In a first approximation, it might be neglected. The last term within the square brackets depends on the way in which the rotational velocity varies with ϖ. We may roughly estimate that, in an actual galaxy, $1 - \overline{(\Theta - \Theta_o)^2}/\overline{\Pi^2}$ would be between 0.4 and 0.8. Adopting an average value, we can now use (9) to derive Θ_c from the measured velocities of rotation, if the velocity dispersion, $\sqrt{\overline{\Pi^2}}$, is known from observation.

It is quite conceivable that with suitable assumptions for the velocity distribution a three-dimensional model of a rotating elliptical galaxy could be constructed in an analogous manner as was sketched for the spherical case. We might again start from the working hypothesis that the distribution of mass density is identical with that of the light, so that the complete potential function $\phi(\varpi, z)$ could be computed except for a constant factor M/L. We could then try to find a velocity distribution which would reproduce the observed densities. The computations would be complicated because we would have to average over three-dimensional orbits. For this reason, in practice, it might prove to be extremely difficult to obtain a reasonably good model. However, such a model, once obtained, would furnish reliable values directly for the last two terms in formula (9).

For the present, we shall have to limit ourselves to the very rough estimates for these terms given above. We can still compare the values of Θ_c so derived with the values corresponding to the gravitational forces computed from the light distribution on the assumption of the equality of the distribution of mass and light. The comparison yields average values for M/L for parts of the galaxy extending to the various distances at which the rotation has been measured and can, therefore, serve to obtain a rough check on the constancy of the ratio.

Once the potential $\phi(\varpi, z)$ is known, the distribution of the light-density in the z-direction can be used to determine the velocity distribution in this coordinate from the condition of a steady state. Strictly, we would have to consider three-dimensional orbits, but a useful approximation can probably be obtained by treating the z-motions as independent of the random motions in the other coordinates.

The only elliptical nebula for which accurate measurements of rotation at various distances from the centre have been made is NGC 3115. These measurements were made by Minkowski, who has, likewise, measured the velocity dispersion and the variation of rotation with distance from the major axis so that the full analysis described above can be carried out. The outcome of a still somewhat provisional discussion follows.

Up to the limit of the rotation measures, at about $\varpi = 80''$, corresponding to 2500 pc (if the distance is 6.5×10^6 pc, as computed from the radial velocity with a value of 75 km/sec/Mpc for the Hubble constant), the data can be well represented by a constant value of 18 for M/L.* An interesting feature is that the curve giving the run of Θ_0 with ϖ shows a rather deep minimum around $\varpi = 1200$ pc, where Θ_0 is only one half of the maximum near $\varpi = 500$ pc. The rotation curve has a second maximum at a much larger distance which cannot yet be fixed, but which may be expected to occur near 3000 pc.

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*Further discussion of the observational material, which was only partly completed at the time of the Symposium, has indicated that though the value of M/L is sensibly constant over the entire brighter part of NGC 3115, it increases strongly beyond $\varpi = 2$ kpc. As a consequence, it appears now that the ratio of total mass to total luminosity must be at least 50. Therefore. it comes close to the results which Page has derived from observations of double elliptical and S0 galaxies.

The dispersion of the z-components of the motions is computed to be about 166 km/sec at $\varpi = 500$ pc, and 123 km/sec at $\varpi = 1000$ pc. These values may be compared with the dispersion of 200 km/sec observed for the *radial* motions.

It is of interest to give an estimate of the times of revolution and relaxation in a typical E7 galaxy like NGC 3115. With the data used in the above analysis, we find that the average time of revolution varies from 6 million years near $\varpi = 300$ pc to 40 million years at $\varpi = 2000$ pc. The number of revolutions since the formation of the galaxy must, therefore, have been of the order of 1000. It is not surprising that a fair degree of regularity has been reached after such a number of revolutions.

The time of relaxation depends on the luminosity function in the galaxy. If we assume, somewhat arbitrarily, that the average mass of the stars, weighted in proportion to their contribution to the energy transfer, is one quarter of the sun's mass, the time of relaxation at the centre would be 2×10^{12} years; at 320 pc from the centre it becomes 2×10^{14} years, while for still larger distances it increases further in approximate inverse proportion to the density. These relaxation times are so much longer than the presumed age that it would appear that the effect of encounters can be safely neglected.

The same conclusion holds for the giant *spherical* galaxies, of which NGC 4486 and 3379 may be specimens. For smaller elliptical galaxies, like the companions of the Andromeda nebula, the relaxation times may be between 10^2 and 10^3 times lower than for corresponding points in NGC 3115. The factor might even become 10^4 for the intrinsically faintest Sculptor systems. In the central parts of these systems, the velocity distribution may, therefore, have been shaped by stellar encounters.

For an S0 galaxy, the interpretation of the data is simpler than for an elliptical galaxy. The extremely flat shape indicates that the random motions in the z-coordinate are small, while the "steps" in the light distribution along the major axis may well be a sign that the other components of the random motions also are small compared with the rotational velocities. If this is so, the differences between Θ_0 and Θ_c will be small, and the uncertainty in the computation of Θ_c from Θ_0 may be negligible.

Another simplification is that, in systems of such flatness, the z-motions may be expected to be practically independent of the motions parallel to the equatorial plane which greatly simplifies the dynamics.

In spiral galaxies, it will often be possible to determine the rotation from emission lines as well as from absorption lines. The former are due to gaseous nebulae situated close to the equatorial plane and having small average random motions, while the latter arise in the more or less smooth central bulge where, in general, the random velocities are considerable. Although usually there are few emission nebulae in the parts where the central bulge is strong enough to allow measurement of absorption lines, there may be galaxies where an overlap occurs. In such cases, a comparison between the value of Θ_c determined from Θ_0 with the aid of formula (9) and the value determined more directly from the emission lines may afford an interesting check on the mean random motions used in (9). It should be noted, however, that the circular velocity of the gas may differ from that derived from stars because it is influenced by magnetic fields. In principle, it might even be possible in a few cases to obtain an indication of the magnetic pressure gradients from comparisons of this kind.

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DISCUSSION

Gold: Is the conclusion of long relaxation times for star-to-star encounters really the important one? A more clustered distribution would have much shorter relaxation times – the star-to-cluster and cluster-to-cluster encounters then being important. A less smooth distribution than these galaxies now have would have resulted in a relaxation towards the smoothness now observed.

Hoyle: It seems unlikely, I feel, that stars would condense singly. Separate condensation would require very low temperatures indeed. Rather would one expect temperatures in the region of 10^4 °K, or even higher in view of the large internal velocities that must have developed. My point is that at 10^4 °K, the minimum mass that could begin to condense would be of the order of that of a globular cluster. If the initial condensations contained perhaps 10^6 stars, relaxation times would, of course, be much reduced. One would expect the initial condensations to be destroyed by encounters until the time of relaxation became as long as the age of the galaxy.

Oort: Purely statistical fluctuations in density have been taken into account in the derivation of the relaxation times. It might be that there are still important real groupings of stars which are sufficiently large to affect the relaxation, but there is no direct evidence of such groupings.

I agree with Hoyle that large condensations may well have been important in the first stages of the evolution of an elliptical galaxy and that these may have been instrumental in producing the observed regularity of these galaxies.

Zwicky: A disturbing new preliminary observation is that of H and K lines far out from an EO galaxy which indicate a relative radial velocity of 1000 km/sec from one end to the other.

Mayall: Globular clusters in M 31 are known out to a distance of nearly 2°.5 from the nucleus.

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Theories of Spiral Structure in Galaxies

The appearance of spiral structure in galaxies of great flattening depends on mass motions in the systems which, from the observational point of view, are of unknown nature. The fact that the systems of spiral structure form a natural prolongation of a series of closed elliptical types of increasing degree of flattening suggests that the mass motions in question may be due to some sort of instability which appears at a certain average value of angular momentum per unit mass, and, consequently, at a certain degree of general flattening. The spiral phenomenon may also be due to a change of physical properties of the systems with increasing degree of flattening, in particular, the growth of mass of a subsystem of interstellar gas and dust close to the equatorial plane which is observed to be intimately connected with the appearance of spiral structure. It is, of course, possible to combine these points of view, which are by no means mutually exclusive.

We shall not consider here early gravitational theories which interpret spiral structure as due to orbital motions of stars starting from a small nucleus, as it is fairly evident that they are not reconcilable with modern evidence concerning the physical nature of the systems. Among earlier theoretical concepts which may still have a certain relevance, we may mention the theory of E. Brown [1] which interprets the observed spiral arms as the envelopes of showers of individual orbits. We may also mention the circumstance [2] that the spiral shape appears at a degree of flattening which corresponds to the limit of "ordinary" stability of a homogeneous MacLaurin ellipsoid against the bisymmetrical sectorial harmonic "Riemann wave." It may further be mentioned that circular orbits in the equatorial plane outside of, but close to, the edge of a rotating ellipsoid of high flattening are unstable, and that circular orbits inside of, but close to, the edge need only a slight individual change in energy in order to be transformed into quasi-asymptotic orbits extending very far from the system. Quasi-asymptotic orbits of this kind may also occur in the case of sufficiently rapid internal density gradients in the equatorial plane [3].

Theories of mass motions in systems deviating slightly from formations of a steady state may give valuable hints concerning the basic phenomena underlying the spiral formation. The gravitational theory of internal density variations has been treated by B. Lindblad [4], Coutrez [5], Langebartel [6], Lindblad and Langebartel [7]. The most important modes of density variation in a region where the angular velocity of rotation does not change rapidly with the distance from the centre appear to be of the type

$$D_1 = A_s D_0 \left(\frac{r}{a}\right)^s \cos(\sigma t + s\theta), \quad s = 1, 2, 3, \ldots .$$

where D_1 is the total variation of the mass D in cylinders of unit cross-section with axes parallel to the z-axis, and where D_0 is the value of D in a steady state, r and θ are polar coordinates in the equatorial plane in a coordinate system following the angular speed of rotation, a is the "effective" radius in the equatorial plane and A_s is a constant. The conditions for instability have been investigated for the waves $s = 1, 2, 3$. The greatest interest attaches to the wave $s = 2$ because it tends to explain the formation of barred spirals. The density variation is accompanied by the development of four whorl motions. Two of these whorls, which occur in the direction of rotation, are situated on maxima of D_1/D_0 on one and the same diameter, symmetrically with respect to the centre at the points $r = 0.775a$. The two other whorls, which occur in the other direction, are situated on minima of D_1/D_0 at $r = 0.775a$ on a diameter at right angles to that of the whorls of positive D_1. The two maxima of D_1/D_0, together with the central condensation, give an immediate explanation of the phenomenon of the "bar." The disturbances due to the four whorls on the motions in a surrounding ring structure explain in a qualitative way the development of spiral structure.

If we consider separately the mass motions in the gaseous substratum, we have to take into account, in principle, besides the gravitational force due to the system as a whole, the hydrodynamic and electrodynamic forces in the gas. The difference in this respect between stellar and gaseous matter causes a great complication in the dynamical problem. It is, therefore, necessary to attack the problem from different sides simultaneously.

In a recent paper, J. Tassoul [8] has gone to an extreme point of view by regarding a stellar system as composed of a thin, completely ionized gas, in which the stars and the gaseous nebulae form and are forced to follow in the rotational motion. The density waves which are possible according to the equations of mass motion are in many ways analogous to those arrived at in the purely gravitational case. In a qualitative discussion of the origin of spiral structure, Tassoul starts from a density wave in which matter concentrates towards a certain diameter to form a "bar." It appears that electromagnetic fields along circles in the equatorial plane about the centre do not essentially affect the density waves considered.

Attention must be paid in this connection to the "turbulence theory" of spiral structure developed by Heisenberg and von Weizsäcker [9]. In the exceedingly thin, slowly rotating, interstellar gas there will appear a hierarchy of turbulence elements and in the outer regions a general expansion according to this theory. Some such turbulence elements may form clouds which disperse by differential rotation and form the spiral arms. Also, in this case, von Weizsäcker refers to a bar formation as a possible original feature of the spiral structure.

In order to create a more definite theory of the development of spiral structure, it is necessary to consider in more detail the statistics of stellar motions and the mass motions in a typical stellar system.

We accept on broad lines the division of the mass of stars into the two population groups introduced by Baade [10] which, from the dynamical point of view, is intimately connected with the theory of rotating subsystems. According to modern ideas, we assume that the stars of population II, which is more widely spread with respect to the equatorial plane, and which has a strong concentration towards the centre of the system, were formed from gaseous matter at an early date before the present phase of the system. The stars of this population group have a strong internal velocity dispersion and, on the whole, somewhat lower mean angular speed of rotation about the centre than the stars of population I, which are mainly confined to the region about the galactic plane. Gas and dust in clouds which have a wide range in size are associated with population I. Various mechanisms have been suggested by which the comparatively young stars of population I have been created from such clouds. These stars are formed in clouds or associations with a velocity dispersion sufficient to make these formations dissolve in time. We can also imagine that a vast cloud of gas fragments into smaller clouds with a certain velocity dispersion.

For a general survey of stellar motions in the equatorial plane, it is convenient to study the "epicyclic" motions in a coordinate system ξ, η, the origin of which follows the circular angular motion ω_0 at a certain selected distance R_0 from the centre [11]. The motion in the coordinate $\xi = R - R_0$ may be described in the first approximation as an harmonic oscillation $\xi = c_1 + c \cos \varkappa (t - t_0)$, where $\varkappa = 2\sqrt{\omega(\omega - A)}$, $A = -\frac{1}{2} R \frac{\partial \omega}{\partial R}$. The frequency distribution may be expressed by the four parameters c, c_1, t_0, θ_0, where θ_0 is the value of θ for $t = t_0$, that is for the apcentrum of the orbit.

A frequency distribution of greater physical significance may be obtained starting from orbits which are called "dispersion orbits" [12]. We arrive at these orbits by introducing a coordinate system with angular speed ω'; where

$$\omega' = \omega - \frac{1}{n} \varkappa ,$$

with

$$n = \frac{d\varkappa}{d\omega} .$$

If the angular coordinate in this system is denoted by θ'; we have to the first order

$$\xi = c_1 + c \cos n (\theta' - \theta'_0) .$$

For neighbouring orbits which differ slightly as to the value of c_1 there will be a differential motion in θ'; but the orbits will stay close together. The members of a dissolving association will scatter along the dispersion orbit of the mean motion of the association. The dispersion orbits will be closed orbits if n is a rational number, and they will have a specially simple form if n is an integral number. It now appears from the $\omega, \varkappa$-diagrams of various systems that in the outer regions of a system we have closely $n = 1$, which is the value of n in a Newtonian field of force, whereas, in the inner parts, constituting the "main body" of higher density, n is remarkably close to 2. These results are borne out [12, 13] by the function $\omega(R)$ in the galactic system derived from observations of the 21-cm. line by Kwee, Muller, and Westerhout [14] and by Schmidt [15]; further, from observations of the 21-cm. line in M 31 by van de Hulst, Raimond, and van Woerden [16], and from the velocity curve in M 81 observed by Münch [17].

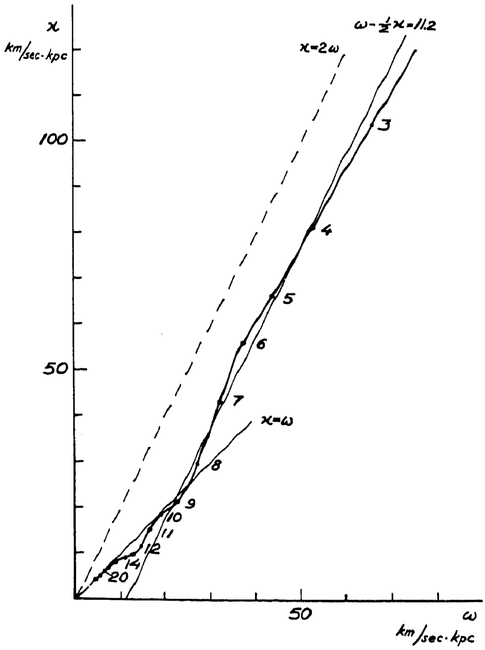


Figure 1. Relation between ω and $\varkappa$ in Schmidt's model of the Galaxy.

We shall consider here specially the case when n is close to 2. For individual stars in such a region, we introduce the ‘‘special

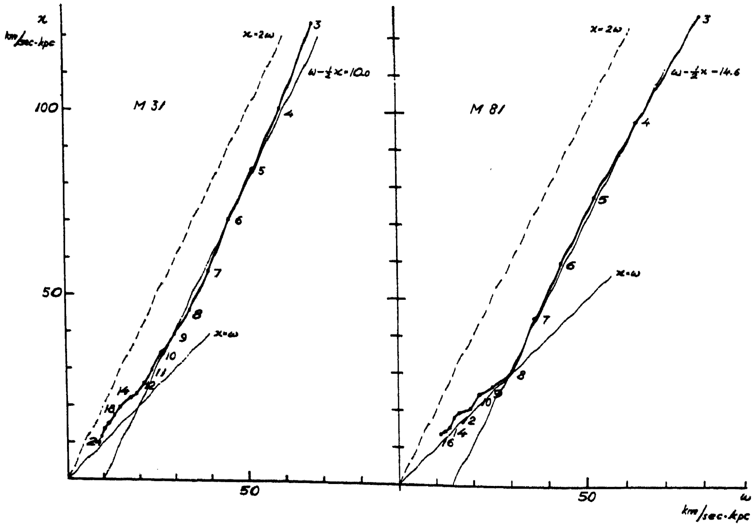


Figure 2. Relation between ω and $\varkappa$ in M 31 and M 81.

orbits" which are defined in individual coordinate systems of angular speed

$$\omega'_s = \omega - \tfrac{1}{2}\varkappa .$$

These orbits are closed bisymmetrical curves of quasi-elliptical form. In order to define a unique coordinate system in the region in question, we choose the angular velocity ω'_0, as a mean of ω'_s defined by

$$\omega'_0 = \omega_0 - \tfrac{1}{2}\varkappa_0 ,$$

where ω_0 and $\varkappa_0$ are the means of ω and $\varkappa$, respectively, in the region about $R = R_0$. The angular coordinate θ' in this coordinate system is given by

$$\theta' = \theta + \tfrac{1}{2}\varkappa_0 t,$$

where θ is the angular coordinate in the system following the circular motion with angular speed ω_0. Starting from the equations for the "epicyclic" motions in the (ξ,η)-system of angular speed ω_0, we find for the orbital motion in the system of angular speed ω'_0, if we write for the longitude θ'_1 of the apsidal line of a special orbit,

$$\theta'_1 = \theta'_0 + (\omega'_s - \omega'_0)\,(t - t_0),$$

$$\xi = c_1 + c\cos 2\,(\theta' - \theta'_1),$$

$$\theta' - \theta'_1 + \frac{c}{R_0 + c_1}\sqrt{\frac{\omega}{\omega - A}}\,\sin 2\,(\theta' - \theta'_1) = \frac{\varkappa}{2}(t - t_0) .$$

We get, further, for the components of motion u,v *relative to the circular motion* at the point where the particle is situated at the time t

$$u = -c\varkappa\sin 2\,(\theta' - \theta'_1)$$

$$v = 2cB\cos 2\,(\theta' - \theta'_1),$$

where

$$B = A - \omega .$$

The constant parameters in this motion are c, c_1, θ'_0, t_0, and we may express the general frequency distribution by

$$f(c, c_1, \theta'_0, t_0)\,d\xi\, R_0\, d\theta'\, du\, dv.$$

We can choose as parameters, instead of θ'_0 and t_0, the expressions $\theta'_0 - (\omega'_s - \omega'_0)\, t_0$ and $\theta'_0 - (\omega'_s - \omega'_0)\, t_0 - \frac{1}{2}\,\kappa\, t_0$, which are independent of t. This gives by the expressions above for θ' and θ'_1 for the frequency function

$$f\,[c,\, c_1,\, \theta'_1 + (\omega'_0 - \omega'_s)\, t,\; \theta + \frac{u}{2BR_0} + (\omega_0 - \omega)\, t]\,.$$

The parameters $c,\, c_1,\, \theta'_1$ are given by

$$c^2 = \frac{u^2}{\kappa^2} + \frac{v^2}{4B^2}, \qquad c_1 = \xi - \frac{v}{2B}\,,$$

$$c \sin 2\theta'_1 = \frac{u}{\kappa} \cos 2\theta' + \frac{v}{2B} \sin 2\theta'\,,$$

$$c \cos 2\theta'_1 = -\frac{u}{\kappa} \sin 2\theta' + \frac{v}{2B} \cos 2\theta'\,.$$

By writing f as follows

$$f\Big\{c,\, c_1,\, c \sin 2\big(\theta'_1 + (\omega'_0 - \omega'_s)t\big),\; c \cos 2\,\big(\theta'_1 + (\omega'_0 - \omega'_s)t\big),$$
$$\theta + \frac{u}{2BR_0} + (\omega_0 - \omega)t\Big\},$$

and limiting ourselves to functions containing u and v in the first and second order, we obtain the most general form of an ellipsoidal distribution with vertex deviation [18].

In the case of the population of clouds of gas and dust, it is, of course, possible to construct on broad lines a similar frequency function for the clouds within various classes of individual mass. The circumstance that intersections between special orbits carrying such clouds will lead to collisions with sudden changes of the relative velocities and loss of kinetic energy means a tendency of the clouds to approach towards circular motions. On the other hand, according to Spitzer [19], the internal velocity dispersion of the clouds may be strongly stimulated by the field of radiation due to highly luminous O and B stars. As to local electromagnetic fields, we may assume that these occur mainly along the dispersion orbits due to the differential motion in neighbouring orbits. Such fields should not interfere perceptibly with the free motion along the special orbits.

We may now consider the entire mass of population I moving in the outer parts of the system. The gravitational force of this mass will, to a great extent, form a part of the general field of rotational symmetry, which may, in general, be supposed to be dominated by the mass of population II. In view of the general expression for f given above, we must assume, however, that the gravitational field due to population I will, to some extent, be variable with the angular coordinates θ' and θ, and, at the same time, slowly variable with the time. The "regular" component of rotational symmetry produced by population types I and II together may be formed as a mean of the radial force for different θ' for any given R and as a mean over a long interval in t. The difference between the actual force and this mean radial force at any given point is considered as the disturbing force at this point. We can now develop the disturbing force along a circle of radius R by Fourier series, thus

$$K_\xi = C_0 + \sum_\nu C_\nu \cos \nu\theta' + \sum_\nu C_\nu' \sin \nu\theta' ,$$

$$K_\eta = D_0 + \sum_\nu D_\nu \sin \nu\theta' + \sum_\nu D_\nu' \cos \nu\theta' ,$$

$$\nu = 1, 2, 3, \ldots .$$

Here C_0, D_0 and the coefficients $C_\nu, D_\nu, C_\nu', D_\nu'$ are in the general case variable with the time. These coefficients are valid for the circle of radius R in the equatorial plane. If they are small quantities and if the same is true for their first derivatives in ξ and η, we can assume that the development is valid also for a closed curve adhering closely to the circle in question.

The variation with time of the coefficients in the development of K_ξ and K_η according to θ' will, of course, be partly due to a dependence on θ, as this angle enters explicitly into the frequency function f. The ω, $\varkappa$-diagrams indicate that the variation with R of the angular motions ω_s' of the special orbits in the main body of the system is considerably smaller than the corresponding variation of the angular speed ω of the circular motions. The explicit variation of f with θ will, therefore, tend to be smoothed out by the differential rotation. We can, consequently, at this point consider the variation with θ to be of accidental nature, and try to follow up separately the "secular" disturbances due to an uneven distribution of the longitudes θ_1' of the apsidal lines of the special orbits.

If F is the regular central force, the differential equations for the disturbances in ξ and η when the (ξ, η)-system is assumed to follow the circular motion ω at R (in this case not ω_0 at R_0) are

$$\frac{d^2\xi}{dt^2} - 2\omega \frac{d\eta}{dt} - \omega^2 \xi + \frac{\partial F}{\partial R} \xi = K_\xi ,$$

$$\frac{d^2\eta}{dt^2} + 2\omega \frac{d\xi}{dt} = K_\eta .$$

We suppose that the disturbing force K proceeds with the angular speed ω'_0, which we take here to be the mean angular motion of the apsidal lines of the disturbing special orbits, and define

$$\frac{\delta}{2} = \omega'_0 - \omega'_s \ , \ \frac{\sigma}{2} = \omega - \omega'_0 ,$$

and then have, because $\omega'_s = \omega - \frac{1}{2}\varkappa$,

$$\delta = \varkappa - \sigma .$$

Measuring here θ in the system of angular speed ω and θ' in the system of angular speed ω'_0, we have

$$\theta' = \frac{\sigma}{2} t + \theta .$$

The value of $\varkappa$ is given by

$$\varkappa^2 = 3\omega^2 + \frac{\partial F}{\partial R} .$$

The bisymmetrical properties of the special orbits which together produce the "secular" disturbing force give, as a natural consequence, a prevalence of terms of even ν in K_ξ and K_η . A steady distribution of matter along a special orbit gives for the matter μ per unit length

$$\mu = \mu_0 \left[1 + k \ \frac{c}{R} \ \cos 2 \ (\theta' - \theta'_1) \right] ,$$

where

$$k = \frac{4\omega}{\varkappa} - 1 .$$

When $\omega'_0 - \omega'_s$ is numerically small, $\varkappa$ is close to σ, and we shall have resonance for those terms in K_ξ and K_η in which $\nu\,\sigma/2$ is close to $\varkappa$. This shows, because $|\varkappa - \sigma|$ is small, that the most important terms are those with $\nu = 2$.

Let us suppose that C_2 and D_2 increase exponentially with the factor $e^{\gamma t}$, where γ is small. The disturbance will then be zero for $t = -\tau$ where τ is large. It may be shown [20] that if we divide the terms of $\nu = 2$ into two components, thus

$$(K_\xi)_{\nu=2} = C \cos 2 \ (\theta' - \phi_1) + C' \cos 2 \ (\theta' - \phi_2),$$

$$(K_\eta)_{\nu=2} = D \ \sin 2 \ (\theta' - \phi_1) + D' \sin 2 \ (\theta' - \phi_2),$$

we can choose ϕ_1 so that we have

$$C' - \frac{2\omega}{\sigma} D' = 0, \quad \phi_2 = 45^\circ + \phi_1.$$

When $|\varkappa - \sigma|$ is small, we then find the following approximation for the disturbances in ξ and η; counting θ' from the direction ϕ_1 and setting $\theta' = \frac{\sigma}{2} t + \theta$.

If $|\varkappa - \sigma| < \gamma$, though γ is small,

$$\Delta\xi = \left(C - \frac{2\omega}{\sigma} D\right)_0 \frac{e^{\gamma t}}{2\sigma\gamma}\left[\frac{\varkappa - \sigma}{\gamma} \cos(\sigma t + 2\theta) + \sin(\sigma t + 2\theta)\right],$$

and if $|\varkappa - \sigma|$ is small, but $|\varkappa - \sigma| > \gamma$,

$$\Delta\xi = \left(C - \frac{2\omega}{\sigma} D\right)_0 \frac{e^{\gamma t}}{2\sigma(\varkappa - \sigma)}\left[\cos(\sigma t + 2\theta) + \frac{\gamma}{\varkappa - \sigma} \sin(\sigma t + 2\theta)\right].$$

It is evident that in regions where the value of σ characterizing the disturbance approaches close to the value of $\varkappa$, there will be considerable disturbing effects tending on the average to elongate the special orbits in a certain direction. The orbits in such a region may be supposed to form a ring with elongation in this direction, partly superposed on special orbits of other orientation and of somewhat different mean distance from the centre. It may be shown [20] that such a ring will have a tendency to assimilate the intersecting orbits within its range of variation in R. It seems, therefore, that the "secular" disturbances due to the variation of the frequency f with the apsidal lines of the special orbits will have a tendency to break up the subsystem of population I into a formation of slightly elongated rings.

Other reasons for a formation of a ring structure in the subsystem of population type I may be found in a possible instability of density variations in the form of zonal harmonics depending only on R. From the empirical point of view, several features in spiral patterns of galaxies of different types strongly suggest a tendency towards rather massive annular formations in the main body of the system.

The distribution of density in a slightly elongated ring may be written [21]

$$\mu = \mu_0 \left[1 + k \frac{\xi_0}{R} \cos 2\theta'\right],$$

measuring θ' from the vertex of the ring. Here k is to be determined from the equation of continuity in the motion. The terms of $\nu = 2$

in the Fourier development of K_ξ and K_η may be written

$$K_\xi = C \cos 2\theta' \, . \; K_\eta = D \sin 2\theta',$$

where $\theta' = \frac{\sigma}{2} t + \theta$, and

$$C = -(3 + k)\, \xi_0 Q, \quad D = -4k\, \xi_0 Q,$$

and

$$Q = G\mu_0 \frac{1}{R^2} \ln\left(\frac{2R}{b}\right),$$

where R is the radius of the ring, b the "effective" radius of a transverse section of the ring, and G the constant of gravitation.

The equations of motion and the equation of continuity give [20] an equation of fourth degree in σ, thus

$$\sigma^2(\varkappa^2 - \sigma^2) = (32\omega^2 - 8\varkappa^2 - 12\omega\sigma + 6\sigma^2)\, Q - 24Q^2.$$

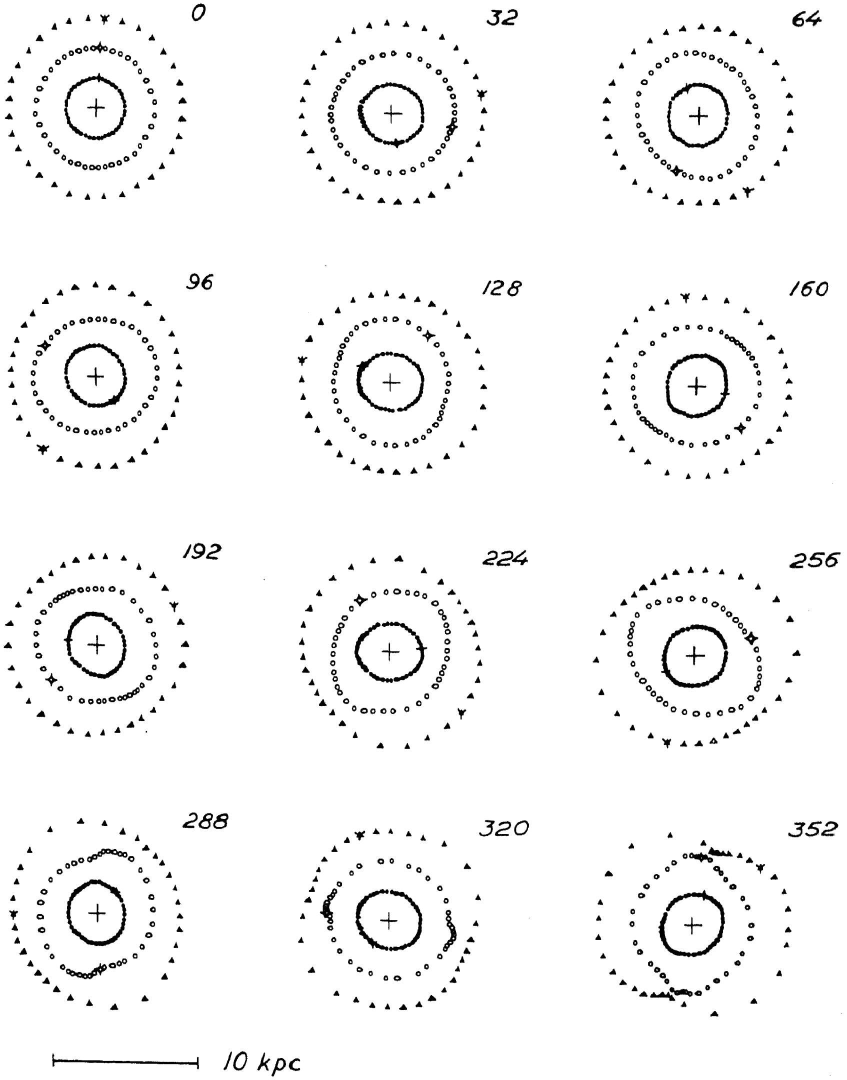
For very small Q, two of the roots σ lie close to $\pm \varkappa$, and the corresponding waves move one slower, one more rapidly, than the circular angular speed ω. The variation of μ along the orbit is comparatively small, and the waves have the character of "waves of deformation." The two other roots σ are small, and the waves follow nearly the circular motion. The density variation along the orbit is strong even for small values of amplitude ξ_0, and the waves have, therefore, the character of "density waves."

The equation in σ has some analogy to Maxwell's [22] equation for the frequency of the disturbing waves in the case of the rings of Saturn. The analogy is still stronger with the equation in σ obtained for a ring in the Newtonian field where $\frac{d\varkappa}{d\omega} = 1$. In the discussion of the stability, however, we can limit ourselves in the present case to the long waves which have principal Fourier terms of $\nu = 2$ in a region where n is close to 2, and of $\nu = 1$ in a region where n is close to 1. As we have seen, there are special reasons based on the general frequency function why "deformation waves" of this kind should arise. Corresponding "density waves" may arise because the secular disturbances of the shape of the orbits will not be immediately followed by an advance towards a steady state in the density distribution along the orbits. The difference against the normal "steady" distribution will appear as density waves propagated along a ring. The shorter waves, which play a part in the theory of Saturn's rings, will, in the case of the present ring formations, tend to be obliterated by the differential motions

along the ring. It is also possible that the presence of magnetic fields along the ring will tend to prevent the development of these waves. Such a conclusion seems to be in accordance with Chandrasekhar and Fermi's [23] results concerning stability in a longitudinal magnetic field.

For the waves of $\nu = 2$ in a region of $n = 2$ the limit of stability, at which a deformation wave and a density wave combine in an unstable formation, occurs when the total mass of the ring amounts to about a few per cent of the entire mass of the system.

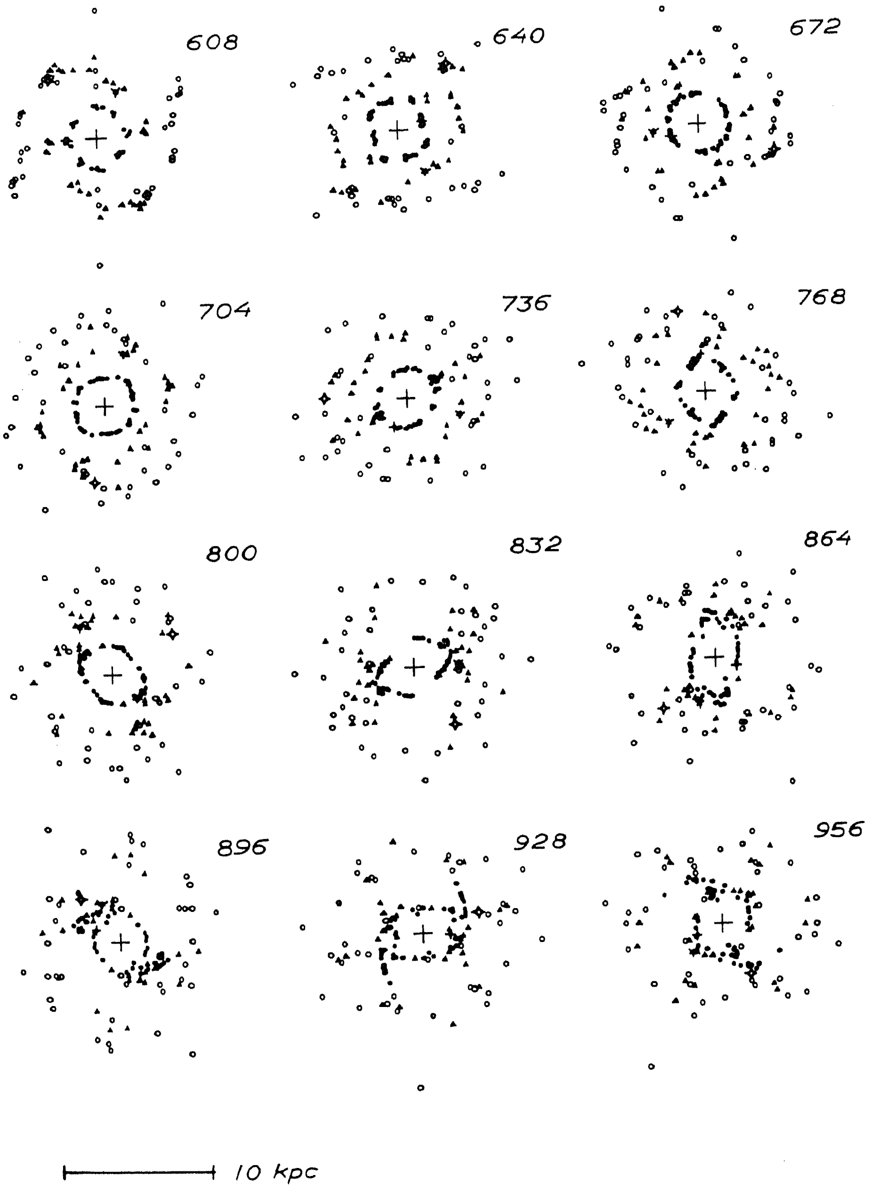
P. O. Lindblad [13] has shown by numerical computations on rings composed of mass points moving in a typical galactic field of force and acting upon each other according to Newton's law that for moderate mass a ring is effectively stable for very long time. By investigating the mutual disturbances in a system of rings, P. O. Lindblad has developed further the gravitational theory of spiral structure. The secular perturbing effects of a dispersion ring are found to be strong in a region where the angular velocity of circular motion ω equals the angular velocity ω_0' of the apsidal line of the disturbing ring. This effect appears to be of importance for the development of spiral structure in the exterior regions. The bisymmetrical pair of density waves along a ring, of which one component moves with higher angular velocity, and the other component with lower angular velocity, than the particles of the ring, will act on the surrounding matter in the equatorial layer. The mutual disturbance of neighbouring rings will lead to considerable displacement of matter and to a characteristic spiral pattern, including a bar-structure in the inner regions. The spiral arms appear at the first stages to be "leading" with respect to the direction of rotation, but become at a later stage, in a process involving a far-reaching exchange of matter between inner and outer regions, "trailing." The spiral structure finally deteriorates by differential rotation, but a central bar-structure remains for a very long time in the inner regions. Under the influence of this bar, a repeated development of spiral structure occurs in the outer regions where, after the deterioration of the first spiral structure, conditions are produced which in several respects resemble the original schematical configuration. A repeated formation of spiral structure appears in this way to be possible. An observed detail which appears in many spiral galaxies is rather sudden turns in the run of the spiral arms, which sometimes gives the pattern a shape of a polygon. This detail seems to be well reproduced in the theoretical model. For one of the typical patterns, P. O. Lindblad has assembled the series of computed pictures into a film which well illustrates the continuous transformations of the system.



(a)

Figure 3. (a), (b), (c). Computed development of a model of a galaxy. The central field of force is that of Schmidt. The total mass of the points in the three rings amounts to 10 per cent of the total mass of the system. The ring of open circles carries the original density wave. The numbers indicate millions of years.

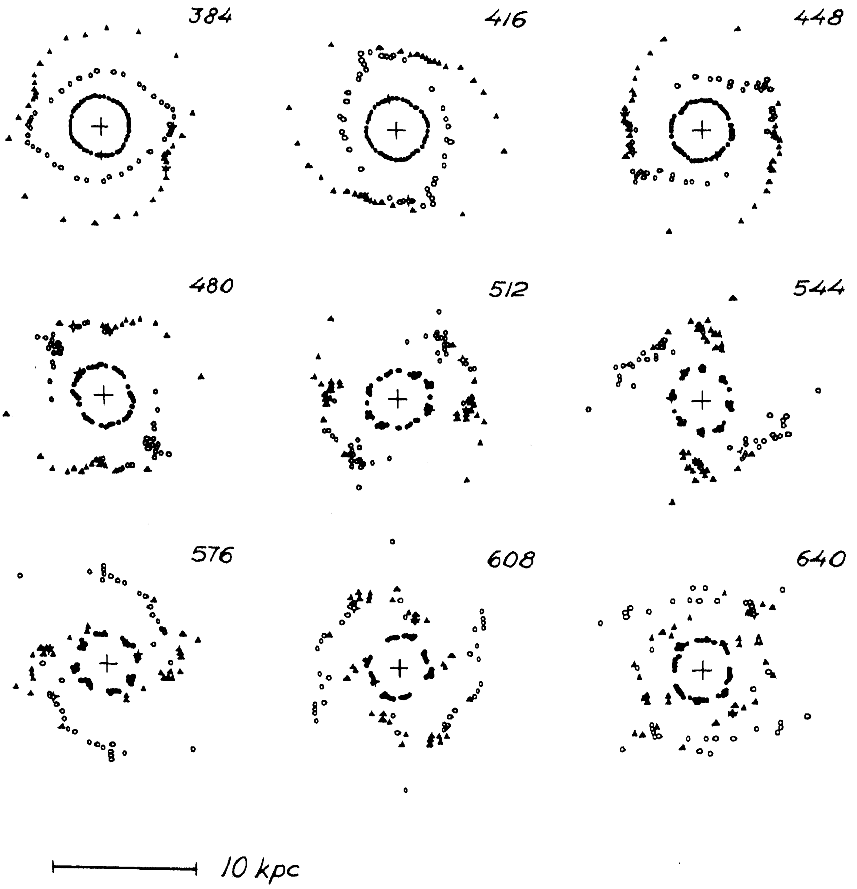
P. O. Lindblad [24] has also investigated the resonance effects to be expected in double galaxies. The secondary component will, under certain circumstances, produce an ejection of matter from a resonance region in the primary component which may reach very



(b)

Figure 3 (*continued*)

great distances. This phenomenon may tend to explain the narrow threads of matter observed by Zwicky [25] which often connect the members of multiple galaxies. Passages of galaxies with large relative velocities and with not too small minimum distance are



(c)

Figure 3 (*continued*)

shown to give only small effects on the internal structure. Pfleiderer and Siedentopf [26] have shown, however, that closer encounters with smaller minimum distance can produce very strong effects leading to extensive spiral patterns. In this case the field of force in the system was supposed to be Newtonian and the stars revolve in circular orbits around the centre. The masses of the particles were assumed to be infinitesimal. The disturbing effect is largest when the angular velocity of the particles in the undisturbed state is equal to the angular velocity at the vertex of the relative hyperbolic orbit of the disturbing system.

It appears that gravitational resonance effects are possibly sufficient to account for a great part of the process of development of spiral structure. Whether other effects contribute sensibly in the process depends on the properties of the interstellar gaseous matter. The spiral arms will contain a mixture of stars, gas and dust. If electromagnetic fields connected with the gas occur preferentially along the dispersion rings, they will tend to check individual motions of gas clouds at right angles to the ring, though at the same time the magnetic pressure will tend to widen the cross-section of the ring. On the whole, the effect may tend to preserve the shape of the spiral arms resulting from the mutual disturbances between the rings and, in this way, make the deterioration of the spiral structure slower. It may also tend to preserve a ring-shaped structure after the deterioration of the spiral and, thus, prepare for a renewed spiral pattern.

It seems to be a natural conclusion that electromagnetic fields will occur along a dispersion ring, sometimes possibly in a kind of helix about the central orbit of the ring. The magnetic lines of force will largely be "frozen" into the plasma, and will, therefore, be extended by differential motion along a dispersion orbit. On the other hand, the individual motions of gas clouds will produce local deviations of the field which can, therefore, show a complicated local structure.

The rate of dissipation of kinetic energy by encounters between clouds has been discussed by Kahn [27] and Spitzer [19]. In substantial agreement with Kahn, Spitzer derives the dissipation rate, assuming standard clouds of radius, 7 pc, and mass, 400 solar masses, with density of neutral hydrogen 10 atoms per cm^3. The average number of clouds in the line of sight per kilo-parsec is 8. A mean velocity of 14 km/sec gives a collision every 9×10^6 years, and the dissipation rate 3.5×10^{-27} erg cm^{-3} sec^{-1}. If the magnetic field is strong and the lines of force are predominantly along the ring, the transverse velocities are suppressed and the dissipation rate gets considerably diminished. According to Spitzer, the main source of power for maintaining the random motions of the interstellar clouds is the radiant energy from the O and B stars which might supply 14×10^{-27} erg cm^{-3} sec^{-1}. Earlier suggestions on this point were made by Spitzer [28], Oort [29], and Schlüter [30], who considered the dynamical consequences of the heating of H II regions by ultra-violet radiation from early-type stars. Oort, in particular, has shown how the birth of an O star may cause an outward flow of the surrounding gas.

These effects will, of course, not change the tendency of the mean motion of the clouds in a certain region to coincide with the circular motion as a consequence of the collisions. Nor will these effects prevent the clouds in the range of R covered by a dispersion ring from tending to be amalgamated with this ring. In the case of a fairly regular magnetic field along the ring, however, this process may be complicated by the additional central force caused by the magnetic field in a ring of partly ionized gas.

Due to the increase in central force by the magnetic field in a partly ionized gaseous ring, this ring will have a tendency to contract and will obtain a higher angular velocity when the centrifugal force balances the resultant magnetic and gravitational force. This idea forms the starting point of the theory of spiral formation given by A. Elvius [31]. This theory has been further developed by A. Elvius and N. Herlofson [32] and has been approached by numerical calculations by A. Elvius and P. O. Lindblad [33]. According to A. Elvius and Herlofson, the magnetic pressure in the ring may be balanced by decreasing density and higher temperature in the surrounding interstellar gas in combination with highly irregular magnetic fields in the surrounding matter. It may be remarked here that, in the case of fairly massive dispersion rings, it appears that the gravitational action of a dispersion ring as a whole can contribute effectively to balance the magnetic pressure. Stars born out of the gaseous ring will have higher angular velocity than that corresponding to circular motions in the gravitational field and will, therefore, be ejected from the ring. A possible ellipticity of the ring and consequent variation of density may cause the ejection to be localized at two opposite points. The young stars in the outgoing stage may then form the spiral structure eventually under combination of the effects for several rings of different radius. The order of magnitude of the field required is estimated to be 2×10^{-5} gauss. From these considerations we may expect that the spiral patterns produced by dispersion rings containing older stellar matter of population I, by the gaseous clouds, and, finally, by stars recently formed out of the gaseous clouds, will not be entirely congruent – which will lead to an additional complication of the structure. The detailed study of the spiral arms of the great Andromeda nebula by Baade [34] is of great importance in this connection. The often observed tendency of dark obscuring clouds to appear, preferentially on the inner, concave side of the luminous arms, would be a natural consequence of a greater central force due to the magnetic field working on the partly ionized gas in which the dust clouds are suspended, which would cause a separation of dark and luminous matter already in a closed dispersion ring. The formation of luminous early-type stars in the gaseous clouds and the separation outwards of such stars would enhance this effect, both in the case of a closed dispersion ring and in a stage when such a ring has been broken up by gravitational resonance effects. It should be pointed out, however, that the tendency of the obscuring clouds just mentioned is by no means general. In many cases, it can be found that dark strings of matter proceed outwards with a greater inclination against the radius vector than an adjacent luminous arm and, therefore, cut over this one in a more or less complicated way. Such details no doubt show that gravitational and electromagnetic forces may interact in ways which are open to many different combinations.

We shall, finally, make a general remark which may be of some importance in this connection. The theory of dispersion rings, and the development of spiral structure by mutual secular disturbances in a ring structure, admits that the luminous spiral arms contain stellar matter of population I of fairly high age and not exclusively very young and massive stars which have recently formed out of the interstellar gas. This conclusion appears to be in accordance with the spectra of luminous spiral arms, especially in the well developed Sc type, which seem to indicate integral spectra of fairly advanced type, about spectral classes A-G. The author's conclusion on this point is based mainly on photographs of spectra published by N. U. Mayall [35]. We are concerned here with the luminous structure which, on ordinary photographs, is accepted without question as the essential part of the spiral structure, as distinguished from population II. Further investigations of the character of the integral spectra of spiral arms should be of great value. The conclusion indicated above is supported by J. Ramberg's [36] results concerning the composition of the spiral arm in the neighbourhood of the sun in the Cepheus-Sagitta region. He finds, for stars of type A and for giants of types G-K in this region, a condensation in space which coincides nearly in distance with the arm defined by the Leiden observers by means of the 21-cm. hydrogen line. Essential parts of the spiral structure can be traced by gaseous arms containing dust and by frequently quite narrow strings of associations of young stars of high surface temperature, as exemplified by the results of Baade's work on M 31. As has been mentioned above, it is quite possible that the structures defined by the gas and the young stars do not coincide exactly with the luminous spiral arms which appear as such on photographs representing the integral light in a wide spectral region.

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Magnetic Fields in Galaxies

Magnetic fields in the Galaxy were first invoked to explain the acceleration of cosmic rays and interstellar polarization. No doubt exists that they are present, but, even in the Galaxy, the strength of the field and, more, its topology are still rather uncertain. The following arguments indicate a magnetic field intensity H of about 3×10^{-5} oersted in the spiral arms near the sun and a four times lower value in the halo:

1. The nonthermal radio emission from the Galaxy depends essentially on the density of relativistic electrons and on the strength of the magnetic field in the emitting region. From the cosmic-ray observations, an estimate of the flux of relativistic electrons can be made and thus H can be determined. The analysis [1,2] is based on the assumptions that the nonthermal radio emission is produced in a continuous (not necessarily smooth) distribution, that the cosmic-ray electrons travel rather freely between different regions of the disc and the halo, and that the electrons observed near the earth are a representative sample of the interstellar cosmic-ray electrons.
2. Biermann and Davis [1] have indicated that it is unlikely that the cosmic rays could be confined within the Galaxy if the energy density of the magnetic field were much less than that of the cosmic rays. The result depends on whether the halo field is wholly anchored to the interstellar gas in the disc or whether gas in the halo is also important. In the first case, the magnetic energy

should exceed one-third of the cosmic ray energy; this leads again to the above magnitude for H. In the second case, equilibrium would seem possible with a very much weaker field, but it is not unlikely that the configuration would then be unstable. Essentially, one would have gas high up in the halo supported by the cosmic-ray pressure and such a situation would lead very likely to some kind of a Taylor instability.

3. If one wishes to explain the systematic deviations from the circular velocity in the interstellar gas in the Galaxy as a consequence of a magnetic field, its strength should be of the order of magnitude indicated above. However, arguments can also be given which suggest a much weaker field ($H \lesssim 0.3 \times 10^{-5}$ oersted).

4. Zeeman-effect measurements at Manchester [3] did not show a magnetic field larger than a few times 10^{-6} in a few dense hydrogen clouds. However, if the cloud fields are irregular, this need not indicate a low value for the field strength in the cloud, since the Zeeman effect measures only the mean field along the line of sight. Thus, if the emitting region contains an equal number of field lines parallel and antiparallel to the line of sight, no Zeeman effect would be detected.

5. Spitzer has emphasized the difficulties connected with star formation in the presence of strong magnetic fields. It is perhaps not impossible that some of these difficulties can be overcome if the fields in the interstellar clouds are not systematic, but, rather, irregular.

6. Spitzer also points out that the origin of a strong systematic field poses a much more difficult problem than the origin of a weak small-scale field. In the second case, amplification by random motions may be important. In the first case, we may have to suppose that the field is primeval. Or perhaps, as has been suggested by Biermann, it could have been formed in the early days of the Galaxy.

Various arguments of lesser weight could be discussed, the most important of which is connected with abnormal radio galaxies, which may, perhaps, be taken as an indication that strong galactic fields are frequently present. Taking all the evidence together, we would give most weight to the determination based on the nonthermal radio radiation. If the field were actually weak, many relativistic electrons would be needed to produce the synchroton radiation, and it is difficult to see how we could avoid observing them from Earth.

The situation concerning the structure of the magnetic field in the Galaxy is still obscure, although some general statements can be made. It can, for instance, be shown that, if the fields in the spiral arms are as strong as we have suggested, the field lines should be helices around the axis of the arm. If this were not the case, the arm would expand in the z-direction where the gravitational attraction is not very effective. Also, if little matter were present in the halo, the mean square field component in the z-direction should exceed the mean square field component parallel to the galactic plane. Thus, though there is little doubt that, near the axis of a spiral arm, the field lines are parallel to the galactic plane, appreciable transverse components in other regions are likely. If much matter is present in the halo, no definite conclusions on the fields in the halo can be reached.

If no matter is present in the halo, the gas in the galactic disc should – on the average – rotate slower than the circular velocity. This follows directly from the virial theorem. Biermann and Davis [1] have shown that this velocity difference may be appreciable. Some models, which have been proposed in the literature, in which the gas rotates with velocities in excess of the circular velocity would, thus, seem to violate basic dynamical principles.

The relation between the dynamics of magnetic fields and the fundamental problems of spiral structure and – if they exist – expansional motions and circulation in galaxies are still quite incompletely understood. It is well to remember here that stationary magnetic fields can do no work and, so, cannot be used to maintain a circulation but only to guide it.

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DISCUSSION

Westerhout: One of the most important assumptions made to arrive at a value of 3×10^{-5} oersted for the galactic magnetic field seems to be the assumption that the synchroton emission at radio wave lengths originates in a smooth medium. The observational evidence for this is very poor because it depends on surveys made with low resolving power. In fact, from recent observations made at Jodrell Bank in the direction of the anticenter, one gets the impression that the so-called smooth background resolves into a number of extended sources. It might well be that we are dealing either with local high-field regions embedded in a low field, or with a low field throughout and supernova-remnantlike sources of relativistic electrons, or both. This holds in particular for the galactic plane, but might likewise be the case in the halo where, because of the long line of sight, it will be difficult to detect individual extended sources, which might overlap on the sky.

Woltjer: The only assumption which is made is that the synchrotron radiation originates in a *continuous* medium, that is, that it is not produced in a number of discrete, magnetically closed, sources like, for example, supernova remnants. This does not exclude that the galactic magnetic field has considerable structure both on a large and a small scale.

Schmidt: In view of the large difference between rotational velocity of gas and the circular velocity implied by a strong magnetic field, I should like to state that solar motions, observed for interstellar gas and for different types of stars, admit a maximum lag of the gas of 4 km/sec.

Woltjer: The velocity difference implied by the magnetic field is an average over the whole galactic disc. Thus, a comparison with the data on the solar neighborhood is not possible. Also the presence of matter in the halo would decrease the predicted values for the difference.

Hoyle: The spectrum of the radio emission shows that one cannot have the high field case, if the magnetic field of the Galaxy is closed, and if the emitted electrons are derived from nuclear collisions of cosmic-ray protons. The latter point can be checked observationally by establishing whether or not electrons and positrons are equally represented in the cosmic rays.

Woltjer: If one accepts the usual estimate that cosmic rays remain in the Galaxy only so long as to traverse on the average 3 grams of matter along their path, no difficulties are encountered unless the field is twice as strong as the fields which I considered.

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Evolution of Stellar Content of Galaxies

The object of this paper should be to discuss the variation with time of the distribution of all kinds of objects throughout a galaxy. Unfortunately, our knowledge of the present distribution of objects in the Galaxy is quite limited. Even in the immediate neighborhood of the sun, a substantial fraction of the mass has not yet been accounted for. Obviously, then, we cannot hope to be able to draw firm conclusions about the evolution of galaxies, and our considerations will be necessarily of a somewhat speculative nature. I should like to start with a brief discussion of the effects of evolution in galaxies, and then proceed to describe how a more detailed picture of galaxian evolution might be obtained.

It has long been suspected that the Hubble classification of galaxies might have evolutionary significance. Observations show that the sequence of types (Irr, Sc, Sb, Sa, E) orders galaxies in terms of increasing color index. It seems fairly safe to assume that the rate of star formation in a galaxy as a function of time does not increase. If it is constant, the population of the upper part of the color-magnitude diagram, where stars evolve rapidly, will also be constant. The population of the lower part, where evolution takes a time longer than the age of the galaxy, will increase. Since the faint stars contribute little to the total light, the integrated luminosity and color index will increase quite slowly. If the rate of star formation is a decreasing function of time, the population of bright stars will decrease too. Since these stars dominate in the total light, the luminosity will decrease in this case. The color index, however, will, again, increase.

Thus, a galaxy seems to become redder as it evolves. This suggests that the sequence Irr, Sc, Sb, Sa, E may represent galaxies in different stages of their evolution, the Irr being in an early stage, the E in a late stage of evolution. Information available from 21-cm. observations indicate that the Irr may contain around 20 per cent of their mass in the form of gas, the Sc around 10 per cent, and Sb galaxies a few per cent only. The rate of star formation in the Irr galaxies must have been much lower than in the Sb types. This leads to the same sequence of evolution as we just arrived at from the colors.

Alternatively, the observed colors and gas contents could be understood if the age of galaxies increased along the sequence Irr to E. This somewhat improbable possibility would be ruled out if it could be shown that the age of the oldest objects in the Magellanic Clouds, and in our Galaxy, are essentially the same.

Sandage [1] has proposed recently that a given galaxy begins as an Irr or Sc, then becomes an Sb and, subsequently, evolves into an Sa galaxy. We cannot be completely certain that an Sc will eventually look exactly like a present-day Sa galaxy. Therefore, an alternative interpretation is possible: that the sequence of galaxian types represents different stages of evolution of systems that were already different when they were formed.

The extent of the central amorphous region in a galaxy increases along the sequence and covers the whole galaxy in ellipticals. Apparently no bright stars are formed in the amorphous regions. Morgan [2] has indicated that spectra of the nuclear regions of M 31 and of ellipticals show evidence of a large population of K-type giants of normal composition. Therefore, metal enrichment of the interstellar gas must have taken place; this suggests that bright stars were formed in the amorphous regions at an earlier stage. One gets the impression that a disease that prevents the formation of bright stars spreads from the center of a system as it evolves and, eventually, affects the whole system.

Dwarf galaxies and small ellipticals do not fit into the evolution scheme discussed above. They are quite blue, yet seem to contain little or no gas and, generally, resemble globular clusters in their stellar population. The reason that globular clusters are so different from, say, giant ellipticals is clear: the gas that was ejected from evolving stars could not be retained by the cluster, consequently, no metal-enriched stars such as are found in old population I could be formed. Possibly, the same, or a similar, explanation applies to dwarf galaxies and small ellipticals.

The progressive reddening of a galaxy as it grows older implies that galaxies with large red-shifts should be intrinsically bluer than nearby galaxies. This expected blue effect for galaxies at large distances is opposite to the original Stebbins-Whitford effect that is now believed to be nonexistent. A very uncertain estimate for the solar neighborhood and for Sb galaxies gives a variation of both B-V and U-B of $0\overset{\mathrm{m}}{.}03$ per billion years.

The foregoing considerations about the evolution of galaxies are of a qualitative nature. It would be most desirable to have some quantitative estimates of the time variation of the luminosity of a galaxy, its gas content, the abundance of heavy elements in the interstellar gas, the rate of supernova explosions, etc. These estimates are extremely difficult to secure and no reasonably certain information for the above desiderata can be obtained at present. Therefore, I shall limit myself to a critical discussion of the problems involved.

The evolution of the stellar content of a galaxy may be described by the formation of stars and their subsequent evolution. Models of the evolution of the solar neighborhood have been developed by Mathis [3] and by Schmidt [4,5] (for the solar neighborhood) and by Salpeter [6] (for the whole Galaxy). The scheme adopted for the evolution of a star was as follows: after its formation, the star remains near the main sequence for a period of time depending on its mass, then evolves into a giant or supergiant, ejects mass, and ends up as a white dwarf of about one solar mass. We know much less about the formation of stars. No information is available from astrophysical theory that would permit an estimate to be made of the rate of star formation in a gas of given physical state, or of the distribution of stellar masses that will be formed. Some assumption has to be introduced at this stage and it usually takes the form

$$\text{rate of star formation} = \mathit{const.} \times (\text{gas density})^n \qquad (1)$$

Values for n ranging from 1 to 3 have been used by different authors. For a given value of n, an evolutionary model can be obtained, and the time variation of the color, the luminosity, the gas content, etc., can be computed.

It seems reasonable that the rate of star formation will depend on the amount of gas available. However, it is by no means certain that gas density is the dominant factor. Such factors as the chemical composition of the gas, the degree of turbulence, or the magnetic field may well be of greater importance. Therefore, let us try to get whatever information on the rate of star formation can be obtained without assuming a dependence on gas density.

The distribution of masses, or luminosities, which is formed (initial luminosity function, introduced by Salpeter [7]), has often been associated with the luminosity function of stars in galactic clusters. If the two are indeed strictly identical, the rate of star formation as a function of time can be deduced directly from the cluster luminosity function and the van Rhijn luminosity function for field stars as has been shown by von Hoerner [8]. However, galactic clusters are now known to be deficient in faint stars (van den Bergh [9]). This might be due to a loss of faint stars or of protostars at the time the cluster was formed. The luminosity function of brighter stars in the clusters may well have been affected too.

Observations at 21-cm. wave length have shown that the radial distribution of neutral hydrogen in the Galaxy is fairly uniform, and does not show a strong concentration toward the center such as is found for the common stars. If this is also the case for, as yet unobserved, interstellar molecular hydrogen, then we have to explain the difference in distribution of gas and the bulk of stars. One possible explanation is that the rate of star formation was much faster in the inner parts than in the outer parts of the Galaxy. Another possibility is that the distribution of gas was essentially made uniform by radial motion of the gas. In this case, most of the interstellar gas in the solar neighborhood may have come from the central region of the Galaxy. The situation would be so complicated that no deductions about star formation could be made at present. We can only discuss the simpler case in which no radial mass motion of gas takes place in the Galaxy. The following considerations are presented in a highly condensed form.

Some idea about the way in which the rate of star formation varies as a function of time is obtained from knowledge of the present rate of formation, and of the average past rate of formation. For *faint* stars the average past rate of formation can be derived from the field luminosity function. G. H. Herbig [10] has obtained a rate of formation of stars with $+2 < M_{pg} < +10$ from his extensive studies of T Tauri stars. It may be shown that this present rate of formation equals 60 per cent of the average past rate if the age of the Galaxy is 16 billion years. An independent estimate may be made for stars with $M_v = +6$ if the reasonable assumption is made that their present rate of formation is larger than that of $M_v = +3$ stars. If so, it may be shown that the present rate of formation of +6 stars is at least half of their average past formation rate. We get the impression that, in the solar neighborhood, the rate of formation of faint stars has decreased only slightly throughout time.

The past formation rate of *bright* stars presents a formidable problem. The present rate is derived simply from the field luminosity function and the life times on the main sequence. The only way to find out about the past formation rate is through the end-products of stellar evolution: white dwarfs, helium, and heavy elements. White dwarfs cool very rapidly, and our observational data, which are only complete for fairly bright white dwarfs, refer, therefore, mainly to recent stellar decay, corresponding to a mostly recent stellar formation. The total number density of white dwarfs in the solar neighborhood is unknown.

Helium abundances of those objects for which they have been determined show a surprising uniformity [11]. Absolute determinations, even in the Magellanic Clouds [13], always seem to lead to values very close to that found in the Orion nebula [12]. In any case, helium abundances that are now available probably do not carry much information about star formation, just because they seem to be so uniform.

The variation of metal abundance of the interstellar gas with time should be related to the past formation of bright stars. If we follow O. C. Wilson [14] in his interpretation of the dispersion in the spectrum-color relation for main-sequence stars later than G5, in terms of metal abundance, very roughly 50 per cent of the faint main-sequence stars must be deficient in metals by a factor of 10 or more. This is a surprisingly large percentage of metal-poor stars which it would be very worth while to check by conventional abundance determinations for a number of stars. If we combine the percentage mentioned with a rate of star formation of faint stars as derived above, we find a roughly constant rate of formation of bright stars. This result should be considered as preliminary; it may well change considerably once we have reliable statistical material on the distribution of metal abundance among faint stars.

In case a physical interpretation of Eq. 1 were admissible, the value of n could be determined by correlating gas density and star counts. I then find for the Small Magellanic Cloud $n = 2$, for the Large Magellanic Cloud $n = 1$. A similar correlation in the Galaxy, but perpendicular to the plane of the Galaxy, leads to $n = 2$ for bright stars [4], but, in this case, the correlation may have been affected by a different dynamical behavior of star-forming gas clouds as compared to gas clouds in general, as was pointed out to me by van den Bergh.

Thus, we see that even in the solar neighborhood we cannot determine the total past production of bright stars with any reasonable certainty. We may eventually be able to do so through a study of the effects of the chemical evolution of the interstellar gas on faint stars for which we may have a fair idea of the rate of formation as a function of time. If the local interstellar gas has been appreciably replenished from the interior regions, the situation is too complicated and practically nothing can be derived about star formation.

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DISCUSSION

Schatzman: I do not think that the time-scale of cooling of white dwarfs is so short. For a luminosity of 10^{-2} solar luminosity, the time-scale of cooling is of the order of 10^9 years, and longer, the fainter the white dwarf is.

Schmidt: The time-scale of cooling will be smaller if helium burning occurred in the star before it became a white dwarf, as was pointed out to me by Dr. R. Weymann. Even a time-scale of 5×10^9 years would be considerably less than the age of the Galaxy.

Holmberg: The theoretical approach by Dr. Schmidt leads to very interesting results as regards the mass-luminosity ratios of stellar systems. It is possible to derive the mass-luminosity ratio of the model system as a function of time, and also the relation between integrated color and time. By eliminating the time parameter, we arrive at a relation between mass-luminosity ratio and integrated color which very closely agrees with that found from observational data.

Bondi: Can you give a lower limit for the figure of $0^{\mathrm{m}}.03$ per billion years, suggested by you with great caution as a likely value for the rate of reddening? Would you regard $0^{\mathrm{m}}.01$ per billion years as definitely too low a figure?

Schmidt: The computations are very involved and it is not possible to give a reliable lower limit without more computations. Even then, the effect of the assumptions made would require extensive investigation. I would not state, therefore, a definite lower limit at present.

Vorontsov-Velyaminov: How do you account for the fact that the irregular galaxies are of much smaller masses than the ellipticals; can the latter originate from the irregulars?

Schmidt: I agree with you that it now seems quite well established that irregular galaxies have smaller masses than ellipticals. In that case, present-day irregulars could eventually not evolve into present-day ellipticals, and the sequence of morphological types must represent different stages of evolution of *intrinsically different* systems unless irregulars are really young and ellipticals old.

Massevich: If the presently observed white dwarfs do not represent formerly existing bright stars, the question arises: where are the remnants of the bright stars? As the rate of star formation is decreasing with time, the total mass of those "black" remnants in our Galaxy must be rather large and could not remain unnoticed.

Schmidt: I consider the presently observed *bright* white dwarfs as remnants of stars formed rather recently. It is only for these bright white dwarfs (with $M_v < +14$) that we have complete information in the solar neighborhood, and, thus, can find the number per unit volume. Data for fainter white dwarfs are very incomplete, but it is only these that could tell us about star formation in the early stages of the Galaxy.

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Multicolour Photography of Galaxies at the Crimean Observatory

I should next* like to read a short communication from I. M. Kopylov, V. B. Nikonov, K. E. Chuvaev, and A. B. Severny which describes an attempt they have made to carry out multicolour photography with the 2.6 m. reflecting telescope of the Crimean Observatory.

Photographs of 58 galaxies were taken last July at the primary focus (f = 10 m.) of the new 2.6 m. reflecting telescope of the Crimean Observatory. Photographs were obtained with the aid of an image tube. Four filters were used: (Table 1).

TABLE 1

Filter	λ *max*	λ
Hα	6580	70
"6100"	6120	150
"V"	5300	680
"B"	4330	900

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*Dr. Ambartsumian made some preliminary remarks of his own but no manuscript of these was received for publication.—Ed.

The main purpose was to obtain data about the structure of the nuclei of galaxies, especially in Hα light.

Some galaxies show jets visible only in Hα, and also show the complex structure of their nuclei, especially in Hα light (NGC 1569, NGC 4490). In other galaxies, these jets are stronger in the continuum, as, for instance, in the radio-source Virgo A (NGC 4486).

In the peculiar nebula NGC 4151 the nucleus is more extended in Hα light than in the continuum and even in blue light.

As a rule, a strong emission in Hα is accompanied by concentrations of hot luminous stars (NGC 4214); however, there are some exceptions in which we cannot find any traces of hot luminous stars at the places of strong Hα emission (NGC 604).

The hydrogen emission is well pronounced in Sc and Sb spirals and is connected with groups of hot luminous stars (for instance, NGC 4736, NGC 6946, and M 51). The strong Hα emission is observed also in most irregular galaxies (NGC 1569).

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Normal Galaxies and Stellar Systems: Summary

The subject of structure and evolution of galaxies is in the unsatisfactory stage where we have observed many intriguing phenomena, but have no good theories to explain them.

THE PHENOMENA

FORMS, COLOURS, SPECTRA

Important advances have been marked by Morgan's new data on classification, by Vorontsov-Velyaminov's *Atlas of Interacting Galaxies*, and by the *Hubble Atlas of Galaxies* which has recently been published in a beautiful form by Sandage. It is, in particular, these collections of pictures that bring home the vivid realisation of how little we understand the processes determining the evolution of galaxies.

An important datum obtained is that, in the blue region, spectra of central bulges of spirals showing concentration to the centre as well as of large ellipticals resemble the spectra of old populations I (K giants). The colour-magnitude diagram of the Draco system obtained by Baade and Swope, and colours of other dwarf ellipticals show, however, resemblance to halo globular clusters. Spinrad has indicated some evidence that, in the yellow region of the spectrum, the amorphous regions of galaxies may show characteristics of late dwarfs.

In general, nuclear regions appear to be redder than outer regions. De Vaucouleurs has pointed to an opposite trend in irregular galaxies.

A very promising line for further research is that of multicolour measures and spectral scans.

STELLAR CONTENT

Particularly important are the data on the occurrence of interstellar matter. 21-cm. measures give indications of total amounts of gas; these are generally small, but vary with type of galaxy. Gold thinks, however, that there may be larger quantities of hydrogen *molecules*. Dark lanes give information on localisation of interstellar matter. In a number of early-type spirals, the dark matter appears to be confined to a central disk, and sometimes to a ring which may be situated in the outer parts of the galaxy. In NGC 4594 a remarkable segregation seems to occur between the luminous stellar matter in the disk and the dark matter, the luminous disk population having its maximum density in a region inside the obscuring ring where the amount of dark matter is negligible. An interesting observation was reported by Münch. He found that none of the luminous knots in the disk shows emission lines.

INTERNAL MOTIONS AND MASSES

There has recently been a great extension in our knowledge of rotations, mainly by the measurements of emission lines by the Burbidges and by the detailed observations of separate emission patches in M 31 and M 81 by Guido Münch. In combination with careful measures of the dispersion of random motions (by Minkowski and by the Burbidges) this has led to a much better knowledge of masses and mass-luminosity ratios. The latter ratio appears to have roughly the same value, of about 15, for all large ellipticals and amorphous parts of Sa and Sb spirals. This value does not apply to *all* population II systems as indicated by the fact that the ratio is less than 1 for globular clusters like M 3 and M 92. For systems in which population I is the most prominent population, the ratio is likewise smaller, probably between 1 and 5.

Extensive data have been obtained on the internal motions in the E7 galaxy NGC 3115, from which a fairly complete dynamical model can be constructed in principle.

Interesting new phenomena relating to the motion of the interstellar gas have been found in the central part of the Galaxy and near the nucleus of the Andromeda nebula. In the nuclear part of the Galaxy, the motions of the gas have radial components up to

100 or even 200 km/sec, which appear to be directed away from the galactic centre. These data provide strong evidence for the existence of magnetic fields of sufficient strength to cause spiral arms to be pushed away from the centre with velocities of this order. The systematic character of the radial motion of the so-called 3-kpc arm and the striking regularity of this arm are, in my opinion, a strong argument for the existence of magnetic fields with large-scale structure and considerable strength (up to 10 γ perhaps). Other evidence for fields of the order of a few γ in the outer parts of the Galaxy has been summed up by Woltjer. But there is also considerable evidence pointing to a field strength that is at least an order of magnitude weaker.

Rotational motions observed in a small disk of neutral hydrogen within 600 pc from the centre indicate an extremely high concentration of stellar mass towards the galactic centre, quite similar to what one finds in M 31.

It is of great importance to investigate fully how much of the observed features can be brought about by the action of the gravitational field alone. In this connection, special reference should be made to the calculations made by P. O. Lindblad. The results were shown on a most interesting film.

Arp has pointed to evidence that age and composition may enter as partly independent factors.

Of evident importance for the problem of colours of very distant galaxies are studies of the way in which the population changes in the course of time. This subject was summarized by M. Schmidt.

SOME QUESTIONS

On the theoretical plane, the most essential problems remain unsolved. Outstanding questions are the following:

What has caused the different forms of galaxies?

Why has star formation been practically completed in elliptical galaxies, in the giant ones as well as in the thinly populated Sculptor-type systems? Why has it *not* been completed in spiral galaxies in which the amorphous parts may be quite comparable to elliptical galaxies?

Why is the constitution of dwarf ellipticals and globular clusters different from that of large ellipticals and central bulges of spirals?

Why has the amorphous part of some galaxies developed into an oblate elliptical shape with strong mass concentration towards the centre, while in other galaxies it has developed into a prolate, bar shape with little or no central mass concentration?

Why is dark matter sometimes localised in central disks (NGC 5866) or in rings (NGC 4594)? And why does not the ring in NGC 4594 coincide with the main concentration of the irregular luminous matter in the disk?

Is spiral structure confined entirely to the gas (including stars recently formed), or does an important fraction of the mass of a spiral arm consist of stars?

Are the spiral arms principally due to gravitation, or do gas dynamics and magneto-hydrodynamical phenomena play the main roles?

Are the magnetic fields in the arms of the order of a few times 10^{-5} gauss, or are they an order of magnitude lower?

How can the spiral or bar structure persist during a time of the order of a few tens of billions of years? And, more particularly: how can a spiral pattern be maintained in systems with rapid differential rotation?

How much gas is there in the halo, and what role does the halo play in the general circulation of gas through a spiral galaxy?

DESIDERATA

ELLIPTICAL GALAXIES

Velocity dispersion at different distances from the centre. One desideratum is to find out whether or not in a spherical galaxy the motions are preponderantly radial. It is also important to investigate this for halo stars in the Galaxy.

Dynamics of three-dimensional orbits in a galaxy like NGC 3115.

More accurate radial velocities in double galaxies, in order to obtain an improved derivation of their total masses.

SPIRALS

Accurate measures of brightness in at least three colours in spiral arms, and comparison with the same observations in regions between the arms.

Situation of stars and clusters of different ages relative to the arms found from 21-cm. observations.

In a galaxy in which spiral arms can be followed throughout the system (M 81 or M 51, for instance) accurate radial velocities for many emission knots all along the arms, so that possible systematic motions in radial direction might be revealed. This might lead to important conclusions concerning the mechanism by which the spiral pattern is maintained.

Measurements of polarization of radio-frequency radiation in galactic disk and halo, in order to determine the average field strength and to find the structure of the magnetic field. Other observations for determining the field strength (cosmic-ray electrons, Zeeman effects, etc.).

Comparison of the gravitational field, as determined from distribution and motions of planetary nebulae and RR Lyrae variables near the galactic centre, with that inferred from the rotation of the small disk of 600 pc radius of neutral hydrogen.

Further numerical computations of the development of spiral structures under the influence of gravitational forces.

PART TWO

Multiple Systems, Clusters, and Radiogalaxies

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Statistics of Double and Multiple Systems

1. IDENTIFICATION OF PHYSICAL ASSOCIATIONS

It is commonly recognized that double and multiple objects represent an important structural feature of the extra-galactic universe. An analysis of the small-scale distribution of nebulae suggests an unbroken transition from single objects, through pairs and small groups, up to the occasional great clusters.

In some investigations on the clustering tendencies, double and multiple systems have been selected according to predetermined definitions, based generally on the apparent distances between the nebulae; all separations which are less than a certain limiting value, dependent on the apparent dimensions or apparent magnitudes of the objects, have been assumed to correspond to physical companions. In order to exclude, as far as possible, optical companions, the maximum separations allowed have necessarily been given comparatively small values. This procedure naturally introduces serious selection effects, and no complete description of the clustering phenomenon can be obtained. The problem can be approached, however, from the opposite direction. The clustering tendencies may be revealed by a statistical analysis of the distribution of the apparent distances between neighboring nebulae in the sky. The analysis leads to a separation, in a statistical sense, of physical from optical companions, and the method thus reveals the double and multiple systems as they actually exist in the sky.

The first method has been used by the writer [1] in a survey of nebular associations by means of the Heidelberg Observatory plate material (Bruce telescope). The survey revealed 827 systems, the maximum angular separation allowed being twice the sum of the apparent major diameters. The relative number of optical companions included in the material probably amounts to only a few per cent. With the definition used, it is estimated that, except for clusters, about 22 per cent of all nebulae are members of double or multiple systems.

In a subsequent investigation [2] based on the Reinmuth catalogue [3], the second method has proved quite successful. The total number of physical companions belonging to nebulae of a certain class, for instance, the magnitude class 12-13, is found by surveying circular areas around the selected nebulae, and noting the distances to neighboring objects. The statistical distribution of these angular separations is easily dissected into two distributions, one referring to optical members (random distribution, represented by a straight line), and one to physical members. The latter distribution curve furnishes complete information about the total number of physical companions and about their apparent distances, but does not allow any definite conclusions as regards the number of objects associated with a given nebula. It is interesting to note that this method leads to the result that, except for clusters, approximately 53 per cent of all nebulae belong to double or multiple systems.

2. *STATISTICAL RESULTS*

In Table 1 we find the results of the second method, as applied to nebulae (Virgo Cluster area excluded) of magnitude 12.1 to 13.0 in the Shapley-Ames catalog [4]. The second column gives the total number of objects (Reinmuth catalogue) in circular strips around the selected nebulae, the width of each strip being 5′. It may be noted that the table has been cut off; in reality, the survey extends to a distance of 120′. In the third column, we find the probable number of optical companions, as corresponding to a straight-line distribution curve; the inclination of this line is based on the frequencies of objects at large distances (where no physical companions are expected). The difference between the figures of the two columns represents the probable number of physical companions corresponding to each interval of angular separation. The maximum separation referring to physical companions, about 45′, seems to be rather well defined. The mean separation amounts to about 15′.

TABLE 1

Distribution of Angular Separations Between Nebulae of Magnitude Class 12.1 to 13.0 and Their Companions

Separation	Total Objects	Optical Companions	Physical Companions
0′ – 4′	35	1.4	33.6
5 – 9	35	5.0	30.0
10 – 14	37	8.6	28.4
15 – 19	37	12.1	24.9
20 – 24	30	15.7	14.3
25 – 29	31	19.2	11.8
30 – 34	27	22.8	4.2
35 – 39	33	26.4	6.6
40 – 44	37	29.9	7.1
45 – 49	31	33.5	(–2.5)
50 – 54	37	37.1	(–0.1)

In order to find the absolute distances corresponding to the angular separations, we may accept the mean absolute magnitude, $\bar{M}_m = -19.3$, as derived by the writer [5] in a recent investigation (app. dist. mod. of the Virgo Cluster assumed to be 30.2). The mean apparent magnitude of the nebulae investigated amounts to 12.2, if reduced to the international system [6], or to 11.9, if the mean galactic absorption is also subtracted. Accordingly, the above mean separation of 15′ corresponds to a projected distance of about 75,000 pc. The maximum projected distance is three times as large.

The maximum projected distance is obviously equal to the maximum distance *in space*; hence, the maximum space distance, R_m, between physically associated nebulae amounts to about 225,000 pc. A determination of the mean space distance, $\bar{R}$, has to be based on the statistical distribution curve, $F(R)$, referring to the space distances. This curve can be derived by well-known methods from the distribution of the projected distances as obtained from Table 1. It has been shown by the writer [7] that, within the limits of the accidental deviations, the sought distribution function in this case is proportional to $(1 - (R/R_m)^3)$. By an integration procedure, we now find that $\bar{R} = 0.4\ R_m$. Physically associated nebulae are thus, on an average, separated by a space distance of 90,000 pc.

A more detailed analysis of the statistical data provides some information about the relative frequencies of single nebulae, pairs, triples, and so on. The result is reproduced in Table 2 which gives the proportions of nebulae that are members of systems of different orders. We thus find that, outside clusters, 47 per cent of all nebulae are single systems (order 1), whereas 53 per cent have one, or more, physical companions. It also follows from the tabulated data that, for all the nebulae included in the analysis, the average number of physical companions amounts to 1.1.

According to [1], an increase in the general space density of nebulae (number of objects per cubic pc) is accompanied by an increase in the relative number of physical systems, that is, the space density of physically associated nebulae increases faster than the general space density. The increased clustering tendency corresponding to a higher density is clearly exhibited by an analysis of the Virgo Cluster. An application of the above statistical procedure gives the result that each cluster member is, on an average, accompained by about 3 physical associates. The mean angular separation amounts to 17′, and the maximum separation to about 30′. With an apparent modulus of 30.2, the distance of the Virgo Cluster would be 9.7×10^6 pc. Accordingly, the maximum space distance (= maximum projected distance) between physically associated nebulae in the cluster would be about 85,000 pc. This distance is considerably smaller than that found above for field nebulae (225,000 pc), a difference that is explained by the assumption that loose groups are easily disrupted by the strong gravitational field of the cluster.

TABLE 2

Relative Numbers of Nebulae Which are Members of Systems of Different Orders

Order of System	Relative Frequency
1	47%
2	24
3	15
4	7
5	4
6	2:
7	1:

An investigation of the grouping tendencies in the Virgo Cluster is being undertaken by the writer. Detailed results are not yet available.

The orientation in space of the orbits of double and multiple nebulae represents a very interesting statistical problem. According to [1], the absolute orbital radii of 570 double systems have an ellipsoidal distribution, the two largest axes of which are approximately the same; the shortest axis points towards gal. long. 92° and gal. lat. +22°. The orbital planes would, thus, tend to be parallel, the mean plane corresponding more or less to the great circle in the sky that joins the areas having the largest apparent densities of nebulae (metagalactic equator, according to Lundmark). Considering the common origin (cf. next section), it seems very likely that the principal planes of the two nebulae in a double system are approximately parallel to each other, and parallel to the orbital plane. No investigation giving statistically significant results has so far been made on this problem. It also seems likely that the two components are rotating in the same direction. In this connection, it may be pointed out that the spiral arms of associated spiral nebulae, as far as can be judged from photographs, generally unwind in the same direction.

In conclusion, it may be recollected that a study of the orbital motions in double systems, as revealed by the differential red-shifts, leads to a determination of the mean mass of the components. The method, first suggested by the writer in [1], has been subjected to a detailed mathematical analysis in [7]. Besides the results to be obtained from measurements of rotation in a limited number of nearby nebulae, a statistical analysis of the dynamics of double systems seems to be the only means available at present to determine nebular masses. In a recent investigation [not yet published], the writer has derived a mean mass-luminosity ratio (solar units) of 80 for 37 pairs of E-SO nebulae. The result is based on the Humason-Mayall red-shifts [8], corrected for systematic errors [5]; it should also be noted that the result is based on a Hubble parameter $H = 90$ km/sec/Mpc. (The mars-luminosity ratio is proportional to the value adopted for H.) The masses of spiral nebulae, especially, type Sc, are so small, and the resulting orbital motions so slow, that the method cannot be expected to yield any significant results in these cases. It is interesting to note that the mean ratio derived by Page [9] for type E-SO is, if reduced to the same distance scale, practically the same as that obtained by the writer.

3. *STELLAR CONTENTS OF ASSOCIATED NEBULAE*

A photometric investigation of double systems leads to the very interesting conclusion that physically associated nebulae have stellar contents that, in most cases, are more or less identical. The observational agreement refers to the integrated color index, and to the mean luminosity density (solar units per cubic pc). The agreement also extends to the nebular type.

In a previous paper [6], the writer has studied the integrated colors total *pg* magn. *minus* total *pv* magn. of the components of 39 pairs of nebulae, the great majority of which are presumably physical systems. The color indices have been derived from magnitude measurements of photographic plates, the material obtained by means of the Mount Wilson 60-inch and 100-inch telescopes. The colors of irregular and spiral objects have been corrected for the effects of internal absorption (type E-SO assumed to be absorption free) by special correction formulae derived by the writer; the corrected indices therefore represent the intrinsic colors of the stellar contents. For 35 of the investigated pairs there is a rather pronounced correlation between the absorption-free colors of the components, the coefficient of correlation amounting to +0.88; the color indices range from $+0^{\rm m}.22$ to $+0^{\rm m}.82$. The agreement indicates, quite definitely, that associated nebulae have stellar contents of a very similar type.

A close agreement between the components is also obtained as regards the surface magnitudes of the 24 pairs of spiral nebulae included in the material. The surface magnitudes have been corrected for internal absorption and for inclination effect. The coefficient of correlation in this case amounts to about +0.90. If the galaxies are assumed to be oblate spheroids of the same relative thickness, the correlation is explained as a result of an agreement between the components as regards mean luminosity density (solar units per cubic pc). Since the integrated color indices, hence, the mass-luminosity ratios, are approximately the same for the associated nebulae, we arrive at the important conclusion that the agreement in luminosity density is accompanied by an agreement as regards mass density. It may, on the other hand, be noted that the observational data available do not indicate any correlation between the total absolute luminosities, or between the total masses, of the associated objects.

The agreement found between types of stellar content, and between luminosity densities (mass densities), strongly suggests that physically associated nebulae have a common origin, that is, that they are parts of one and the same mother system which, because of dynamical instability, has disintegrated. Double and multiple nebulae would thus form an interesting parallel to stellar associations. It may be noted here that there are a number of nearby nebulae in which a disintegration seems to be in progress at the present time; the most interesting cases are NGC 2366, NGC 4088, NGC 4485-90, and NGC 4656.

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DISCUSSION

Neyman: When studying the statistics of multiple systems of galaxies, it is very essential to have a clear-cut definition of the entities concerned and of the method by which these entities are counted. Would Dr. Holmberg explain what he means by a "double" galaxy and by a "triplet"?

Also, I would like to see the formula whereby the expected number of optical doubles and of optical triplets has been calculated. Finally, it would be interesting to know how the actual number of doubles and triplets on a photographic plate was ascertained.

Holmberg: Since a complete answer to these questions would take up too much space, I wish to refer to my paper "On the Clustering Tendencies among the Nebulae".* It may be pointed out that the theoretical method of analysis used by me does not lead to any definite information as regards the number of physical (or optical) companions in *individual* cases; we only get mathematical expectations.

Morgan: There is a spectacular difference in the evidence from the colors and spectra of NGC 5194 and 5195; the spectral types of the two galaxies are similar, while the color of NGC 5195 is much too red for its spectral type. This may be due to internal reddening material in NGC 5195.

Zwicky: We should make statistics of *(a)* multiple galaxies interconnected by luminous inter-galactic formations; *(b)* close galaxies which show mutual distortions; and *(c)* close galaxies for which symbolic velocities of recession are known. To my knowledge there is no evidence that there is any particular selection of types of galaxies among the component galaxies.

Holmberg: Since I do not know how Dr. Zwicky has selected his physical systems, I have no comment to make.

Von Hoerner: You said that the similarity of the members in multiple systems means that they apparently have been formed by a disintegration process. I would like to add that this might be true, of course, but they might also have been formed more or less simultaneously out of the same larger original cloud, similar to the formation of multiple stars.

Holmberg: Yes, of course.

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*See E. Holmberg, *Ap. J.* **92**, 200, (1940).

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Interaction of Multiple Systems

Zwicky has drawn attention to double galaxies that are connected by luminous filaments; some of these had already been noticed in the nineteenth century. He discovered, or reminded us of, some spectacular systems. The interest in these systems was sustained by de Vaucouleurs – in particular, by his study of the appendages of the Magellanic Clouds. In 1958, I issued Part I of the *Atlas* of 355 interacting galaxies. I must note some misunderstandings that have arisen in reference to that catalogue. Leaving aside the objects which may have been included erroneously, especially owing to the difficulty of the correct interpretation of those systems which are seen in projection, the *Atlas* contains galaxies which display a *visible interaction*. That is, their forms are distorted by mutual influence. The other systems, which I also call the "interacting" galaxies, do not show distortion, but they are embedded in a common haze or they form a chain.

I strongly object to calling "interacting" galaxies "peculiar" galaxies or "multiple" galaxies. There are many isolated galaxies that are called "peculiar," for instance, the elliptical galaxies with dark matter. The "interacting" galaxies are physical pairs where no question of accidental projection arises.

Two fundamental facts have been established concerning the nature of the connecting filaments and tails. The first fact is that *they consist mainly of stars, sometimes with some amount of luminous gas*. This was shown by me by the discovery of systems where, as in M 51, the spiral arm plays the role of a connecting filament. Condensations similar to those observed in Sc spirals were found in the filaments. Spectroscopically, the same fact was proved by Carpenter and, especially, by Zwicky. *The appendages in interacting systems are, in all respects, essentially of the same nature as the spiral arms.*

The second fact is that *the connecting filaments and the tails have nothing to do with tidal phenomena.* I have demonstrated this by the following observations.

1. Tails are much more frequent than the connecting filaments. When both are present, the tail is usually brighter and, probably, more massive.

2. The tails and the filaments are often so thin and so long (up to 100,000 pc) that no tidal forces could produce them.

3. There exist pairs with two more or less parallel connecting filaments. No tides could produce such forms.

4. Many thousands of close pairs are observable. Many of them must be neighbouring in space. However, no tidal distortions of the expected form are known, and relatively few pairs are interacting in our sense.

5. The percentage of interacting systems in clusters is not higher, and sometimes is less, than in the general field.

6. The dependence of disturbances on true separation is not known. However, the disturbances are often striking at separations several times greater than the dimensions of the components. It can be said definitely that the disturbances do not grow at the same rate as the tidal forces when the separation diminishes.

The study of the interacting galaxies adds the strongest argument (to the many already existing) that the spiral arms cannot originate from tidal phenomena combined with rotation. This idea, traceable to Jeans, is still circulating, but it has to be abandoned once and for all. In fact, we observe galaxies with one, two, three, and many arms, and galaxies with inner and outer arms. There are galaxies with arms winding in opposite directions. Though they are few in number, a single exception is sufficient to disprove our old conceptions. The spiral arms do not only originate from the nucleus or from the ends of the bar. Very often they start far from the central lens or from the bar in the disc, or they start from the rim of a ring. Sometimes they are observed only well off the main body as somewhat independent features. In some cases, the two arms emerge from one and the same place and are not opposite to each other.

As was stated above, *the nature of the spiral arms, of tails, and of the spanning filaments is much the same. So must be their origin* *and the reason for their stability.* One must accept that their stability is much greater than could be admitted when it was believed that the spiral arms were a spectacular, but a short-lived, phenomenon. By compiling the morphological catalogue of some 50,000 galaxies, I get the impression that – while the disc galaxies are apparently the predominant form – the spirals really predominate in each unit volume among giant galaxies, rather than the ellipticals.

The conclusion that the ellipticals predominated, relative to the spirals, was made when the S0 and E galaxies were counted together. The great frequency of systems with hot giants suggests the long duration of their presence.

Rotation would quickly disrupt the spiral arms unless there were a very specific law of rotation or they rotated like solid bodies.

We have given the following additional argument for the long persistence of the spiral forms. There seems to be no diminution in the proportion of spirals at the limit of recognition by the 200-inch telescope. But the spirals at this limit are seen as they were a billion years ago. It seems quite improbable that the formation of such distant spirals began much earlier than in our neighbourhood. where the spirals we now observe are found. It seems more probable that spirals appeared everywhere more or less simultaneously in the same proportion and that the nearer spirals are, on the average, not older than those which we see as they were a billion years ago.

The persistence of spiral arms shows the great viscosity in galaxies. This viscosity is demonstrated even more by the interacting galaxies. A long connecting filament suffers, at its centre, equal attractions from both galaxies. So it is not influenced by external gravitation. The smallest peculiar velocities of stars there are hyperbolic. Yet the filaments are very thin and they do not disperse. For a half-width of filament equal to 1 kpc, a star would traverse this distance in 10^8 years at a speed of 10 km/sec. At such an average speed under certain forces, the star needs 5×10^9 years to travel along a filament 50,000 pc long. This also shows that the filament cannot be a short-lived phenomenon.

The Burbidges have obtained photographs on a larger scale of some of the galaxies which I have found on the Palomar Sky *Atlas* and reproduced in my *Atlas*. In some systems, which I call "nests of galaxies," and in the system "Mices," they discovered exceptional quantities of gas and drew many interesting and valuable conclusions. But I cannot share their opinion that all these systems and their appendices are very short-lived, that the chains of galaxies described in my Atlas are very young formations. For instance, one of the chains consists mainly of elliptical galaxies, whose stars are believed to be very old. This configuration must be equally old.

The interacting galaxies are systems for which the physical connection is evident without measuring their distances and velocities. The difference of the velocities of the components is entirely compatible with orbital motion under the law of gravitation. Using among other pairs the interacting ones, Page has obtained quite reasonable masses. (It is superfluous to argue further that the interacting galaxies have a common origin.) Important contributions to the knowledge of some properties of interacting systems were made by Humason and Zwicky.

So it seems that gravitation still governs the motion of all galaxies. But the forms of galaxies certainly are influenced also by some other powerful forces besides rotation. The "disruption of the facade" phenomenon occurs when no spiral structure is seen on that half of a galaxy which is turned toward its companion, while the spiral structure is seen distinctly on the other side. It must be remembered that the spiral arms are composed of hot giants and of gas. Therefore, it appears that, under the influence of the companion, there is no general repulsion of matter. The companion either prevents the formation of spiral structure on the nearer side from the already existing giants and from gas, or it repels the gas alone to the farther side where the stars in spiral arms can condense.

The case of straight bright tails is a more convincing item of evidence for repulsion. But, even here, it is not all the matter in the galaxy that is repelled – only the matter constituting the tail.

Meanwhile, many physicists suggest that there is a general repulsion. They try to explain this by the most recent and popular theories of antimatter, of negative attraction, of negative masses and so on.

What is the nature of the forces that govern the new phenomena displayed by the interacting galaxies, if the tides, and, consequently, gravitation, are ruled out? P. O. Lindblad has succeeded in obtaining some forms of interacting galaxies from a mechanical model, but, taking the evidence as a whole, one is compelled to look for other explanations. Surely the first thought that occurs is that magnetic forces might be responsible. The idea is attractive and has been applied to the explanation of the stability of spiral arms. Yet it is valid only for the gaseous arms and not for the stellar arms. It was supposed that the hot giants originated from the gaseous arms. Not being retained by the magnetic force, the giants leave the arm and disperse into the disc. But at this time they cool and are no more recognizable as hot giants.

Can the spiral arms persist in this way for sufficiently long periods? Let us assume that the luminosity produced by the hot stars in the arm is 0.1 of the total. For a supergiant galaxy with $M = -20.5$, the partial luminosity of giant stars would correspond to -18. This luminosity can be created by 10^5 stars with $M = -5$. Their total mass correspondingly would be 4×10^6 $M_\odot$.

The store of gas in an Sb galaxy is 0.01 of the total mass, or of the order of 10^8 $M_\odot$. The whole gaseous mass cannot be converted into stars; stars of different masses and luminosities are formed. An extreme upper limit for the gas that could be converted into hot giant stars is 10^7 $M_\odot$. It suffices to create $10^7/(4 \times 10^6) \cong 2$ generations at most. With a duration of life equal to 10^6 years, this would maintain the spiral arms only for 2×10^6 years. This time is too short.

The magnetic nature of the interaction is doubtful owing to the fact that the tails and the filaments are also observed among the ellipticals which are short of gas and consist of old stars. Their appendages seem to be composed also of old stars and cannot be young formations recently developed from the gas. The high viscosity and stability of galactic structures must be accepted inevitably. We are at the threshold of discovery of new properties of galaxies in the same way as we were when we began studying nuclear interactions. It is impossible that, in the infinite universe, the systems of higher order are governed only by the same laws which prevail in our neighbourhood. Personally, I am convinced that the double galaxies and clusters have properties and produce effects rather different from those which would be expected for systems of material points of the same total mass.

The dimensions of nuclear and, even, of molecular systems are much closer to the dimensions of living bodies than are the dimensions of galaxies. Yet the fundamental properties of these small systems and the laws that govern their behaviour are quite different from those in our immediate neighbourhood. Meanwhile, the astronomer sticks to the idea that nothing but gravitation governs the larger systems ad infinitum – only recently has the cosmic importance of magnetism been realized.

In the infinite universe, the scale of systems is infinite and so, also, is the number of their properties and of the natural law.

DISCUSSION

B. Lindblad: I believe that, when talking of tidal forces, Dr. Vorontsov-Velyaminov ignores the effects of resonance by which ejection, in showers or streams, may occur up to very great distances from the disturbed systems, both on the near and far sides, reckoned from the disturbing component.

Vorontsov-Velyaminov: I have already said in my report that Dr. Lindblad has succeeded in explaining some forms of interaction by means of a certain mechanical model. But, taking the evidence as a whole, it appears that some other explanation must be looked for in order to understand the variety of interactions between galaxies. Of course, one must try to use the mechanical models, and I appreciate very much the researches carried out in Stockholm.

Page: The existence of chains of galaxies, bridges, and filaments that cannot have long duration under ordinary Newtonian mechanics may still have happened as a "chance result" of boundary conditions set by an earlier phase in the history of the Universe. Before a chain of galaxies can be said to be "improbable," we need a theory of the *formation of galaxies.*

Ambartsumian: In reply to Dr. Page's question, may I state that we can really understand the origin of chains only if we have a general theory of the origin of galaxies. But probabilistic considerations are important when we have to prove that the *chains are physical* and not optical.

Hoyle: Is it your impression that spiral arms end at the edge of a galaxy? Or could they go out into inter-galactic space in the manner of filaments?

Vorontsov-Velyaminov: Sometimes the spirals have faint outer arms, but, generally, the spiral arms end in a surprisingly abrupt manner. You can see badly overexposed galaxies, yet the spiral arms and the structure have the same boundary as on normally exposed plates. There is, sometimes, a faintly luminous structure outside, but with no definite structure.

Gold: While gas and magnetohydrodynamics are probably responsible for the long tails and bridges, one has to consider what the dominant force will be once the material is in stellar form. A long string of stars would be gravitationally unstable about breaking up into segments. Do we really know that the stars are old? In that case, we need to find an explanation not only of the setting up, but also of the avoidance of, the instability.

P. O. Lindblad: According to the picture of gravitational resonance effects, the connecting bridges would not have to be stable configurations, but would rather be streams of matter supplied from the narrow resonance region within the main body. So we would still have the possibility of finding old stars in the bridge.

G. Burbidge: I would like to ask Dr. Zwicky how many spectra he has obtained of luminous bridges?

Zwicky: The luminous bridges between interconnected galaxies are young, by testimony of the relative velocities. Less than 10 per cent show emission lines due to fluorescent gases and hot, embedded giant stars; 90 per cent are smooth, showing no indication of resolution, no supergiant hot stars, and they are mostly blue. It is thus indicated that most of the bridges are composed of ordinary, or even subluminous, old stars.

Harwit: Professor Gold's comment is in agreement with some work which Professor Hoyle and I have just completed. The relative recession velocities and lengths of filaments indicate an age of the order of 10^{16} sec. The dynamic breakup time also appears to be at this order of magnitude. One may expect bridges to break up into smaller lumps – presumably the observed chain galaxies.

Vorontsov-Velyaminov: The observed chain galaxies are too large to be formed from the filament. Only very dwarfish galaxies can originate when the filament breaks. Probably they exist but we cannot observe them at present.

Zwicky: Gravitational tidal actions and effects from magnetic fields should not be neglected in trying to account for spiral arms and filaments between galaxies. Certain interactions and phenomena, not yet considered in the literature, however, can account for both the filaments and for parallel star streams.

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Identification with Optical Objects

The uniform distribution of radio sources away from the galactic plane has always been considered as an indication that most of these sources are extra-galactic. This interpretation found quantitative support in a discussion by Ryle [1] who could show that the small sizes of many unidentified sources indicate that most sources are intrinsically strong and at large distances. That not all sources at high galactic latitudes are extra-galactic, however, is now firmly established by the identification of the source 3C48 with a galactic, possibly stellar, object [2]. Caution is, therefore, needed in the interpretation of data from unidentified sources, in particular, as far as the small sizes are concerned which are shown by a small fraction of the sources [3].

The great majority of the extra-galactic objects with which sources have been identified seem to be completely normal, both in their appearance and in their spectra, where these are available. This is in sharp contrast to the early identifications, of which many led to highly peculiar objects. Most of the identifications now are not supported by peculiarities of the objects but rest entirely on positional coincidence.

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The size and structure of a source can be important supporting evidence, but it is well known that a source may differ widely in size and structure from the optical object. The size of a source, particularly if measured only in one coordinate, can be misleading. Of particular importance for the identification problem are the observations of the brightness distribution of Fornax A (NGC 1316) by Wade [4]; the galaxy is single, but the source is double and asymmetrical and its center of gravity does not coincide with the center of the galaxy. A pronounced asymmetry might make it difficult to find the proper identification [5]. There is no reason, however, to believe that difficulties of this kind play a major role in the identification problem.

Detailed individual studies of the many objects of normal appearance that are now known as intrinsically strong sources cannot be expected to contribute much to our understanding of the physical conditions that lead to strong radio emission. The investigation of the size and structure of the sources is more likely to give relevant information. But the number of identifications now available is adequate to make the first tentative attempts at a statistical exploration.

Sound statistical conclusions can be obtained only from data that refer to a unit volume. It is possible to derive the space density of sources from lists of identifications which refer to a defined area of the sky in which the identifications may be assumed to be complete to definite limits of apparent photographic magnitude and of apparent radio magnitude [6]. The number of identifications now available is too small to establish the space density for separate types of sources. If one wants to differentiate between different types, the best that can be done is to use the data in the selected lists. Two such lists have been prepared: one, from sources of the declination zone -10° to +20° of the survey at 86.5 Mc/s by Mills, Slee, and Hill [7]; the other, from the 3C-survey at 159 Mc/s [8]. The identifications for the MSH survey come from searches by the author and by Mills [9], those for the 3C-survey from searches by Dewhirst [10] and by Bolton and his group [11].

The results from the two sample areas are summarized in Table 1. While there are differences in detail, the general trend of the data is much the same for both areas. The differences that exist can be easily understood as consequences of the difficulty to classify faint galaxies correctly and to recognize double galaxies fainter than 18^{m} on plates of the 48-inch Schmidt telescope [12].

Many double ellipticals have been identified as sources. This has been considered as indication of stimulation of radio emission in double galaxies as long as single elliptical galaxies seemed, in

TABLE 1

Identified Sources in Selected Areas

	MSH	*Notes*	*3C*	*Notes*
m_r limit for selected sources	$9^{m}.2$		$8^{m}.5$	
m_p limit of search for single galaxies	16^{m}		18^{m}	
m_p limit of search for double galaxies	18^{m}		20^{m}	
Size of area	3 ster.		3 ster.	
Number of sources	230		80	
Number of identifications	23		25	
Apparently normal single galaxies	10		15	
E, SO	4		7	
Sa, Sb	1		...	
Sc	4		1	
faint, type uncertain	...		7	
Apparently normal double galaxies	9		6	
Peculiar single galaxies	2	(1), (2)	2	(3), (4)
Peculiar double galaxies	2	(5), (6)	2	(5), (7)

(1) NGC 1068, Sb, strong nuclear emission (Seyfert galaxy).
(2) 05 +02, Sa, with heavy absorption lane, possibly similar to NGC 5128.
(3) 3C 66, EO, very faint "jet": no emission.
(4) NGC 4486, EO, jet, strong and broad $\lambda 3727$ in nucleus.
(5) 00 – 015 = 3C 26, SO + Sb, distorted, emission.
(6) 09 – 014, Hydra A, double E with strong emission.
(7) NGC 1275, Sc + Irr, absorption and emission lines of both systems, = 3000 km/sec.

general, to be weak emitters. Since a substantial number of intrinsically strong sources has now been identified with single elliptical galaxies, the argument is no longer valid. The apparently large number of double galaxies has been considered as another indication of peculiar effects in such systems. Actually, the high incidence of doubles is only due to a selection effect arising from the fact that the limit of search for identifications was set fainter for double galaxies than for single galaxies [12].

The limit of search has been chosen by all observers so that there is only a small chance – usually less than 10 per cent – of accidentally finding an object of a given type in the error area of a source position. Since double systems are less frequent than single galaxies, a fainter limit can be set for doubles. Holmberg's data [13] on the frequency of double systems and independent tests by several observers lead to a limit for doubles about 2 magnitudes fainter than for single galaxies. Under the circumstances of the search, the number of identifications of single and of double galaxies refer to different volumes chosen so that they contain equal numbers of both types, i.e., twice as many galaxies in double systems as in single galaxies. If membership in a double system has no effect on the radio emission and doubles are sources because one component is a source, the number of identifications with doubles should be twice that of identifications with single galaxies. The number of identified double galaxies, thus, is actually less than might be expected. This deficiency is easily explained as a consequence of imperfections of the search. In any case, there is no valid evidence of an excess of sources connected with double galaxies of normal appearance. If normal-appearing double galaxies exist in which interaction between the components stimulates radio emission, they must be rare. Most double galaxies seem to be sources because one component is a source.

The data for doubles of normal appearance have no bearing on peculiar doubles such as Cygnus A, Hydra A, and NGC 1275 where appearance and spectrum show unusual conditions. It seems clear that sources connected with such peculiar doubles must be rare. The sample in Table 1 contains only one new source (00-015 = 3C 26) that may belong in this group. The source 3C 295 is not included in the sample since it was identified by a special search. However, it may belong in this group; it is closely similar to Cygnus A as to the strength of radio emission and as to the optical spectrum, but too distant to show its structure.

Equally rare as peculiar doubles appear to be peculiar single galaxies. But the peculiarities of single galaxies are relatively subtle. The "jet" in NGC 4486 is a very peculiar feature, but it is not to be expected that it would be visible at a much greater distance, nor is it certain that it should occur in all galaxies in which the physical conditions are similar to those in NGC 4486. Only one source, 3C 66, was found that shows a faint feature possibly similar to the "jet." Such features as the dust clouds in NGC 1316 or the absorption lane in NGC 4474 (M 84) are also difficult to see at much larger distances. Many, or even all, of the galaxies with normal appearance and strong radio emission might be in one class with the peculiar single galaxies.

Another question is that of whether there is a difference between the "normal bright galaxies," i.e., nearby definitely normal galaxies with weak radio emission such as, e.g., NGC 224 (M 31), and the galaxies of normal appearance and strong radio emission. Some evidence favors the existence of such a difference. P. R. R. Leslie's results [14] show that, of 35 "normal bright galaxies," at most one has $m_r - m_p < -1$, and the discussions by Long and Marks [15] seem to show that there are very few sources with $m_r - m_p$ in the range -2 to -4. This would be an indication that two classes of sources exist, but the numbers of sources involved are too small for statistically significant results. A possible systematic difference between the radio spectrum of "normal bright galaxies" and galaxies with strong emission has been reported by Heeschen [16], but recent observations by Goldstein [17] do not seem to show such a difference.

An association between radio sources and clusters of galaxies seems to be well established. Mills [9] finds a statistically significant number of coincidences between radio sources and clusters cataloged by Abell [18]. The same conclusion has been reached for 3C-sources by van den Bergh [19]. The identified sources in Table 1 also seem to show the association. Forty-two per cent of the normal-appearing galaxies from the MSH list and 33 per cent of those from the 3C-list are in clusters (including the Virgo cluster). The lower fraction for the 3C list may arise because some of these objects are more distant than Abell's clusters. On the other hand, only 10 per cent of all galaxies are in the rich clusters listed by Abell [12], [20].

The association with clusters is possibly more pronounced for the sources identified with peculiar objects. Altogether 11 such sources are now known. Seven of them are in clusters, four are not. It is tempting to think that it is no accident that Cygnus A and 3C 295, the intrinsically strongest sources known, are in clusters of galaxies.

Spectroscopic observations are available for 13 objects included in Table 1. With a Hubble constant of 75 km/sec/Mpc, the mean absolute photographic magnitude of these 13 galaxies is -20.8 (±0.3 m.e.). This seems to indicate that the galaxies that are radio sources are brighter than average. It is necessary to emphasize, however, that the value suffers from two selection effects. The apparently brighter galaxies were observed only spectroscopically and the apparently brighter galaxies are preferred in the search for identifications; both selections could easily lead to a preference for absolutely bright galaxies. There is no doubt that some sources are very luminous systems, such as NGC 5128, $M_p = -22$, and NGC 6166, $M_p = -22.3$.

The luminosity function of sources can be determined if the possible existence of different classes of sources is disregarded and all sources are grouped together. Attempts made by Mills [9] and by Long and Marks [15] do not derive the space density of sources. In both investigations, the data do not refer to a unit volume; this would require limiting distances, but only limiting apparent photographic magnitudes could be used. The space density can be derived in a simple way if the distance of individual sources is known.

For the objects with known red-shift, the distance modulus and the absolute radio magnitude are known. For the other objects, the difference between apparent photographic magnitude and the mean absolute photographic magnitude -20.8 gives distance moduli and absolute radio magnitudes that are statistically correct. It is then possible to count, in an interval M_r to $M_r + dM_r$, the number of sources with absolute radio magnitude M_r that are stronger than the limiting apparent radio magnitude m_r. For these sources, the maximum distance modulus is $m_r - M_r$; they are thus contained in a known volume so that their space density can be computed. The number $N(M_r)$ per Mpc^3 of sources with M_r in the interval $M_r - \frac{1}{2}$ to $M_r + \frac{1}{2}$ thus derived is

$$\log N(M_r) = 6.9 + 0.48\, M_r \qquad (1)$$

between $M_r = -21$ and $M_r = -34$ [6], [11]. In the range from $M_r = -24$ to $M_r = -30$, the data are fairly reliable; for sources stronger than $M_r = -30$ or weaker than $M_r = -24$, considerable uncertainty exists. Eventually, the luminosity function might offer a way to decide whether different classes of sources exist. With the present data, any indications of structure – such as a possible inflection at $M = -22$, which corresponds to the minimum shown by Long and Marks at $m_r - m_p = -3$ – are not statistically significant. Identifications of about 200 or more sources and determinations of red-shifts for at least a large fraction of them will be necessary to establish the presence or absence of structure in the luminosity function.

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DISCUSSION

D. S. Evans: Results of absorption-line velocities in the elliptical component of NGC 5128 by Evans and Harding give a mean of about +430 km/sec compared with the central velocity of +600 km/sec found by the Burbidges from emission lines. There is no marked rotation. The relatively small difference of velocity is probably against the hypothesis that the system consists of two interacting systems. Star counts suggest a deficiency of numbers near NGC 5128, but there are other deficient areas nearby. In view of the large size of the radio source, it is not certain whether all these sparse areas are significant. Star counts should be undertaken on Schmidt plates to decide whether the object is distant or not.

Matthews: I show slides of ten identified radio sources which summarize the California Institute of Technology results on accurate positions and brightness distribution. The majority are unequal radio doubles separated by several minutes of arc. In most cases, the position of the radio source falls on the suggested optical identification.

Gold: In view of the fact that the great majority of radio galaxies are of normal appearance, unusual appearance of a galaxy is not a strong criterion for the identification any more. Therefore, how certain do you feel of the identification of the Cygnus A source?

Minkowski: There is no doubt about the identification of Cygnus A. The chance of an accidental coincidence of a highly peculiar object with a source whose position has been determined with very high accuracy is less than 10^{-5}.

Ambartsumian: What is the meaning of your statement that many of the sources are normal galaxies? Have you observed all listed galaxies by means of the 200-inch?

Minkowski: Yes.

Dewhirst: Dr. Minkowski is, of course, correct in saying that the apparent preponderance of double and multiple galaxies in a list of all available identifications is due to observational selection arising from the method of search. If one analyzes a list of objects down to a suitable apparent magnitude one should largely remove this effect, but the numbers become rather small for statistical interpretation. They do not exclude the idea that the excess of radio galaxies which are members of double or multiple systems may still be real. The brighter galaxies tend to be very close pairs: in the selection of fainter doubles a much stricter criterion of "double" has been used than was used by Holmberg. The percentage of galaxies that are members of *close* pairs is much less than Holmberg's 53 per cent that are members of *all* pairs on his less rigorous criterion, so that less than half the identified radio galaxies as members of double systems could still constitute a significant excess.

Minkowski: The difference of 2^m between the limits of search for single and double galaxies is not based on Holmberg's 53 per cent that are members of all pairs on his less rigorous criterion, but on the frequency of those doubles of the type used for identifications. This frequency was established by sampling of 48-inch Schmidt plates; it is in agreement with the frequency of close doubles of appropriate type in Holmberg's catalog.

The frequency of the close doubles was also checked on a random sample of 200-inch plates covering about 2.5 square degrees. This random sample consisted of plates obtained in inspecting the positions of radio sources which later were found to be elsewhere or not to exist. It is, thus, a sample which contains no strong sources.

While it seems certain that there is no excess of double galaxies of normal appearance, it is not at all excluded that interaction occurs in some of these objects.

Oort: I should like to obtain some clarification on the fraction of bright radio galaxies that can now be identified. Is it of the order of 10 per cent? Or does it appear possible to identify the majority of the sources in the 3C Catalogue?

Minkowski: The fraction of sources that can be identified depends on limiting radio magnitude and accuracy of positions. The data in Table 1 indicate that about 30 per cent of the 3C sources with flux density larger than 25×10^{-26} w m^{-2} $(c/s)^{-1}$ have been identified. With the positional accuracy obtained by Ellsmore, Ryle, and Leslie, and the Owens Valley Observatory, about 80 per cent of these sources might be identified; the complicated brightness structure of many sources is likely to reduce the possibilities of identification. The fraction of sources that can be identified decreases rapidly for fainter sources.

Bondi: Can you tell us a little more about the spurious identifications, and especially whether in positions later found to contain no radio sources any suspiciously peculiar galaxies were found?

Minkowski: The fraction of identifications once considered as definitely established, but later found to be spurious, is of the order of a few per cent.

G. Burbidge: I think that it is better to use the term "peculiar" to describe a radio source only if the galaxy associated with it shows strong high excitation emission features. Surface peculiarities will not be recognized if the galaxies are faint.

Minkowski: Where peculiarities of structure are clearly recognizable, there is no reason not to speak of a "peculiar" object. The narrower definition of peculiarity based on peculiar emission spectra may be useful, but it is not very practical in view of the difficulty of obtaining spectra of galaxies fainter than the 18th magnitude.

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Physics of Radio Frequency Radiation

The main physical properties of abnormal radio galaxies seem to be:

1. The large extent of the emitting region which forms a gigantic halo whose extent may be anything from 30 to 500 kpc.
2. The complex character of the distribution of radio brightness, which, however, can often be interpreted as being produced by two, usually unequal or asymmetrical, components. (Some radio galaxies show two equal components, others a halo and core, still others have a strongly elongated contour.)
3. The general similarity of the radio spectra and spectral indices, as far as these are known.
4. No particular correlation exists between diameters and radio luminosities.
5. Emissions range from less than 10^{43} ergs/sec ($M_r = -27$) up to (1/2) 10^{45} ergs/sec ($M_r = -35$), the numbers in erg/sec being more uncertain – possibly on the low side – than are the M_r.
6. Volume densities range from about 10^{-5} (Mpc)$^{-3}$ for radio galaxies with $M_r = -24$ to 10^{-8} (Mpc)$^{-3}$ for those with $M_r = -30$.

The synchrotron mechanism is the only one which seems to explain the observed phenomena at the present time. The rest of my discussion will, therefore, be based on the assumption that it is the prevailing mechanism of emission. It should not be forgotten, however, that new observations may well change the weight of the arguments for or against this assumption.

It may be concluded from (4) and (6) above that the magnetic field may very well be a more permanent feature than the relativistic electrons, which, especially in the sources of largest intrinsic brightness, are possibly of a more transient nature. The appearance of the sources as described in (2), would then be a consequence of the geometrical features of the magnetic field. A toroidal configuration having, for example, screw-type features as in force-free magnetic configurations, is probably too simple a model of such a field. Following Burbidge [1], one can estimate the strength of the magnetic field: if the energy densities of the magnetic field, of the relativistic electrons, and of the relativistic photons are of the same order of magnitude, then values of a fraction of 10^{-5} gauss to some multiple of 10^{-5} gauss are obtained. The emitting volumes being of the order of 10^{68} to 10^{71} cm^3, total energies of the order of 10^{57} to some 10^{59} ergs are found. For a mass density in such a halo of $\leqq 10^{-26}$ gr/cm^3, a lifetime for energetic protons (as determined by nuclear collisions) of about 10^{10} years or more can be determined. This suggests the possibility of an early origin for the relativistic protons and for the magnetic field [2].

The properties of the associated galaxies have been described by Minkowski in the preceding paper. According to Bolton [3], they are, on the average, of high luminosity with M_{pg} almost equal to −21. The average mass-to-light ratio should also be large because of the number of E and S0 galaxies among the radio sources. Thus, masses of the order of one or more times 10^{12} $M_\odot$ might be common. This would lead to a gravitational energy density of the same order of magnitude as that of the magnetic field. Hence, the virial theorem would not necessarily introduce a difficulty. The total energy of the relativistic electrons, and so on, of the order of 10^{58} ergs would be equivalent to 1 per cent of the energy gained in converting 10^8 $M_\odot$ of hydrogen into helium. The lifetime of the relativistic electrons found from the data given above is 10^8 to 10^7 years.

The following conclusions may be drawn regarding the origin of the relativistic electrons. A primary origin due to an acceleration process acting on all charged particles with similar efficiency, as in solar flares, seems most probable. A secondary origin, due to the collisions of the protons and the gas in the halo, seems less likely. In this case, the energy requirements would also be much more stringent because of the larger number of protons required. The relativistic electrons might come from supernovae or from active stars, as is being currently discussed in the theory of the origin of cosmic rays. The amounts involved, though large, do not seem to be impossible on general grounds. The very brightest objects, such as Cygnus A, might all be special cases. One might speculate that perhaps one out of every hundred, or every thousand galaxies has a halo with the required properties, and that, of these, one out of a thousand has, at the present time, relativistic electrons of the appropriate energy. I remind you that the lifetimes of the relativistic electrons run from 10 million to 100 million years. It is not known whether a transient and intense production of energetic electrons is connected with any specific stage in the evolution of the optical galaxy.

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Membership of Clusters of Galaxies

Twenty-five clusters of galaxies are listed in Shapley's (1933) catalogue [1]. By 1948, there were still only several dozen known clusters. In the subsequent decade, however, clusters of galaxies were found in large numbers on photographs taken for the Lick Astrographic Survey [2,3,4] and for the Palomar Observatory Sky Survey [5]. Neyman, Scott, Shane, and Swanson [6,7,8,9] have shown that the galaxian distribution on the Lick Survey is compatible with the hypothesis that all galaxies belong to clusters. The writer's catalogue of rich clusters identified on the Palomar Sky Survey lists 2712 entries [5]. The comprehensive catalogue of galaxies and clusters being prepared by Zwicky and his associates will list many thousands of clusters [10,11]. It now appears that clusters of galaxies may be fundamental condensations of matter in space.

GENERAL APPEARANCES OF CLUSTERS

Clusters of galaxies range from rich aggregates of at least ten thousand members to the relatively poor groups, such as the Local Group, which contains only 17 known members, or even to double and triple systems, if these can be classed as clusters. Groupings of galaxies that are probably physical associations can be recognized by the tens of thousands on the Palomar Sky Survey photographs. Only a few nearby groups, however, have been studied in detail; our knowledge of the rare, relatively rich clusters is actually more complete.

The rich clusters of galaxies can be roughly classified into two categories: *regular* clusters and *irregular* clusters. The regular clusters possess spherical symmetry and show marked central concentration. They tend to be very rich clusters and most of them probably contain at least a thousand members in the brightest 8-magnitude interval. The regular clusters have structures resembling those of globular star clusters; they are sometimes referred to as *globular clusters of galaxies*. Examples are the Coma Cluster and the Corona Borealis Cluster – Numbers 1656 and 2065, respectively, in the writer's catalogue. The irregular clusters, sometimes called *open* clusters, have a more nearly amorphous appearance, and possess little or no spherical symmetry or central concentration. The irregular clusters sometimes, however, have several small subcondensations or nuclei and resemble loose swarms of small clusters. They range in richness from small groups, like the Local Group, to rich aggregates of more than a thousand members – e.g., the Virgo Cluster. There also exist large clouds with linear dimensions of the order 50×10^6 pc (for an assumed Hubble constant, $H = 75$ km/sec/Mpc). The clouds seem to consist of several distinct clusters [2,5], and are classed by the writer as second-order clusters.

Zwicky [10,12] classifies clusters of galaxies as "*open*," "*medium compact*," "*compact*," and "*very compact*." The first two of Zwicky's classes are irregular clusters; many of the clusters in his *compact* and *very compact* classes are regular.

The brightest members of the regular clusters are usually all, or nearly all, elliptical or S0 galaxies. Few, if any, of the member galaxies show the spiral characteristics of interstellar matter, although forbidden emission lines are often present in their spectra, particularly the 3727 doublet of [O II]. Frequently, but not always, a regular cluster is centered upon one or two giant elliptical galaxies; the Coma Cluster, for example, is centered on a point between its brightest members, NGC 4874 and NGC 4889 [=NGC 4884]. The absence of spiral galaxies in the rich regular clusters may be explained by collisions between member galaxies and the consequent removal of interstellar material from them – a hypothesis proposed by Baade and Spitzer in 1951 [13]. Irregular clusters, on the other hand, contain many spirals among their brightest members. Spiral galaxies are also common among the brightest members of the smaller groups. The three brightest members of the Local Group, for example, are the Galaxy, and the spiral systems, M 31 and M 33. Irregular galaxies of the Magellanic Cloud type are also common in irregular clusters. The faintest galaxies in all kinds of clusters are probably dwarf ellipticals of the Sculptor type, unless inter-galactic globular clusters or individual stars can be considered galaxies.

THE COMA CLUSTER

As an example of a regular cluster, we consider the Coma Cluster of galaxies (Figure 1), which is Number 1656 in the writer's cluster

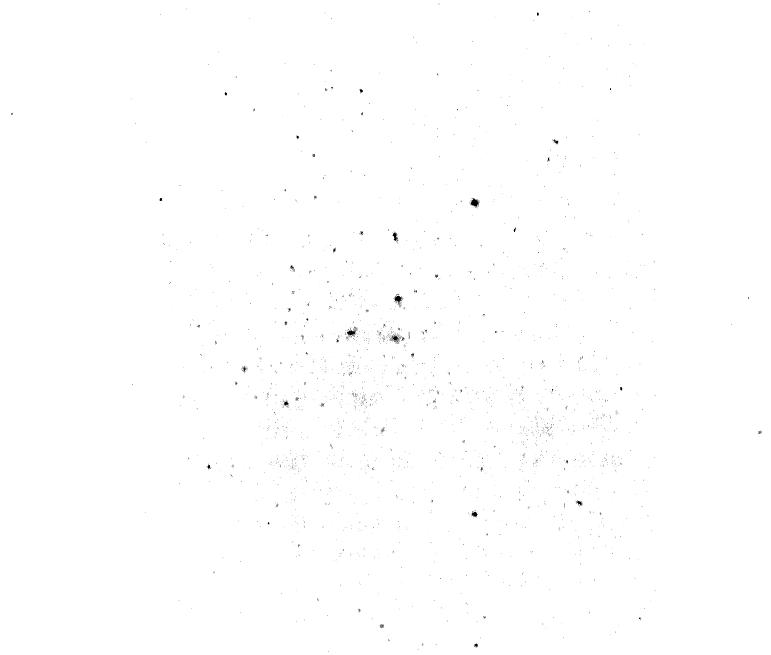

Figure 1. The Coma Cluster of galaxies (No. 1656). North is at the top, East to the left, (Scale: 1 mm = 00″).

catalogue, and has equatorial coordinates: (1950) $\alpha = 12^h\ 57^m\!.3$, $\sigma = +28^\circ\ 14'$. According to Zwicky [14], there are 804 galaxies brighter than $m_{pg} = 16.5$ within 160′ of the cluster center, and 29,951 galaxies brighter than $m_{pg} = 19.0$ within 360′. He estimates the actual cluster population (after correction for the field galaxies) to be 10,724 members brighter than $m_{pg} = 19.0$. In a region of 7 square degrees centered on the cluster, the writer has counted 1722 galaxies brighter than $m_{pg} = 19.3$, of which about 1000 are probably cluster members. The total number of galaxies counted by Zwicky in the same region is about 3400. It is probable that Zwicky's estimate of the limiting magnitude of his plates is in error. If his counts refer to all visible galaxies on photographs obtained with the 48-inch Schmidt telescope, the faintest of them are probably near $m_{pg} = 20.0$.

The angular size of the Coma Cluster is a subject of controversy. According to Zwicky [14], the diameter is at least 12°. On the other hand, T. Noonan [15] has pointed out that Zwicky's counts of the total numbers of galaxies within projected circles of radius r, centered on the cluster, increase as r^2 for r greater than 100′, which suggests that those regions may represent a background distribution, in which case, Zwicky's large diameter and, thus, his large population, for the cluster may result from irregularities in the background at large distances from the cluster center. In any case, all observers agree that the Coma Cluster has an angular radius of at least 100′.

Zwicky [16] has also shown that the projected distribution of galaxies in the inner part of the Coma Cluster can be fitted, approximately, to the projected mass distribution in the isothermal gas sphere. It has not been shown that the isothermal polytrope uniquely represents the structure of the cluster, but the available evidence is compatible at least with the hypothesis that its inner parts are in statistical equilibrium. If the cluster is assumed to be, actually, in a steady state, its mass can be calculated from the virial theorem. The mean red-shift of the cluster members is 6850 km/sec [15]; for an assumed value of the Hubble constant, H, of 75 km/sec/Mpc, the distance to the cluster is about 90×10^6 pc, which corresponds to a distance modulus of about 35 magnitudes. Red-shifts are available for 50 galaxies that are probably cluster members, 22 of which have been published by Humason, Mayall, and Sandage [17] and 28 of which are unpublished observations of Mayall. For these 50 objects, the root-mean-square deviation from the mean red-shift is 1050 km/sec. The velocity data, when combined with the potential energy of the cluster, computed by T. Noonan [15], lead to a total cluster mass of 2×10^{15} solar masses.

The writer's luminosity function of the Coma Cluster gives for the total photovisual luminosity 4.5×10^{12} solar units, which results in a mass-luminosity ratio of 470. This ratio will be higher than the true value if:

1. Some of the galaxies whose velocities were used in computing the kinetic energy of the cluster are foreground or background objects of radial velocities quite different from that of the cluster.
2. The distance to the cluster is greater than 90×10^6 pc.
3. The cluster is expanding.

Regarding the latter case, however, Noonan has pointed out that if the mass-luminosity ratio is to be reduced by, say, a factor of 20, the potential energy of the cluster would be only 1/10 of its kinetic energy; i.e., gravitational forces would be negligible and the cluster would dissipate in a time of the order of 2×10^9 years.

The brightest members of the Coma Cluster are at $m_{pv} \sim 12$. If $m - M = 35$, those brightest *E* galaxies are at $M_{pv} \sim -23$, or about 10^{11} times the solar luminosity.

THE VIRGO CLUSTER

As an example of an irregular cluster, we consider the Virgo Cluster. The major part of the cluster covers an elliptical region with dimensions about 10° by 12°, centered near: (1950) $\alpha = 12^h\ 28^m$, $\sigma = +11°$. According to Sandage [18], the distance modulus is probably between 30.6 and 30.8. For $m - M = 30.7$, the distance to the cluster is 14×10^6 pc, and the linear diameter is about 3×10^6 pc. Of the 34 extra-galactic objects in Messier's catalogue, 16 are Virgo Cluster members. According to Zwicky [19], the cluster has at least one thousand members bright enough to be photographed with the 48-inch Schmidt telescope.

The Shapley-Ames Catalogue [20] lists 205 galaxies that are members of the Virgo Cluster. Of these 139 are spirals, 39 are ellipticals, and 27 are irregular or unclassified. Of the 272 brightest galaxies in a 6.6-degree square field centered at (1950) $\alpha = 12^h\ 25^m$, $\sigma = +13°$, 53 per cent are spirals and 47 per cent are elliptical and S0 galaxies. Elliptical and S0 galaxies are less dominant in the loose, irregular clusters than in the highly concentrated regular ones, but are, nevertheless, relatively more numerous than they are among the bright galaxies in the "general field"; only 15 per cent of the Shapley-Ames galaxies are ellipticals. G. Reaves [21] has shown that the Virgo Cluster contains many galaxies of low surface brightness and small angular size; most of these are probably dwarf ellipticals, similar to the dwarf ellipticals in the Local Group. Of 76 dwarf galaxies studied by Reaves, only 11 appear to be definitely not of the elliptical type.

The brightest galaxies in the Virgo Cluster are, according to Holmberg [22], and Sandage [17], at $m_{pv} = 8.0$ to 8.5. For a distance modulus of 30.7, we find that those galaxies lie in the range $m_{pv} = -22.5$ to -23.0.

Among the 178 elliptical and spiral Shapley-Ames galaxies in the region of the Virgo Cluster, the distribution of 139 spirals and 39 ellipticals east and west of the cluster center are shown in Table 1. The tabular data illustrate: (*a*) the low central concentra-

TABLE 1

*East-West Distribution of 178 Virgo Cluster Galaxies**

Distance (degrees)	*1*	*2*	*3*	*4*	*5*	*6*	*>6*	*Total*
Spirals	39	28	23	22	14	7	6	139
Ellipticals	19	4	5	8	1	1	1	39
Totals	58	32	28	30	15	8	7	178

*Adapted from Zwicky [19].

tion of the Virgo Cluster of galaxies; and (*b*) that the elliptical galaxies show greater concentration to the cluster center than do the spirals. A χ^2 test shows that the probability that the east-west distribution of the ellipticals is a random fluctuation of the east-west distribution of the spirals is less than 0.02. De Vaucouleurs [23] has also called attention to the different spatial distributions of the spiral and elliptical galaxies in the Virgo Cluster, and has suggested that the cluster may be, in reality, two clusters seen in superposition, one consisting largely, or entirely, of elliptical and lenticular galaxies, and the other of spiral and irregular galaxies.

Humason, Mayall, and Sandage [17] list radial velocities for 66 galaxies in the Virgo Cluster (not including a southern extension of the cluster). Sandage gives for the mean velocity of the cluster +1,136 km/sec. There appears, however, to be a systematic difference between the mean velocities of the elliptical and spiral galaxies; de Vaucouleurs [23] quotes +962 km/sec for the ellipticals and +1362 km/sec for the spirals. This systematic difference seems to lend support to de Vaucouleurs' hypothesis that the Virgo Cluster is not a single system. The total mass of the cluster can be computed by application of the virial theorem or by assuming that half the total velocity range of 3000 km/sec is the velocity of escape from the cluster. In either case, the mass is found to lie within a factor of two of 5×10^{14} solar masses. The total luminosity of the cluster is difficult to determine, but if the above mass is correct, the mass-light ratio can hardly be less than 400. The total mass of the cluster is not changed if, following de Vaucouleurs, the masses of the elliptical and spiral components are computed separately.

The improbably high values of the mass-light ratio obtained for many groups and clusters of galaxies have led Ambartsumian and others (see, for example, the Burbidges [24]) to conclude that these systems are unstable and are dissolving. If such systems are, indeed, unstable, they must be very young; the Virgo Cluster, for

example, must be less than 9 × 10^8 years old if gravitational forces are not holding it together [25]. The Burbidges [24] encountered a similar problem in the case of the Hercules Cluster (Figure 2) (Number 2151 in the writer's catalog), which is morphologically similar to the Virgo Cluster. There is also evidence that the Hercules Cluster is part of a much larger cloud of groups and clusters (that is, a second-order cluster). The cloud is the last of six such features described by Shane and Wirtanen [2], and can be seen in Figures 7 and 10 of the writer's study of the distribution of rich clusters [5].

Figure 2. The Hercules Cluster of galaxies (No. 2151). North is at the top, East to the left, (Scale: 1 mm = 00″).

RICHNESSES OF CLUSTERS

It is impossible to specify the total "richness" of a cluster of galaxies. We do not even know the total population of the Local Group. The four dwarf elliptical galaxies discovered on the Palomar Sky Survey [26] bring the number of Local Group systems that can definitely be classed as galaxies to seventeen. However, there may very well be undiscovered dwarf galaxies in the Local Group, particularly near the plane of the galactic equator. Moreover, some of the globular clusters also discovered on the Palomar Sky Survey [27] appear to lie at distances comparable to those of the Magellanic Clouds [27,28]. If these clusters are representative of systems distributed more or less throughout the entire Local Group, and are not merely outlying members of the system of globular clusters associated with the Galaxy, then they can properly be classed as galaxies, as Zwicky [29] and others have proposed. It is not known, of course, how many systems of lesser richness may exist in inter-galactic space, or whether there may even be individual stars.

An appropriate operational definition of the "richness" of a cluster must specify: the magnitude interval which is to be included; the angular radius over which the cluster is assumed to extend; the fraction of the observed galaxies that belongs to the field (foreground and background objects that are not physically associated with the cluster); and the account which should be taken of the possibility of clusters of different distances being projected along the same line of sight.

The problem of "field" galaxies requires special comment. It now appears possible that *all* galaxies belong to clusters. Certainly, many small clusters and groups will not be identified as discrete systems when seen projected against the surface distribution of other galaxies. To be able to speak of a cluster as a physical association of galaxies, therefore, we must restrict our attention to those rich aggregates which stand out conspicuously, and for which there is a negligible probability of their being accidental coincidences of smaller groups or individual galaxies in the same line of sight. We regard these clusters as being superposed upon a general "field" of galaxies, the density of which may vary considerably from point to point, but in a smooth and continuous manner. We shall not concern ourselves with whether this "field" is actually composed of isolated individual galaxies, of clusters of galaxies overlapping in projection, or both. Of course, it is never possible to state with certainty whether a *particular* galaxy belongs to a cluster or to the field.

CLUSTER POPULATIONS IN THE CATALOGUE OF ZWICKY, HERZOG, AND WILD

As an example of an operational definition of the total richness or "population" of a cluster, we describe the procedure employed by Zwicky, Herzog, and Wild in the preparation of their extensive catalogue of bright galaxies and rich clusters [10,11]. The apparent photographic magnitude of the brightest member galaxy in a cluster is denoted m_{max}. A rich cluster is one which contains more than 50 members in the range m_{max} to $m_{max} + 3$. On limiting exposures obtained with the 48-inch Schmidt telescope, around the central concentration of each such rich cluster, an isopleth, or equal population contour, is drawn (presumably by eye-estimate) along which the numerical density of galaxies is about twice that of the field. The "population" that is catalogued for the cluster is the number of galaxies visible on the photograph within the area enclosed by the contour, minus twice the number of galaxies enclosed within the same area in the nearby field.

In Table 2 is shown the frequency distribution of clusters of various "populations," so defined, according to data for 119 clusters in four Palomar Sky Survey fields as published by Zwicky, Herzog, and Wild [10]. The figures are not directly related to true cluster populations, however, because a nearby cluster will show more galaxies above the plate limit than will a distant cluster of the same richness. The expected correlation is exhibited also in Table 2, in the breakdown of the numbers of clusters of various populations into Zwicky's five distance categories: "near," "medium-distant" (MD), "distant" (D), "very distant" (VD), and "extremely distant" (ED). According to Zwicky's estimates of the relative distances represented by these categories, if the brightest members of "very distant" clusters are at least 3^m brighter than the plate limit, those of "near" clusters are at least $6\overset{m}{.}6$ above the plate limit; for these near clusters, the "populations" corre-

TABLE 2

Numbers of Clusters of Various Populations as Given for 119 Clusters by Zwicky, Herzog, and Wild

Total Population	Number of Clusters (total)	Numbers of clusters in different distance categories				
		Near	MD	D	VD	ED
less than 50	19	0	0	0	7	12
51 - 100	49	2	4	11	23	9
101 - 150	28	0	5	16	7	0
151 - 200	12	0	2	5	5	0
201 - 250	3	1	2	0	0	0
251 - 300	3	3	0	0	0	0
301 - 350	0	0	0	0	0	0
351 - 400	1	0	1	0	0	0
401 - 450	2	1	0	0	1	0
451 - 500	2	0	1	1	0	0
Totals	119	7	15	33	43	21

spond, therefore, to the numbers of cluster galaxies within the contours to which the counts were made in the interval of the brightest 6^m or 7^m.

The clusters in the Zwicky-Herzog-Wild catalogue do not comprise a homogeneous statistical sample for at least three reasons in addition to the selection effect discussed in the last paragraph:

1. The isopleth contour about a cluster center that represents twice the field density contains a larger fraction of the total membership of a nearby cluster than it does of a distant cluster of the same richness and of similar structure (see Figure 3);

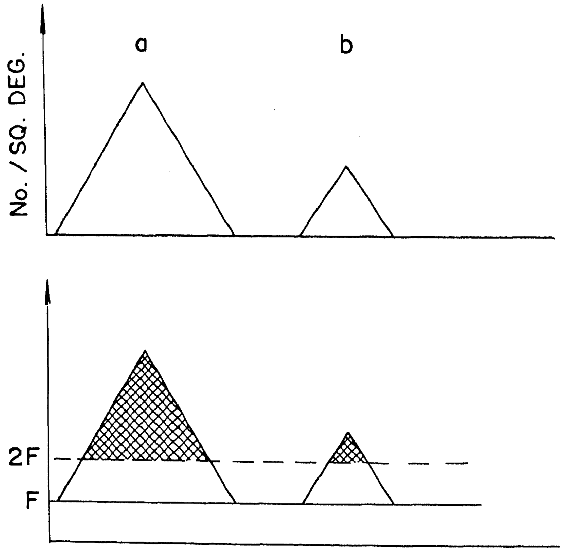

Figure 3. *Above*: Numerical density profiles are shown for two hypothetical clusters, *a* and *b*, which are presumed to have identical structures and total populations. Cluster *b* is twice as distant as *a*, so appears smaller, and fewer of its galaxies are visible above the plate limit. *Below*: When the densities of the galaxies of the two clusters are added to that of the field, *F*, it is seen that a smaller fraction of the visible members of *b* lie within the region where the density of the cluster + field $= 2F$ than in cluster *a*. The shaded regions represent the populations actually recorded.

2. The fraction of the cluster population included within the isopleth also depends upon the structure of the cluster (see Figure 4); and

3. The absolute magnitude of the brightest cluster galaxy can be expected to vary considerably from one cluster to another, and is more likely to be brighter in a richer cluster.

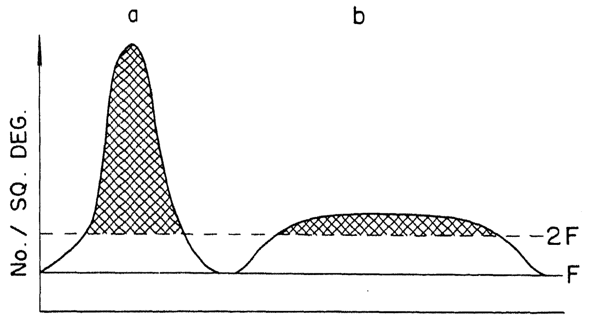

Figure 4. The two clusters *a* and *b* are presumed to have the same distance and total populations of visible galaxies, but different structures. It is seen that the fraction of visible galaxies that lies in the region bounded by the isopleth that denotes twice the field density, and which contributes to the recorded population, depends upon the structure of the cluster, and in particular, upon its central concentration.

Moreover, it is not generally possible to pick out the brightest cluster member unambiguously and be certain that it is not a field galaxy. The number of galaxies in the interval of the brightest three magnitudes is very sensitive to the point chosen for the bright end of that interval, for the luminosity function of galaxies in clusters increases rapidly in the magnitude region near three magnitudes from the brightest members.

CLUSTER POPULATIONS ACCORDING TO ABELL'S CATALOGUE

In his study of the distribution of rich clusters [5], the writer attempted to find an operational definition for the richness of a cluster that would be relatively free from systematic effects that depend upon the distance or structure of the cluster. His "richness" for a cluster is defined by a procedure that requires eight steps:

1. After a cluster is identified (in this case on the red Palomar Sky Survey plates), the tenth-brightest appearing galaxy in that cluster is picked out.

2. The red-shift of the cluster is estimated by comparing that tenth-brightest galaxy to a series of step-scale images that have previously been calibrated against the tenth-brightest galaxies of clusters of measured red-shift.

3. A circle is centered on the cluster image, whose radius, in millimeters, is equal to a constant divided by the symbolic velocity of recession that corresponds to the estimated red-shift, $cd\lambda/\lambda$. In the writer's study of clusters, the constant had the value, 4.6×10^5 km/sec.

4. The tenth-brightest galaxy within the circle is identified. If it differs from the galaxy picked out in step 1., steps 2. and 3. are repeated.

5. The third-brightest galaxy within the circle is identified. Its magnitude is estimated with a calibrated step scale of galaxian images.

6. Several galaxies within the circle and in the surrounding field are marked that, according to the step scale, are two magnitudes fainter than the third-brightest cluster galaxy.

7. All galaxies inside the circle are counted that are not fainter than the images marked in step 6.

8. Similar counts are made of galaxies brighter than the marked images, within circles of the same size, at two or three points in the field surrounding the cluster. The mean of the latter counts is subtracted from the count obtained in the circle centered on the cluster. This difference is taken as an indication of the cluster richness.

For clusters with $d\lambda/\lambda \leq 0.2$ (the limiting distance for those clusters included in the writer's survey), deviations from a linear velocity-distance relation are small. Therefore, the angular diameters of the circles within which cluster galaxies are counted are very nearly in inverse proportion to the distances of the clusters; i.e., galaxies are counted over about the same volume of space for all clusters. The constant (4.6×10^5 km/sec) was chosen such that the diameter of the sphere of space considered is small compared to the mean separation of cluster centers, but large compared to the main concentration of galaxies at the center of a cluster. Consequently, the counts of galaxies belonging to a cluster, after correction for the field, are quite insensitive to the sphere diameter; the recorded "richness" of a cluster would seldom be affected significantly if the radius of the counting circle were changed by 10 or 15 per cent.

A total of 1682 clusters was chosen from the writer's catalogue in such a way as to comprise a homogeneous statistical sample. The distribution of these 1682 clusters according to richness, as defined above, is exhibited in Table 3. Clusters of richness less than 50 cannot be assumed to have been completely identified and are omitted from the statistical sample. It is seen that the numbers of clusters of increasing richness drop off rapidly. On the other hand, no maximum is indicated in the distribution function of clusters according to richness. These data support the subjective impression gained by the writer as he inspected all of the Survey photographs: the most common clusters appear to be those of relatively low total population; the vast majority of systems of galax-

TABLE 3

Distribution of Clusters According to Abell's Richness Definition

Richness	*Number of Clusters*
50 - 79	1224
80 - 129	383
130 - 199	68
200 - 299	6
over 300	1

ies are probably sparse groups, of which the Local Group may be a typical example.

SIZES AND STRUCTURES OF CLUSTERS

It is a very difficult problem to define the total size or extent of a cluster. We have already discussed the problem of the size of the Coma Cluster. The maximum extent of a cluster is a datum of great importance for it is related to such problems as relative extragalactic distances and tests for cosmological models, the luminosity function of the galaxies in the cluster, the degree to which equilibrium has been reached within it, etc. For example, if the Coma Cluster extends to an angular radius of 6° or more, as Zwicky proposes, we must conclude, with Zwicky, that the fainter cluster members are distributed over a significantly larger volume of space than are the brighter ones [14]. Since, presumably, the luminosity of a galaxy is an indication of its mass, it would be implied that galaxies of lower mass have a larger mean distance from the cluster center than do those of higher mass, thus, larger mean velocities – a condition that would be expected if the energy of the cluster were equally distributed among its members. This is, in fact, one of the principal arguments advanced in favor of the hypothesis that the rich clusters are in statistical equilibrium. On the other hand, if the cluster extends only 100 minutes of arc in radius, then the widely spaced fainter galaxies belong to the field, and both bright and faint cluster members have similar distributions [15].

It is possible that the maximum diameters can be specified for the irregular clusters and smaller groups, e.g., the Local Group, but probably even for them, and certainly for the rich regular clusters, the total size cannot be determined unambiguously at present. It seems hopeful, however, that the regular clusters, which show similar structure to one another, can be represented by a single model. The factors by which that model must be scaled to match actual clusters may provide a satisfactory indication of at least their *relative* sizes.

THE STRUCTURAL INDEX OF A CLUSTER

Zwicky [30] has been able to fit the density distributions of several spherically symmetrical regular clusters to the density distribution of an isothermal gas sphere, arbitrarily bounded to avoid the difficulties of its infinite radius and mass. Emden's original solution of the structure of the isothermal polytrope [31,32] is in terms of dimensionless variables which can be scaled up or down by a homology transformation to fit an actual isothermal gas sphere. Zwicky has called the scale factor, a, by which Emden's independent variable, ξ, must be scaled up to produce the radius vector to the corresponding point in an actual cluster (i.e., $r = a\xi$) the *structural index* of the cluster.

The structural indices for four regular clusters are given below. The figures in the second column are the angular values published by Zwicky [30], and those in the third column are linear values for an assumed value of the Hubble constant of 75 km/sec/Mpc.

Structural Indices

Clusters in	*Minutes of Arc*	*parsecs* $\times 10^{-4}$
Coma	2	5.2
Hydra	4	5.4
Perseus	3.33	6.9
Cancer	1.77	2.9

We reiterate that it has not been shown that the distribution of galaxies in rich regular clusters is *uniquely* represented by the bounded isothermal polytrope, and we cannot conclude with certainty that the clusters are in statistical equilibrium. The isothermal distribution can be used as a *model,* however, which *describes* the structure of the clusters. Many other models are possible. In any case, an appropriate scale factor, such as the structural index, may be a satisfactory measure of the *relative* size of the cluster. It is probably the sort of "radius" that must be used in the application of cosmological tests involving linear sizes of systems of galaxies [33].

If a cluster *is* assumed to be in statistical equilibrium, and if its member galaxies interact only under the influence of gravitational forces, it can be shown [30] that the structural index, in linear units, is related to the galaxian velocity dispersion, $\overline{v^2}$, and and the "smoothed" central density of the cluster, ρ_0, by

$$\alpha = \left(\frac{\overline{v^2}}{12\pi G\rho_0}\right)^{1/2}, \tag{1}$$

where G is the universal gravitational constant. Zwicky has used the structural indices of clusters of known velocity dispersion to determine ρ_0, which determines also the total mass of the cluster. The approach is, of course, equivalent to the application of the virial theorem.

SUBCLUSTERING IN CLUSTERS

It has been demonstrated that clusters show a tendency to be found in groups or regions of high cluster density (second-order clusters) [2,5,34,35,36]. In addition, there is growing evidence for subsystems within clusters, at least in the irregular clusters. For example, two such subgroups in the Local Group are the Andromeda triple, M 31, M 32, and NGC 205, and the triple system composed of the Galaxy and the two Magellanic Clouds. The surface distribution of the Virgo Cluster is also quite irregular. Van den Bergh [36] and others have presented evidence that physical double and multiple galaxies exist in the Virgo Cluster. Reaves [37] reports that in the Virgo Cluster, as well as in some other groups, galaxies of low and intermediate luminosity most frequently appear to be associated with galaxies of high luminosity. Van den Bergh also has shown that small projected separations between galaxies in the Coma Cluster exist in greater numbers than would be expected if double and multiple systems did not exist – evidence that subclustering may possibly exist even in the rich regular clusters.

LUMINOSITIES AND COLORS OF CLUSTER GALAXIES

GALAXIAN PHOTOMETRY

The photometry of galaxies presents special problems that do not exist in stellar photometry. The various observers are not yet agreed even upon how the total magnitude (or luminosity) of a galaxy should be *defined,* much less, how it should be measured. Because elliptical galaxies are the most common members of the rich, regular clusters – which are the clusters of greatest interest in the study of the large-scale distribution of matter in space and in cosmological tests – we shall restrict our discussion here to E and S0 galaxies; the discussion may apply also, however, to the spheroidal components of spiral galaxies.

The difficulty of galaxian photometry arises from the fact that the light of a galaxy is not concentrated at a point, as is that from a star, but, rather, is spread over a considerable area. The most common standard photometric techniques do not allow properly for the light contributed from the outer, often unobserved, parts of the galaxy. It is true that the light from the highly centrally condensed nucleus of a typical elliptical or S0 galaxy goes appear almost stellar on focal photographs obtained with short focal-length cameras, and it dominates the extrafocal and square images obtained by extrafocal and schraffier photography. In the latter two cases, however, the larger the out-of-focus distance or the throw of the jiggle-camera, the larger is the fraction of the light from the outlying parts of the galaxy that contributes to the image and the brighter it appears; in no case is *all* of the galaxian light recorded. The excellent photometry on large-scale photographs by Holmberg [22,38] involves the integration of the light in the image by the analysis of several microphotometer tracings made across it. Holmberg has achieved highly consistent results, but the lowest light levels are difficult to calibrate accurately by the photographic process and, in any case, the amount of light emitted by the outer parts of the galaxy that are too faint to record on the photograph remains unknown. Similar difficulties are encountered when photoelectric measures are made of many points on the telescopic image of a galaxy. Photoelectric measures through a diaphragm large enough to admit sensibly all of the galaxian light are not feasible, for the aperture would have to be so large that many stars and other objects would also be recorded. It is necessary, therefore, to find an operational definition for the ''total magnitude'' of a galaxy in such a way that this quantity can be measured in a consistent manner.

An attempt has been made by some investigators to define a galaxian magnitude by the amount of light contained within the region bounded by a standard isophote [39]. Sandage [17], for example, has published a table of galaxian magnitudes (based upon photoelectric measures of Stebbins, Whitford, and Pettit) that have been defined by a variation of the standard isophote concept. These magnitudes have proved very useful, but, nevertheless, do not correspond to the same fraction of the total light emitted by all galaxies. For example, a smaller fraction of the light is counted in a galaxy of low surface brightness than in a galaxy of the same angular size but higher surface brightness (Figure 5).

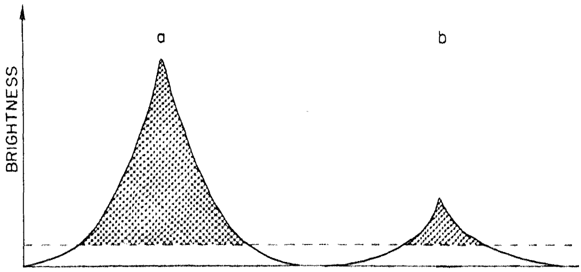

Figure 5. Two galaxies, *a* and *b*, have the same size but unequal surface brightnesses. A smaller fraction of the light of the fainter galaxy (*b*) lies above the standard isophote (dotted line) and contributes to the adopted magnitude less than for the brighter galaxy (*a*).

AN OPERATIONAL DEFINITION OF GALAXIAN MAGNITUDE

There is general agreement that the projected light distribution in the inner parts of elliptical galaxies is well represented by Hubble's interpolation formula:

$$\frac{I}{I_0} = \frac{1}{(1 + r/a)^2}, \tag{2}$$

where I/I_0 is the ratio of the observed surface brightness to that at the center of the nucleus, r is the distance from the center, and a is a parameter. Photoelectric measures of M 87 by Baum [40] seem to confirm Eq. (2) to very large values of r. The integrated light predicted for a galaxy from Hubble's formula, however, is infinite. Other investigators [41,42,43] have found it possible to fit the observed surface brightnesses of elliptical galaxies to interpolation formulae for which the integrated luminosity does converge, but in all cases, the formulae deviate only negligibly from Eq. (2) for small values of r/a.

An investigation of about a dozen E and S0 galaxies by the writer indicates that the projected light distribution for most galaxies can be represented satisfactorily by the interpolation formulas

$$\begin{aligned} \frac{I}{I_0} &= \frac{1}{(1 + r/a)^2} \quad \text{for } \frac{r}{a} \leq 21.4; \\ \frac{I}{I_0} &= \frac{22.4}{(1 + r/a)^3} \quad \text{for } \frac{r}{a} > 21.4. \end{aligned} \tag{3}$$

If the above form for $I(r/a)$ is adopted, and if the parameters a and

I_0 can be found from observation, the total magnitude of the galaxy can be obtained by integration. The writer and D. M. Mihalas [44] have prepared tables from which total magnitudes may be derived from two or more observations of an elliptical galaxy made in any of the following ways:

1. Photoelectrically, with different sized diaphragms.
2. Photographically, with the schraffier method, in which different plates are obtained with different-sized square images.
3. With extrafocal photographs in which plates are obtained that are different amounts out of focus.

To date, the last method has been applied the most extensively by the writer and, for illustration, we shall describe it briefly. Figure 6 shows a sketch of an in-focus image of an elliptical galaxy. Because of the high central concentration of the light in the nucleus, its light dominates the extrafocal image and produces a round image (circle *O*), which usually appears nearly as sharp and well defined as a star image. However, other parts of the galaxian image that are not farther from the nucleus than the diam-

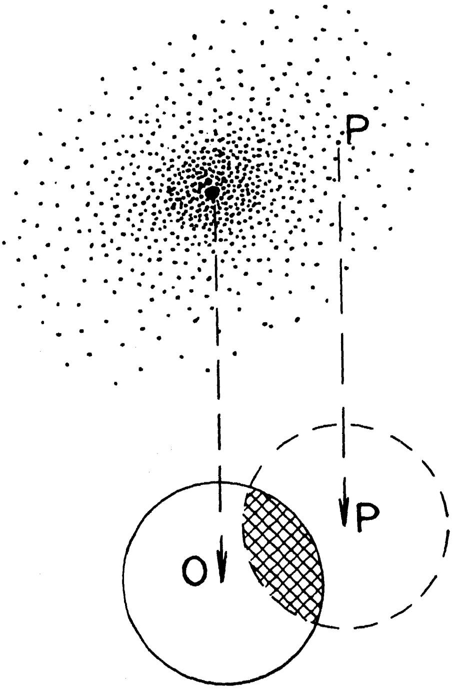

Figure 6. The contribution to the light of an extrafocal image of an elliptical galaxy.

eter of the extrafocal image also contribute some light to the central image. For example, the contribution from point P is the fraction of its extrafocal image (circle P) that overlaps circle O. By numerical calculation, the fraction, in magnitudes, of the total light of the galaxy that is observed in the central extrafocal image was found as a function of r/a, the ellipticity of the galaxy, and and the size of the extrafocal image (in units of a). The "magnitude" of an extrafocal galaxian image can be found by comparing it to the extrafocal images of stars of known magnitude. Magnitudes from images corresponding to two or more extrafocal distances are sufficient to determine a and the correction required to obtain the total magnitude of the galaxy. It turns out that the derived value of a is very sensitive to the ellipticity of the galaxy, but, fortunately, that the total magnitude is not. In practice, it is seldom that light is actually observed from parts of a galaxy more than 20 units of a from its nucleus. Consequently, any error in the adopted interpolation formula for the surface brightness of the outermost parts of a galaxy enters only as a zero-point error in the magnitude scale.

The method is more useful in a statistical sense when magnitudes of many galaxies are to be compared, than for obtaining the magnitude of an individual object, because all galaxies cannot be expected to fit exactly to the same formula for surface brightness. The magnitude of a galaxy, defined by the foregoing procedure, is strictly the total magnitude of a hypothetical galaxy whose surface brightness is given exactly by Eq. (3), and for which the integrated brightnesses within the several apertures used, or extrafocal or square images measured, most nearly match those of the observed galaxy.

MAGNITUDES AND COLORS OF BRIGHTEST CLUSTER GALAXIES

It has generally been supposed [see, e.g., 17] that the brightest cluster galaxies in the rich clusters approach a definite limiting luminosity. It is not yet possible to say positively whether this is the case. The brightest individual galaxies in six different clusters in which the writer has measured magnitudes by the method described above differ from each other by at least five-tenths in absolute photovisual magnitude. The most luminous galaxies yet encountered are giant ellipticals of M_{pv} about -23 (for $H = 75$ km/sec/Mpc).

Several investigators have measured fairly precise color indices for cluster galaxies. The brightest appear to have the same colors as normal "field" galaxies. Table 5 (adapted from published data of Holmberg [22] and de Vaucouleurs [45]) exhibits the approximate

TABLE 5

Color Indices of Brightest Cluster Galaxies

Type	*P - V*	*B - V*	*U - B*
E and S0	0.85	0.9	0.5
Sa	0.72	0.8	0.3
Sb	0.6	0.8	0.25
Sc	0.35	0.55	−0.1

mean color indices found for bright galaxies in nearby clusters. However, at least three observers (Baum [46], Hodge [47], and de Vaucouleurs]45]) have found that fainter elliptical galaxies tend to be bluer than the brightest E giants. $B - V$ color indices of ellipticals 5^m fainter than the brightest limit are as low as 0.6.

THE LUMINOSITY FUNCTION AND A POSSIBLE DISTANCE CRITERION FOR CLUSTER GALAXIES

Hubble [48] reported that the luminosity function of galaxies is Gaussian, with a mean absolute magnitude of −15.1, and a standard deviation of $0^{m}_{\cdot}85$. When corrections are applied to his early distance scale, the mean absolute magnitude is brightened by at least two magnitudes. However, Hubble's investigation suffered selection effects in that he considered the brightest *appearing* galaxies rather than *all* galaxies in a given *volume of space* [49]. Of at least 17 galaxies in the Local Group, for example, only 4 have absolute magnitudes brighter than −17, while 11 are dwarf systems fainter than $M_{pg} = -14$. Similarly, the Virgo cluster contains many dwarf galaxies [21].

Zwicky has derived the luminosity function of cluster galaxies from the numbers of discernible galaxies in clusters as a function of their angular diameters [50], and finds that in the mean the number $N(M)$ of galaxies of absolute magnitude M is given by

$$N(M) = constant \times 10^{M/5} . \qquad (4)$$

Zwicky's counts of Coma Cluster galaxies to different limiting magnitudes (Section Ia) confirm his general conclusion that the luminosity function increases rapidly with absolute magnitude.

The need for more detailed information concerning the nature of the luminosity function of cluster galaxies has led the writer to initiate the investigation of the magnitudes of galaxies in about 30 rich clusters. In all cases, magnitudes are determined by extrafocal photographic photometry (with photoelectric calibration), as described. At the time of writing, provisional luminosity functions are complete for six clusters. In four of these clusters, magnitudes are available in two colors; in a fifth cluster, in three colors. Except for scale factors that depend upon the total richnesses of the clusters, the luminosity functions are all similar.

For illustration, we shall consider the photovisual luminosity function of the Coma Cluster. Figure 7 shows the logarithm of the integrated apparent luminosity function, $\int N(m_{pv})dm_{pv}$, as a function of apparent photovisual magnitude; the observations extend to $m_{pv} = 18.3$. To compare the present results with Zwicky's formula, Eq. 4, it is necessary to find the constant s in the interpolation formula:

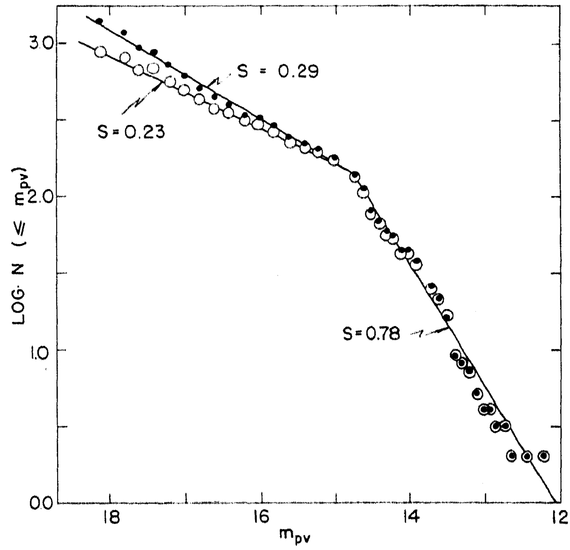

Figure 7. Integrated logarithmic luminosity function of the Coma Cluster of galaxies. Abscissae are apparent photovisual magnitudes. The points refer to case 1, (no "field" correction), and the open circles to case 2 (100 field galaxies per square degree brighter than $m_{pv} = 18.3$).

$$\log \int_0^{m_{pv}} N(m_{pv})dm_{pv} = constant + s \log m_{pv} \,. \tag{5}$$

Unfortunately, the results are rather critical to the assumed density of field galaxies which, in turn, depends upon how far the cluster is assumed to extend. In Figure 7, the luminosity function is shown for each of two cases:

1. No correction is made for the field over the region surveyed (a square region of 7 square degrees centered on the cluster); i.e., the field is assumed negligible – an assumption that certainly overestimates the number of objects in the observed region that belong to the cluster;

2. The total numerical density of field galaxies brighter than the limit surveyed is 100 galaxies per square degree – a density of field galaxies that is consistent with the hypothesis that the cluster extends to an angular radius of 100′ – and that the field objects are distributed in magnitude according to

$$\log N(m_{pv}) = constant + 0.6\, m_{pv}, \tag{6}$$

that is, according to the assumption that field galaxies of all absolute magnitudes are distributed uniformly in space. Case 2 certainly represents an upper limit to the numerical density of field galaxies, since all investigators agree that the cluster extends at least to 100′.

It is seen in Figure 7 that the integrated logarithmic luminosity function shows a sharp change of slope at $m_{pv} = 14.7$. At brighter magnitudes, the field galaxies contribute only negligibly, and the constant, s, has the value 0.78 both for cases 1 and 2. At fainter magnitudes, the value of s must lie between 0.29 for case 1 and 0.23 for case 2. For magnitudes fainter than $m_{pv} = 14.7$, therefore, the writer's results are in at least fair agreement with Zwicky's formula, which predicts a value of s of 0.20.

The luminosity function does not, however, increase monotonically with magnitude. The most prominent feature in the luminosity function is a maximum at $m_{pv} = 14.7$, that gives rise to the change of slope observed in Figure 7. Other, less marked maxima, corresponding to the "waves" of the points about the straight lines in Figure 7, are less certain, but, nevertheless, seem to appear also in the luminosity functions of the other clusters, and may be real features. These features in the bright ends of the luminosity functions of the different clusters make it possible to fit those different luminosity functions together and, thereby, to determine relative distances to the clusters. Mean red-shifts are available for five of the clusters investigated so far (Numbers 1656, 2151, 2199, 151, and 2056, in the writer's cluster catalogue); the logarithms of those red-shifts are plotted against the relative distance moduli for the clusters, as found from their luminosity functions in Figure 8. The root-mean-square velocity deviation about a straight line is less than 600 km/sec, and most of this dispersion comes from two clusters (Numbers 2151 and 2199) that appear to be parts of large clouds of clusters (or second-order clusters), in which differential

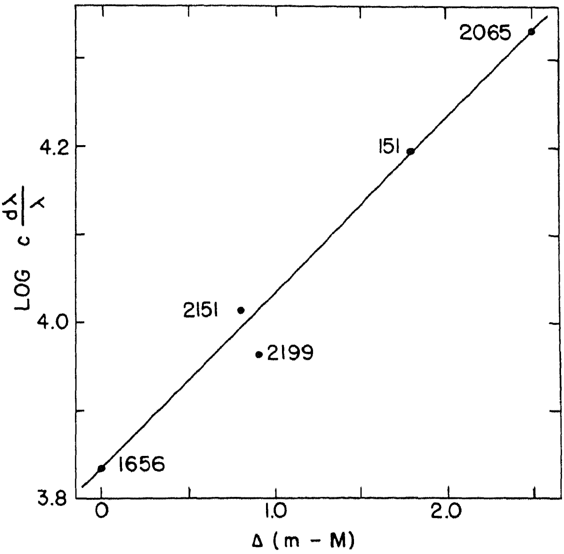

Figure 8. Relation between $\log(cd\lambda/\lambda)$ and the distance modulus (referred to that of the Coma Cluster) obtained by fitting together the luminosity functions of five clusters.

gravitational accelerations may exist. On the other hand, when relative distances to the same five clusters are determined from their ten brightest galaxies [17], the observed dispersion about a straight line is several thousand kilometers per second. Indeed, the results obtained by fitting the luminosity functions together suggest that the tenth-brightest galaxies of different clusters differ from each other in absolute magnitude by at least $0^{m}_{\cdot}5$. Features in the luminosity functions of the rich clusters may, therefore, provide a new and more reliable means than any available today of determining relative distances to the clusters.

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DISCUSSION

Holmberg: Have you done anything to eliminate the optical members, which may cause a serious disturbance, especially in the case of low-concentration, distant clusters?

Abell: By "optical" cluster members, I presume that you mean field galaxies. Correction for the field is a very difficult problem because the field is highly irregular and may even consist entirely of foreground and background clusters. I have, therefore, as yet, not attempted to make corrections for the field; my luminosity functions include field as well as cluster galaxies. Unless a near foreground group happens to lie directly in front of a cluster, the bright end of a cluster luminosity function is not substantially affected, and can still be used for the determination of relative cluster distances. Without making a field correction, of course, I can say little or nothing about the absolute ratio of the number of bright to faint galaxies in a cluster.

Zwicky: To get away from the "quaint" maxima in the luminosity function of galaxies, one must take very large samples. The luminosity function then rises monotonely.

Abell: To date, I have luminosity functions (in two colors) for six clusters. All functions show the "quaint" features you refer to. I agree that it is desirable to consider a very large sample, but these luminosity functions result from photometry on each individual cluster member (including field galaxies) above the plate limits, which is a time-consuming project, requiring months of work. To date, detailed luminosity functions do not exist for a "large sample" of clusters, so your conclusion that they "rise monotonely" is purely hypothetical. One can not, of course, detect the features in a luminosity function by merely counting all galaxies on two or three plates taken with different telescopes with different limits, as you describe in *Morphological Astronomy*.

Whitrow: What is the present position of our knowledge as regards the correlation of richness of clusters and distance as considered by K. Just?

Abell: I know of no objections to Dr. Just's analysis. However, his conclusions (that distant clusters tend to be richer) are very sensitive to minor observational selection effects, and I am not yet convinced that the conclusions are statistically significant.

Just: The correlation between the richness of Abell's clusters and their distance from us is very significant even when we take account of some systematic errors which might have entered his survey and can be traced in his catalog itself. But he might have introduced into his classification still other errors which can only be estimated by repeating at least a part of his counts.

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Kinematics and Dynamics of Clusters of Galaxies

HISTORY

It is my purpose to present a summary and very brief critical review of the work that has been carried out by means of a restricted kind of analysis of the internal dynamics of clusters of galaxies. In the type of analysis in question, one can attempt to apply the virial theorem to a cluster and to derive information concerning the masses of the individual galaxies within the cluster and/or information concerning the stability of the cluster to expansion or contraction in a time-scale characterized by the dimensions of the cluster and the velocities of the member galaxies. In the following discussion such a time-scale will be termed the *"characteristic time-scale"* for the cluster.

This procedure and equivalent ones have been used for more than a quarter of a century [1, 2, 3]. Until very recently, it was tacitly assumed in all such studies that the groups and clusters were stable in the sense just mentioned. On this assumption, and on the assumption that the individual galaxies in a given cluster behave as point masses, estimates for the total mass of the cluster follow from such analyses. If the radial velocities of the individual members have been determined with sufficient accuracy, and if it can be further assumed that the relative masses of the member galaxies can be estimated from their relative luminosities and nebular forms and that there is no appreciable mass of unseen matter within the cluster, then it is possible to obtain from the dynamically derived total mass of the cluster estimates for the masses of the individual members.

While estimates for the masses of individual galaxies in a few relatively large clusters (Coma and Virgo) were being derived in this way, estimates for the masses of a number of the nearer galaxies were being obtained in a more direct manner from studies of the motions within individual galaxies. From accumulating data concerning the masses of galaxies as derived in this latter way, it has become possible in the course of time to obtain estimates for the masses and mass-to-light ratios for a number of galaxies having different nebular types. The analysis of such data has brought to light what appear to be certain regularities relating the masses of galaxies to their absolute magnitudes and nebular forms. However, it has been known since the early work by Zwicky and by Smith that the estimates derived by means of the virial theorem for the masses of typical galaxies in large clusters were about an order of magnitude larger than would have been estimated from the approximate relationship that appears to hold between mass, luminosity, and nebular form as derived from the available studies of individual galaxies.

Ambartsumian [4] pointed out several years ago that this same kind of disagreement was found for several smaller groups as well. In view of this discrepancy, he suggested that it was reasonable to assume that the masses for the member galaxies in clusters were those that would be inferred from the interpolation and extrapolation of the data for the masses of individual galaxies. On this assumption, it followed that, unless large fractions of the total masses of the clusters in question were in the form of unseen matter, these clusters had positive energy and were disrupting in time-scales of the order of the characteristic time-scales for the clusters. It was Ambartsumian's view that the clusters that he had studied were to be interpreted in terms of positive energy and instability.

Since Ambartsumian's suggestion was made, these clusters and others have been analyzed in still greater detail with regard to their stability for what appear to be reasonable independent estimates for the masses of the member galaxies. It is the results of these analyses and the interpretation to be given to them with which my remarks will be concerned.

STABILITY CRITERIA

To this end, it may be useful first to describe briefly the virial theorem and the assumptions implicit in its application to clusters of galaxies. In the form in which it is usually applied to such systems, the virial theorem rests on the assumption that the member galaxies in a given cluster can be considered as point-masses in so far as the dynamics of the cluster is concerned. Where this is true, the virial theorem states that, if the cluster—under the mutual attractions of its member galaxies alone—is in a quasi-equilibrium in which there is no significant net expansion or contraction during an interval of the order of, or greater than, its characteristic time-scale, then twice the time-average of the total kinetic energy of the galaxies with respect to the center of mass of the cluster will equal the absolute value of the time-average of the gravitational potential energy of interaction of the galaxies; that is,

$$2 < T > = - < \Omega > , \tag{1}$$

where the two time-averages are averages over the given time interval and where the kinetic and potential energies here are to be calculated on the assumption that the member galaxies are to be treated dynamically as point-masses. If, on the other hand, the cluster is stable—but only by reason of the presence of an appreciable mass of unseen intracluster matter, then a modified form of the virial theorem [5] relates the velocity dispersion of the galaxies, the relative spatial distributions of the galaxies and of the unseen matter, and the masses in the form of the member galaxies and of the unseen matter.

It is from the virial theorem in one of these two forms, and under the above conditions, that one can derive an estimate for the total mass of the cluster in terms of the appropriate observational data. It is clear that the derivation of masses in this way for the clusters involves the same kind of assumptions and must lead to the same kind of results as do derivations based upon circular and/or escape velocities. Finally, it should be mentioned that, if a cluster is to be stable against unlimited expansion through the mutual attractions of its member galaxies alone, then, from quite general considerations, the algebraic sum of the total kinetic and potential energies of the system of galaxies—considered as acting as point-masses—must be negative at any instant, that is,

$$T + \Omega < 0 . \tag{2}$$

This less restrictive condition is sometimes more useful than the virial theorem in that it does not involve the sometimes troublesome time-averages for the kinetic and potential energies. This more general energy condition is seen to give a lower limit to the total

mass of the cluster whenever it can be safely assumed that the system is stable and does not contain an appreciable mass in the form of unseen matter.

THE APPLICATION OF THE VIRIAL THEOREM TO PARTICULAR CLUSTERS OF GALAXIES

METHOD OF ANALYSIS OF THE BASIC OBSERVATIONAL DATA

The above forms for the virial theorem and for the energy condition can be used with either of two aims. If it can be assumed on some grounds that the cluster in question satisfies the appropriate stability condition, then these relations can be used to derive the total mass of that cluster. Alternatively, if independent estimates for the masses of the member galaxies can be made somehow, it then becomes possible to use these relations as tests for the stability of clusters. In either approach, it is necessary to analyze the observational data to obtain the required expressions for the total kinetic and gravitational potential energies.

Here it is important to note that, even if there were no uncertainty concerning the masses of the individual member galaxies in a given cluster, the available observational techniques do not provide directly all the remaining data that are required for the evaluation of the total kinetic and potential energies due to the galaxies within the cluster. In each case, it is necessary to supplement the direct observational data by means of estimates for the relevant projection factors. In the case of clusters of galaxies having only a few members, the uncertainties in such estimates, and hence in T and Ω, can be large.

THE RESULTS FOR PARTICULAR CLUSTERS

With these preliminary remarks as background, let us now consider the results that have been obtained from this kind of analysis for a number of particular clusters. In the accompanying Table 1, are given certain results of such analyses for nine groups and clusters that have been treated in the literature. References to the relevant papers are given in the last column. In most cases, I have redone parts of the analyses in order to achieve a somewhat greater degree of homogeneity in the material as a whole than would otherwise have been possible; a value for the Hubble constant of 100 km/sec/Mpc has been employed in deriving distances whenever the velocity-distance relation has been used.

The columns in this table contain the following information. Column 1 gives the familiar name or a useful characterization of the particular group or cluster, while column 2 gives an estimate

TABLE 1

*Data from the Analysis of Clusters of Galaxies**

Cluster or Group	*n*	$(\mathfrak{M}/\mathfrak{M}_\odot)_{total}$	$(\mathfrak{M}/\mathfrak{M}_\odot)_{\text{average bright galaxy}}$	$\left[\frac{\mathfrak{M}/\mathfrak{M}_\odot}{(L/L_\odot)_{P_g}}\right]_{total}$	τ_c (*years*)	*Reference*
LARGE CLUSTERS:						
Coma	800.	8.3 <+14	1.0 <+12	350.	1.0 <+9	[1,6,7]
Virgo	500.	4.5 <+14	9.0 <+11	750.	8. <+8	[2,4,7]
Hercules	75.	5.5 <+13	7.5 <+11	100.	6. <+8	[8]
SMALLER GROUPS:						
Stephan's Quintet	5.	5. <+12	1.0 <+12	70.	4. <+7	[4,9,10,11]
NGC 5846 Group	20.	5. <+13	2.5 <+12	500.	1. <+9	[12]
NGC 7619 Group	5.	2. <+13	4.0 <+12	300.	2. <+8	[13]
Sculptor Group	6.	1.5 <+13	2.5 <+12	1000.	1.3 <+9	[14]
TWO SMALL GROUPS OF PARTICULAR INTEREST:						
M 81 Group	4.	1.9 <+12	1.2 <+12	200.	2. <+8	[4,15]
Local Group	2.	4.0 <+12	2.0 <+12	200.	3.4 <+9	[16,17,15]

*Here and in the following tables, a number *B* that is set off to the right of a second number *A*, $1 \leq A < 10$, by a single angular bracket < is the power of ten by which the number A is to be multiplied. Thus 1.234×10^{-7} will be written 1.234 <−7.

for the number of relatively bright galaxies in the group. The results in the next three columns have been derived on the assumption that the cluster is in the required quasi-equilibrium under the mutual attractions of its member galaxies alone and that the virial theorem in the form of Eq. (1) is applicable. Column 3 gives the total mass of the cluster as derived from this form for the virial theorem, while column 4 gives the corresponding average mass per bright galaxy. In column 5 are given estimates for the total mass-to-light ratios for these clusters. Because of uncertainties in the total luminosities of the clusters, some of these latter values are rather uncertain. Finally, in column 6 are given the characteristic time-scales, τ_c, for the several clusters. The time-scale has been defined so that; in a time τ_c, a galaxy, moving with a constant component of velocity in the plane of the sky equal to the presently estimated root-mean-square value for this component, will travel a distance in the plane of the sky equal to the present mean projected distance of the members from the center of mass.

INDEPENDENT DATA CONCERNING THE MASSES OF GALAXIES

If there were no independent data available concerning the masses and mass-to-light ratios of individual galaxies, we would have no particular reason to question the assumption that, in so far as they are physical, these groups and clusters must satisfy the virial theorem in the form given in Eq. (1). From this assumption, it would follow that the masses and mass-to-light ratios given in columns 3, 4 and 5 of Table 1 provide useful and meaningful information concerning the masses and mass-to-light ratios of the individual galaxies that make up these clusters. However, at the present time, there are more than a dozen estimates for the masses and mass-to-light ratios of galaxies from the more direct analysis of the internal motions within individual galaxies. Table 2 presents a compilation of these results, reduced, where appropriate, to distances corresponding to the same Hubble constant that was used in Table 1.

A comparison of the masses derived in this latter way with those derived in Table 1 from the virial theorem for the typical brighter galaxies in clusters shows that the average mass, as derived from the internal motions within individual galaxies, is more than an order of magnitude smaller than the typical galactic masses derived by means of the virial theorem as applied to clusters. In no case is the determination of a mass from internal motions within a galaxy as large as the values derived from the analysis of the clusters in Table 1 on the assumption that the virial theorem in the form given in Eq. (1) is valid. It is this seeming discrepancy between the masses of galaxies as determined in these two ways that has led Ambartsumian and others to question the stability of such clusters.

Here, two points should be noted. First, our knowledge of the masses of the large elliptical galaxies as derived from studies of internal motions is somewhat meager at the present time. The mass discrepancies for several of the clusters in Table 1 would be removed if such elliptical galaxies had masses slightly in excess of $1 \times 10^{12}\mathfrak{M}_\odot$ and mass-to-light ratios of several hundred. Neverthe-

TABLE 2

Masses and Mass-to-Light Ratios from Internal Motions

Galaxy	*Type*	$(\mathfrak{M}/\mathfrak{M}_\odot)$	$\left[\frac{\mathfrak{M}/\mathfrak{M}_\odot}{(L/L_\odot)_{P_g}}\right]$	*References*
NGC 3379	E0 (kE1)	1. <+11	16.	[18]
NGC 3115	E7 (kD4)	2. <+11	55.	[6]
NGC 221 (M 32)	E2 (kE3)	3. <+ 9	15.	[19]
NGC 2146	Sa p (–)	1.5 <+10	3.	[20]
NGC 3623 (M 65)	Sa - Sb (gS5)	2.0 <+11	20.	[21]
NGC 224 (M 31)	Sb (kS5)	3.2 <+11	18.	[22]
NGC 3031 (M 81)	Sb (kS4)	1.4 <+11	19.	[23]
NGC 4594	Sb (kS6p)	2. <+11	3.	[24]
NGC 5005	Sb (gS5)	8. <+10	8.	[25]
NGC 5055	Sb (gS4)	6. <+10	3.	[26]
NGC 1068	Sb (gS3 - gD3)	3. <+10	2.	[27]
NGC 598 (M 33)	Sc (fS3)	2. <+10	5.	[28]
NGC 2903	Sc (fS4)	5. <+10	4.	[29]
NGC 3556	Sc (aS6 or aI)	1. <+10	2.	[30]
Large Magellanic Cloud	Irr (∼aI)	1.3 <+10	5.	[31]
NGC 1097	SBb (gBp)	7. <+ 9 (nucleus)	. . .	[32]
NGC 3504	SBb (fgB1?p or gB1?p)	7. <+ 9	. . .	[33]
NGC 7479	SBb (fB2)	1.6 <+10 (bar)	. . .	[34]
NGC 1365	SBc (fgB)	2. <+10 (nucleus)	. . .	[32]

less, it appears that there would still be serious discrepancies for the majority of the clusters listed in which elliptical galaxies are not found or can scarcely play a significant role. The second point is related to this question. The analysis of the dynamics of double galaxies which was begun by Holmberg [3] and which has been continued by Page [35, 36] leads, statistically, to estimates for the masses of different types of galaxies. Page's most recent published results for the mean masses and mass-to-light ratios for members of the double galaxies are summarized in Table 3. It is seen that the mean mass as derived from the 14 spiral and irregular systems is consistent with the values obtained for such systems from the analysis of internal motions; however, the mean mass-to-light ratio for the spiral and irregular galaxies as derived from double galaxies is about an order of magnitude smaller than the corresponding value as derived from studies of internal motion—a result which may or may not be significant. Still more interesting is the fact that Page's results contain what may well be the best data available concerning the masses of elliptical galaxies. He obtains values of about $6 \times 10^{11}\mathfrak{M}_\odot$ and 100 for the mean mass and mass-to-light ratio from his analysis of 13 double systems containing only elliptical and SO galaxies. These values are within factors of about 2 of resolving the mass discrepancies for those clusters in which elliptical galaxies can play a significant role. It is important to increase the statistical significance of these results through the study of additional systems.

However, the use of double galaxies to provide independent estimates for the masses of galaxies with which to test the applicability of the virial theorem to clusters of galaxies is subject to a logical difficulty. This type of analysis is equivalent to the application of the virial theorem to double galaxies and is, therefore, based upon the tacit assumption that these double galaxies satisfy the virial theorem in the form given in Eq. (1). So long as the applicability of the virial theorem to the groups and clusters listed in Table 1 is in question, there seems to be no a priori reason for concluding that it is safe to apply it to the double galaxies in Page's list—except that here appreciable excesses of positive energy would correspond to very short critical time-scales and therefore appear to be unlikely.

INTERPRETATION OF THE RESULTS

GENERAL

Let us now return to the basic question as to the correct interpretation to be given to the results of the analysis by means of the

TABLE 3

Data from the Analysis of Double Galaxies

Type of Galaxy	Number of Pairs	$(\mathfrak{M}/\mathfrak{M}_\odot)_{\text{average}}$	$\left[\dfrac{\mathfrak{M}/\mathfrak{M}_\odot}{(L/L_\odot)}\right]_{P_g}$
All pure pairs	33	$2.6 < +11$	8.7
Spirals and irregulars	14	$2.1 < +10$	0.33
Ellipticals and SO's	13	$5.9 < +11$	94.
Mixed systems	14	$3.1 < +11$	48.

virial theorem for the clusters of galaxies listed in Table 1. From the foregoing discussion, it is clear that the correct interpretation of these results is of the very greatest importance to our understanding of the nature of galaxies and of clusters of galaxies.

The diagram on p. 249 presents schematically the apparently most reasonable chains of alternative interpretations for the results derived from the application of the virial theorem to any of the particular clusters listed in Table 1. In the present discussion, I shall leave out of consideration the formal possibility that the correct interpretation may lie in the abandonment of Newtonian mechanics and/or gravitational theory; however, it may not be out of place to remind ourselves that neither has been subject to any real verification over the scale of distances that is pertinent here.

On the basis of Newtonian theory, the possible interpretations fall into one of two general classes: either a given cluster is essentially stable, or it is unstable over the relevant characteristic time-scale. The possible interpretations in terms of stability can be further broken down into three alternative possibilities:

1. The cluster is stable for the relatively small masses for its member galaxies that are suggested by the studies of the internal motions in galaxies, the much larger masses given by the virial theorem being the result of errors and uncertainties in the observational data and in their interpretation;

2. The cluster is stable under the mutual interactions of its member galaxies alone and the masses of the member galaxies are as given by the virial theorem—the galaxies in clusters being systematically different from those in the sample from which masses have been derived from internal motions;

3. The cluster is stable only by reason of the presence of a relatively large mass of unseen intracluster matter in the form of faint galaxies, inter-galactic stars, gas, or particles—the masses of the brighter galaxies being significantly smaller than those derived from the straightforward application of the virial theorem in the form of Eq. (1).

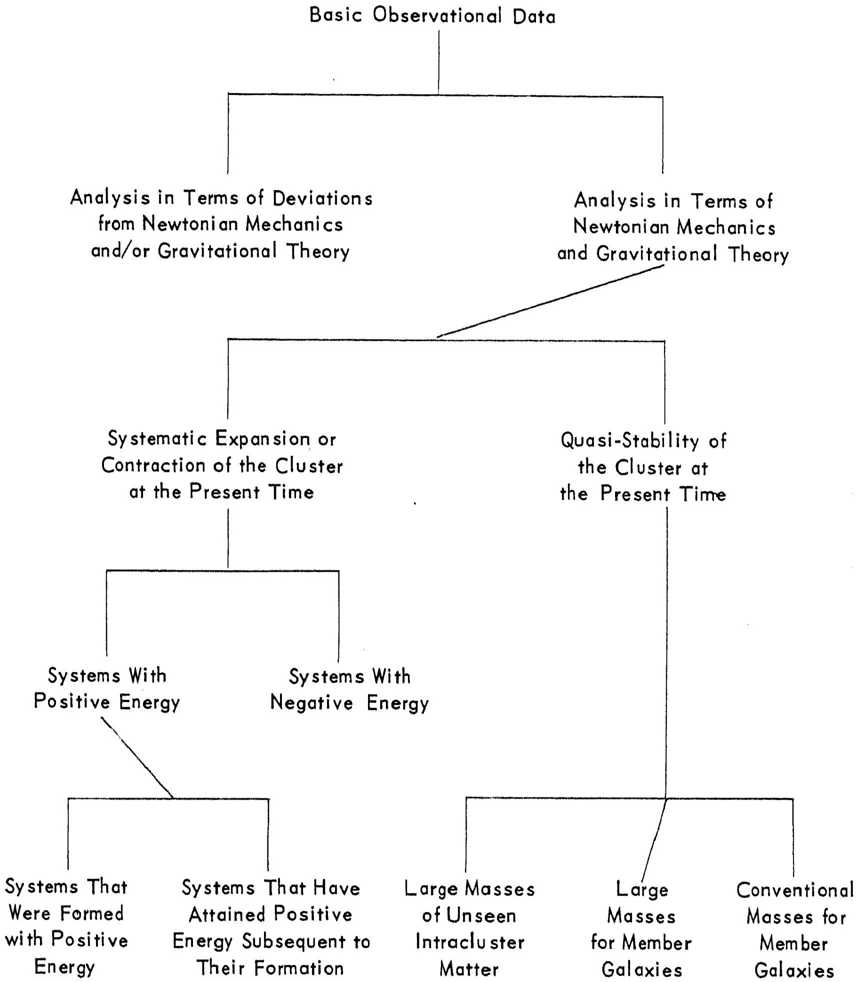
The general case of instability can be similarly subdivided into two further subclasses: either the system has positive energy, or it has not. In the latter case, the system should approach equilibrium in a time of the order of the appropriate characteristic time-scale. However, from the results given in Table 1, it is clear that this possibility cannot explain the mass discrepancies by itself, and, for this reason, it will be considered no further. Finally, in the breakdown of the chain of alternatives, it is convenient to further subdivide the possibility that a cluster has positive energy into the two following cases: either the system had its present positive energy at the time of its formation as a cluster, or it received its present positive energy subsequent to its formation.

Let us now turn to a brief critical discussion of the arguments for and against the several foregoing possibilities as explanations for the apparent mass discrepancies derived for the clusters listed in Table 1.

ARGUMENTS FAVORING AN INTERPRETATION IN TERMS OF INSTABILITY

At present, probably the most compelling argument in favor of instability for the clusters considered in Table 1 is that no direct determinations of the masses of galaxies lead to values for the masses and mass-to-light ratios that are nearly as large as those that follow in these cases from the virial theorem in the form of Eq. (1). The force of this argument rests upon the reasonable assumption–valid or not–that the sampling of galaxies whose masses have been derived from internal motions provides an appropriate sample with which to compare the galaxies in the clusters listed in Table 1. Here, however, it must probably be admitted that values of about 70 for the mass-to-light ratios for the two galaxies that have been classified as elliptical galaxies in Stephan's Quintet are sufficient to make the stability of this particular cluster a reasonable possibility. Such a relatively modest mass-to-light ratio for the elliptical galaxies would correspond to masses for the elliptical galaxies in this cluster of slightly more than $1 \times 10^{12} \mathfrak{M}_{\odot}$, since the members of the Quintet appear to be highly luminous. Further, our lack of direct data concerning the masses of elliptical galaxies may also make it possible that an elliptical-rich cluster, such as the Coma Cluster, may also be stable under the mutual attractions of its member galaxies alone. However, it does not seem possible to use our present uncertainty concerning the masses of elliptical galaxies to argue for the stability of the other clusters in Table 1.

Finally, if the analysis that has led to the results given in Table 1 for the Local Group and for the M 81 group is not greatly in error,


Basic Observational Data
Analysis in Terms of Deviations from Newtonian Mechanics and/or Gravitational Theory
Analysis in Terms of Newtonian Mechanics and Gravitational Theory
Systematic Expansion or Contraction of the Cluster at the Present Time
Quasi-Stability of the Cluster at the Present Time
Systems With Positive Energy
Systems With Negative Energy
Systems That Were Formed with Positive Energy
Systems That Have Attained Positive Energy Subsequent to Their Formation
Large Masses of Unseen Intracluster Matter
Large Masses for Member Galaxies
Conventional Masses for Member Galaxies

Schematic representation of the several alternative chains of possibilities concerning the proper interpretation of the results contained in Table 1.

then these two groups represent cases in which the masses that result from the application of the virial theorem in the form of Eq. (1) are much greater than the corresponding masses of the key members as determined directly from studies of internal motions. I should mention that the mass discrepancy for the M 81 group is reduced below the value that would be inferred from the data in Table 1 by a factor of about 0.6 as a result of a new determination of the radial velocity of M 82 through a 21-cm. study. I am indebted to Professor Oort for bringing this result to my attention.

The argument that these two groups, and, indeed, all of the other clusters in Table 1, are stable because of the presence of relatively large masses in the form of unseen intracluster matter [1, 2, 16] can only become convincing if compelling direct or indirect evidence for its existence is available.

ARGUMENTS FAVORING AN INTERPRETATION IN TERMS OF STABILITY

On the other hand, there seem to be two very compelling, although indirect, arguments in support of the view that most of the clusters listed in Table 1, at least, are essentially stable. First, there is the matter of time-scales. If the masses derived from the virial theorem are too large by nearly an order of magnitude or more, then it must be expected that the clusters cannot have had their present positive energies for more than periods equal to the characteristic times τ_c given in Table 1. In every case but one, these characteristic times are about 1×10^9 years and less. And, although our understanding of the evolution and present ages of galaxies is anything but detailed, it is very difficult to believe that those clusters that contain members of the kind that Morgan classifies as *k* can be nearly so young. I, personally, find it difficult to believe that so short a time-scale can apply to many less extreme systems, as well. It seems no more satisfactory to attempt to explain these short time-scales in terms of the clusters having been given their present positive energies subsequent to the times of their formation, since it appears very nearly impossible to accelerate these galaxies to the required degree once the greater part of their masses are in the form of stars.

The second argument for the stability of these clusters is related to the argument just mentioned. Here it should be emphasized that the nine clusters which are listed in Table 1 and which show mass anomalies do not represent only a small fraction of the clusters analyzed. These nine clusters include essentially every cluster for which the available observational data are sufficient to warrant an analysis—leaving out of consideration the double galaxies. Thus, it is not a case of the mass anomaly's turning up for a small fraction of the clusters analyzed: the anomaly has been found for essentially every cluster that has been analyzed—from the largest clusters to the smallest. There, thus, seems to be every reason to believe that most of the other clusters in the sky, too, would show this same kind of mass anomaly if they were analyzed. If this is the case, then the argument for the instability of all, or most, of the clusters listed in Table 1 becomes an argument for the instability for all, or most, clusters in the sky in times of the order of the characteristic times given in Table 1, and such an argument becomes exceedingly difficult to accept. Unless one rejects the view that the universe is significantly older than 1×10^9 years, then the argument that most clusters are, at present, disrupting in a time-scale of 10^9 years or less would seem to require as a necessary corollary that the number of galaxies that are now seen as field galaxies, but which are the remnants of previous generations of disrupting clusters, should be much greater than the number of galaxies now in clusters. The evidence presently available concerning the clustering tendency of galaxies seems to be completely at variance with this conclusion. Studies by Zwicky [37, 38, 39], by Neyman, Scott, and Shane [40], and by myself [41] have all indicated that the general properties of the spatial distribution of galaxies cannot be explained in terms of a substratum of field galaxies that includes most galaxies and upon which are superposed clusters containing a relatively small number of galaxies. On the contrary, these analyses indicate that belonging to groups and clusters is the rule rather than the exception for galaxies at the present epoch.

This argument is not new. In the paper in which he first put forward the case for the instability of several clusters, Ambartsumian [4] noted that just for this reason one should not expect to find that more than a small fraction of all clusters are unstable.

Finally, it should be noted that although the analysis of the local system appears to raise serious difficulties concerning its stability in terms of the presently available direct determinations of the masses of its key members, the alternative interpretation that the local system is unstable itself encounters difficulties. On this latter view, most of the mass in the local system is contained in M 31 and in the Galaxy, and the total kinetic energy significantly exceeds the absolute value of the potential energy. However, in spite of this, the observations show that the relative radial velocity of M 31 with respect to the Galaxy is one of approach. On the view that the local system is unstable, this seems to imply that M 31 and the Galaxy were considerably farther apart at the time of their formation than at present. Such a conclusion raises several difficult questions for the interpretation assumed, one of which is that it is difficult to see how, under the circumstances, these two galaxies can have interacted so as to conserve momentum when the positive energy was made available to the system. The alternative possibilities for conserving momentum do not seem to be any more attractive to an interpretation in terms of instability.

If, as seems to be the case, there are several strong arguments against the interpretation of the data in Table 1 in terms of instability, it is important to examine the merits of the several possible ways of accounting for the mass anomalies in terms of stability. First, in so far as the explanation of these anomalies in terms of errors and uncertainties in the observational data and in its interpretation are concerned, it is clear that errors in the radial velocities and the misassignment of nonmembers as members of the relevant clusters will act in the direction of producing the observed anomalies. Holmberg [42] has very recently suggested that large and previously unsuspected errors in the published radial velocities are present. He has argued that this effect, together with the effects of the assignment of nonmembers as members and the misapplication of the virial theorem in the larger clusters containing subgroups, are responsible for the mass anomalies. It is important that these possibilities be examined.

There are two further alternatives for those who argue for stability: either the masses of galaxies in many, or most, clusters are significantly larger than would be inferred from the available data concerning direct determinations, or there must be relatively large masses of unseen intracluster matter in many or most clusters. It does not appear out of the question to explain several of the mass anomalies in Table 1 in terms of the first of these possibilities (e.g., Stephan's Quintet and the Coma Cluster), but it is more difficult to explain most, or many, of the discrepancies in this way. In particular, if the analyses given are reasonably accurate, it will be difficult, indeed, to account for the mass anomalies for the local system and for M 81 group in this way.

The explanation of the data in terms of large masses of unseen intracluster matter has little direct evidence in its support. Kahn and Woltjer [16] have argued that, in the case of the Local Group, there is some evidence for the required intracluster matter; but, in the case of the other systems, there seems to be little, if any, independent evidence for the existence of the required masses of intracluster matter. Such matter, if it exists, could be present in the form of galaxies of low luminosity, in the form of an intracluster substratum of stars, or in the form of gas and/or particles. Searches for it up to now have been carried out through photometry [12] and through searches for 21-cm. radiation [43]. In neither type of search has any evidence been found for the existence of the required amounts of matter. Some evidence has been found for the presence of continuous radiation from intracluster matter in the Coma Cluster at a wave length of 75-cm. [44], but there is no evidence as yet that the mass of this matter can account for the mass anomaly for this cluster.

SUMMARY

In summary, the proper interpretation to be given to the results derived from the application of the virial theorem to clusters of galaxies is by no means obvious. It is my own view that, although the arguments for instability cannot be lightly dismissed, the arguments for the stability of at least a goodly majority of clusters of galaxies are still stronger. However, the case for this latter interpretation would be considerably easier to accept if more direct evidence could be found in its support. In any case, it should, perhaps, be kept in mind that no one explanation may be sufficient to explain the mass anomalies for all the clusters considered.

Finally, although it is perhaps already quite evident, it should be acknowledged that I have not been able to do justice to a number of the particular and detailed arguments on both sides of the question under consideration. Any uniform attempt to do so would have required a much longer review.*

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DISCUSSION

Holmberg: It is possible to reduce the exceptionally high mass-luminosity ratios found for cluster galaxies (from the red-shift dispersion) by *(a)* a correct application of the virial theorem; *(b)* correcting for the disturbance caused by optical members; and *(c)* correcting for the systematic errors in the measured red-shifts. I refer to my paper in the Conference on the Instability of Systems of Galaxies [cf. *A. J.* 66, 620 (1961)].

Limber: If the effects mentioned by Professor Holmberg are as large as he has suggested, then they become of very great relevance to the problem under consideration. It is important that these effects be considered more thoroughly.

Zwicky: The difficulty of the mass-luminosity ratio in groups and clusters of galaxies has a good chance to be resolved as follows:

1. In small groups like the Stephan Quintet there are more members than is usually admitted. Stephan's Quintet has definitely a sixth member (SBO far to the north east. $V_S \simeq 6000$ km/sec). There is much luminous debris which, integrated, has probably a luminosity ten times greater than the actual members.

2. The evidence is for far more luminosity in the way of intergalactic stars between the galaxies in a cluster of galaxies than in the galaxies. The evidence is: visible large clouds in clusters; existence of matter far from galaxies because of the appearance of supernovae.

Baum: Photoelectric pulse counts in inter-galactic regions in two large clusters of galaxies seem to show an inter-galactic population above that in surrounding regions, of the same total order of luminosity as that normally assigned to the galaxies in those clusters. In short, I agree qualitatively with Zwicky but would favor a more conservative correction factor for the detectable part of any inter-galactic population.

De Vaucouleurs: In the case of the Coma Cluster, it can be shown that in order to account for the virial theorem mass of the cluster, the mass-luminosity ratio of the intergalactic stellar population must be in excess of 1000.†

E. M. Burbidge: Dr. Zwicky has mentioned the addition of far-out members to Stephan's Quintet; in the Conference on the Instability of Systems of Galaxies, I reported on the probable subtraction of one of the five close members of the group, NGC 7320, which turned out to have quite a small red-shift, so that it may be a foreground object. I would also like to mention, in the face of the long list of groups and clusters for which the virial theorem does not appear to be satisfied, that there is one group, VV 116, in which the virial theorem does give masses in agreement with the mass independently determined from the rotation of one spiral galaxy in the group.

King: If the additional mass in the Local Group is in the form of inter-galactic stars, how many stars would you expect to find with hyperbolic velocities in the solar neighborhood?

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†See *Ap. J.* **131**, 585 (1960).

Limber: One would expect a number of such stars of order unity within 10 pc of the sun.

Von Hoerner: May I draw your attention to two points connected with the application of the virial theorem? I did some experiments with random clusters by integrating the equations of motion, and I found a strong tendency to build *pairs* or small close *groups* of stars. They get destroyed after a while, but new ones are built. If we apply the virial theorem to the observed projected distribution, we still see the high velocities of this effect, but we lose the mutual, smaller than average, distances. This might give us a factor of 2.

The second point is, that the virial theorem is valid only for the average over a long time. In my experiment, the instantaneous values of $2T/(\Omega)$ scattered over the range 0.5 to 1.8, which is another factor of 2, but this time without a bias—so it might help in some cases only.

Limber: Effects of the kind that Dr. von Hoerner has mentioned can certainly be important. The first of the effects that he mentions is taken into account in the analysis of the smaller groups where the separations between individual galaxies are used. The uncertainty resulting from the variation in the instantaneous value of $2T/(\Omega)$ has been included with those resulting from our uncertainty concerning the projection factors.

Pishmish: I would like to make a comment on a statement that Dr. Limber made as a sideline but which is generally referred to and seemingly accepted. It concerns the proof given by Kahn and Woltjer concerning the existence of inter-galactic matter causing the curling-up of the spiral arms of the Galaxy. The criterion they give to disprove a primeval effect seems, however, not to be satisfactory. For, if we take, instead of their time-scale, a shorter time-scale, say, 10^9 years or less, it would not be necessary to postulate the existence of inter-galactic matter.

Massevich: Before Dr. G. Burbidge sums up the contents of the papers in Part II, I would like to draw your attention to the following points. It is generally accepted that star formation has ceased in elliptical galaxies, a point stressed by Dr. Oort in his summary of the Part I papers. But from what we have heard so far about the spectral features of galaxies, about radio galaxies, and about the distribution of galaxies in clusters, it would seem that there are as many arguments for the opposite view—that star formation is just starting at a very rapid rate in elliptical galaxies.

What would we expect a galaxy with an initially rapid rate of star formation to look like? There should be many late type supergiants contracting towards the main sequence. Blue and white stars should be absent, but a number of faint stars somewhat above the main sequence should be present. There should be almost no gas present as most of it would have been used up for star formation. The gas will reappear in the later stages when it has been ejected by bright stars in their last (or perhaps their penultimate) stages of evolution. A situation such as this seems to describe the stellar content of giant elliptical galaxies.

The newly formed stars should have an appreciable abundance of metals representing the metal abundance of the interstellar gas in the Galaxy. We have been told that the helium abundance in the gas is similar in different galaxies. There is, therefore, no reason why the metal abundance also should not be the same. If this is so, we should look for spectral features representing late-type (say K) stars and, as Dr. Morgan has told us, this is, indeed, what the spectra of elliptical galaxies look like.

In view of the rather short lifetime of relativistic electrons, to which Dr. Biermann referred, we could consider that radio galaxies represent a short period in the evolution of some galaxies. Now, it is reasonable to assume that such periods would occur either at a very early, or at a very late, stage in the evolution of a galaxy. There are, as Dr. Minkowski has shown, many E galaxies among the optically identified radio sources. This serves to indicate that the arguments in favor of an elliptical galaxy being either very young or very old are equally balanced.

We would expect a young galaxy to occupy a rather central position in a cluster of galaxies. This is what Dr. Abell has found for giant elliptical galaxies and this, in turn, is an argument in favor of youth.

I do not want to insist that elliptical galaxies are very young and that star formation has just begun in them. I am not sure that I believe in this thesis myself! What I am trying to do is to show that there are a number of arguments on both sides and that we should take this into account when drawing general conclusions.

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Multiple Systems, Clusters, Radiogalaxies: Summary

The papers in Part II have fallen into two categories. First, those consisting of discussions of the properties of multiple galaxies and clusters of galaxies, and, second, those by Minkowski and Biermann on the strong radio sources. It appears to me that it is not clear at present that there is a strong connection between these two topics. The only possible connection is, as was discussed earlier by Mills and others, that there are a number of identifications of radio source positions with clusters of galaxies. However, this may be due only to the fact that there are more galaxies in clusters than outside them. Thus, I shall treat these two groups of papers as two separate topics to be summarized.

Let us first consider the papers on groups and clusters of galaxies. The first paper, by E. Holmberg, gave us some details of his investigations of the relative frequencies of multiple systems and he drew the following conclusions. Fifty-three per cent of all extra-galactic objects outside clusters are members of double or multiple associations. The mean space distance of double systems is about 90,000 pc. Twenty-four per cent of the objects are double, 15 per cent triple, and 14 per cent members of systems of higher order. He believes that there is evidence for subclustering in the

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Virgo Cluster. His major conclusion, and, certainly, to my mind, his most important conclusion, is that there is a pronounced correlation between members of an association as regards integrated color index and mean surface magnitude. This does not mean, of course, that the galaxies have the same brightness; in many cases, one member of a system is very much brighter than all of the others; hence, this does not mean that the systems are equally massive or nearly equally massive. What it does mean is that there is some correlation in a group between mass and mean density. Holmberg concludes from these results that galaxies in the same groups have the same type of stellar content and the same mean surface luminosity. He thinks from these arguments that such groups have formed by some disintegration process. However, it was pointed out in the discussion by von Hoerner that systems of similar type in this respect would result simply if the systems formed at the same time, though not necessarily all in the same condensation. This means, in turn, that the group must have aggregated together in some way, perhaps by capture, though I would comment here that the time scale for such processes may be exceedingly long.

The second paper in Part II, by Dr. Vorontsov-Velyaminov, described the work that he has been carrying on, on multiple galaxies and connected galaxies – those systems which appear to have bridges between them and tails extending from them. He described a large number of important problems in connection with some groups. He argued that, probably, tidal effects were not able to explain the occurrence of the strange features between and outside the galaxies in such groups. He suggested that perhaps other forces – by this, he apparently meant forces other than gravitational forces or magnetic forces – were at work. In the discussion of his paper, a number of comments were made about the composition of these bridges. Dr. Zwicky stated that they were made up of stars, and there was some discussion as to whether they were old systems – as would apparently be the case from the results that Dr. Zwicky quoted. Our own results of investigating such bridges suggest that, in many cases, stars and, particularly, old stars are not involved; i.e., we believe that in many cases gas is present. To obtain spectra of such features is exceedingly difficult, and I doubt whether we have enough good information as yet to allow us to decide what the compositions are in general.

Dr. Abell has discussed the forms and distributions of large clusters of galaxies. He divided them into two categories: (*a*) large symmetrical compact clusters, such as the Coma Cluster, which contain mainly E and SO galaxies; and (*b*) irregular clusters containing many spiral galaxies. He pointed out that clusters in category (*a*) have distributions which resemble equilibrium configurations. It is important to recall that, in an earlier conference, Dr. Abell discussed the existence of superclusters. I should mention that I think that Dr. Zwicky does not believe that superclusters exist. According to Dr. Abell, these superclusters have diameters of about 50 Mpc. Examples are: the Virgo supercluster, for which Abell has provided more evidence, and the supercluster containing the Hercules Cluster, the NGC 6166 cluster, and several other irregular systems. Dr. Abell discussed the methods by which it may be possible to use clusters as distance indicators. This is a very important problem that was also discussed by Zwicky in Part I, and it is connected with the discussion that comes in Part III, so I shall not go into it here.

The important problem which formed a large part of our earlier discussion, which was brought up again by Dr. Limber, concerns the application of the virial theorem to the clusters and groups. Since there was a large amount of discussion which followed Limber's paper, it is not necessary for me to repeat many of the viewpoints which have been expressed. Briefly, it has been found that, for a fair number of groups and clusters (all but one or two in about fifteen), if the virial theorem is to be satisfied, we need to suppose that the galaxies have much higher masses and mass/light ratios than we believe possible from the measures of the masses of single and double galaxies. This result appears to be fairly general and applies to systems ranging from the small compact groups up to the large, symmetrical Coma Cluster. The results obtained for the masses of double galaxies by Page are in fairly good agreement with those for the few single galaxies for which we have measures of mass from internal motions. Hence, I suppose we would believe that, on the basis of the present-day evidence, double systems do not have positive total energy. For all the other groups, a number of possible explanations have been given:

1. The systems are expanding as was originally proposed by Dr. Ambartsumian. This implies that time scales of order 10^8 to 10^9 years are involved and, since galaxies of all types are present, this means that some or all of the galaxies in such clusters are young. If only some are young, then they must have been formed in material associated with the galaxies already there but with positive total energy.
2. The systems are stable because there is a large amount of dark material present – from about 80 per cent to 98 per cent in different groups.

3. Holmberg has argued that much of the discrepancy as far as the Virgo Cluster is concerned is due to large systematic errors in radial velocity measurements. Pending a further test of these arguments, it seems that we should accept the original radial velocity results at their face value. In any case, his arguments cannot apply to many of the groups discussed.

It seems that there are probably systems which are stable and, also, systems which do have positive total energy. There is certainly no objection to our invoking the presence of unseen inter-galactic material except that we have no direct evidence for its presence, and this argument is, in fact, the only evidence for it at the present time. Zwicky suggested in the discussion that there was evidence for inter-galactic material in objects such as the Coma Cluster, and he suggested that a very large amount of mass could be deduced to be present from these observations. However, I am inclined to accept the remarks made by Baum and de Vaucouleurs who said that they thought that, although there was some evidence of measurable surface brightness between the galaxies in such a cluster, it could only reasonably be thought to account for about as much mass as there is in the galaxies. Clearly, objects such as the Coma Cluster do suggest by their configurations that they are stable systems. Thus, I think that the consensus at the present time is that, for systems of this type, inter-galactic material is present in sufficient quantities to stabilize them. In other cases, the problem seems to me to be more difficult. It may be that there is always sufficient material present. However, for the very small compact groups, this presents a problem in that the very high densities which would be required would, one might expect, lead to star formation on a large scale between the galaxies in such a group. On the other hand, the time-scale arguments are certainly very severe. I shall have something to say about those shortly.

It should be mentioned that we now have a considerable number of arguments relating to the situation in the Virgo Cluster. It is, perhaps, instructive to list the various suggestions which have been made concerning this cluster. First, we have the arguments, originally due to Ambartsumian, that the cluster has positive total energy and is expanding. Second, we have the paper of de Vaucouleurs who has concluded from the radial velocities that there are two separate clusters involved – one containing predominantly elliptical galaxies and the other containing predominantly spiral galaxies. Both of these clusters are probably unstable, according to him, though there is no reason to believe that they make up a pair. Third, we have the arguments of Holmberg to which I have

referred above. Fourth, we have the possibility that a large amount of dark matter is present.

It is difficult to draw any final conclusions from the discussion of the stability of the clusters. For any particular group or cluster, it seems, at the moment, to be very much a matter of taste as to which explanation of the apparent dynamical state of the cluster is assumed to be correct.

There are three other points which are important for the discussions raised by these four papers and they were not mentioned, or only briefly touched on. The first concerns the possible existence of inter-galactic magnetic fields. If fields are present in the inter-galactic medium in clusters, and if they have strength greater than about 10^{-7} gauss, then the situation as far as the virial theorem is concerned gets very much worse, since the presence of magnetic fields will, in general, tend to add to the instability of the cluster. Since in some clusters we have radio sources which may be associated with particular galaxies and which will be discussed below, I think that this fact should not be overlooked.

Second, I have already mentioned Dr. Holmberg's arguments concerning the radial-velocity measurements. He also argued in the previous conference that the calculations in applying the virial theorem to the Virgo Cluster had not been made correctly because the orbital motions of the galaxies in the subgroups had not been taken into account. I should like to emphasize that this is not the case in many of the systems. For example, in the Hercules Cluster, the potential energy was calculated by measuring the apparent separations between all pairs of galaxies on a plate. Systematic errors in the red-shifts, even if they are present in some of the early work, and this would seem to be an open question at present (cf. the comments by Mayall, de Vaucouleurs, and E. M. Burbidge on pages 404–407), are certainly not present in general.

The third point, which is perhaps quite new and unacceptable to many at the present time is that, at least twice during these conferences, it has been proposed that some elliptical galaxies may be young. There are a number of reasons for suggesting this. If some clusters are expanding, then probably the galaxies would require comparatively short time-scales. Furthermore, on Shklovsky's picture of the origin of radio sources in galaxies, these galaxies would have to be young in evolutionary terms, i.e., they must contain large numbers of stars which are evolving to the supernova stage quite rapidly and quite frequently. I would simply like to stress that this question should be looked into in detail. In the case of fairly nearby ellipticals and other systems like M 31 and M 81, we have evidence from the spectroscopic investigations of Morgan and Mayall that a population of giants arising from the region of the main sequence near the sun is present. Thus, we must conclude that the systems are old. However, for more distant systems these criteria have not and, at the present time, can not be applied, and it is not clear to me that the integrated color alone will allow us to decide. However, this point is open to detailed investigation. It should be stressed that we have no information as to the initial luminosity functions in galaxies of this type. There is certainly no theoretical reason for believing that all galaxies have the same initial mass spectra and luminosity functions. It would appear to me that these would depend in large part on the initial conditions of angular momentum, magnetic field, turbulent velocity, etc. The results of Limber and, also, intuitive arguments suggest that there is a wide range in initial luminosity functions. Thus, I feel that we should not dismiss lightly the possibility that some galaxies which, apparently from very scanty investigation, are classified as elliptical galaxies, may be young.

Finally, in this category of problems, something should be said about the possible age of an object like the Coma Cluster. If it is a system in equilibrium, then its age may be very great. The relaxation time in the inner part is at minimum about 10^{10} years and, not very far out, it is about 10^{12} years. Zwicky has argued that the cluster is exceedingly old – 10^{15} years – since he believes that the cluster must have been formed by capture. Though this is probably not the case, it may very well be that an age of the order of 10^{11} years may be required for the galaxies to reach the configuration they are in today. Much depends on the initial conditions, in fact, all depends on these conditions, and, as was discussed in another connection in Part I, the relaxation times for multiple systems – in this case, subcondensations in the cluster larger than single galaxies – might reduce the times appreciably. There is still a very considerable problem concerning the formation of galaxies and clusters. However, it should be stressed that very long time-scales of the order of those discussed by Zwicky seem to be entirely out of the question, not from the point of view of cosmology, but from arguments based on stellar evolution. Even ages of 10^{11} to 10^{12} years may present some difficulties, though, if the clusters are as old as this, we would expect there to be present, in the form of unseen or very low-luminosity material, evolved galaxies made up completely of white dwarfs. These, of course, would contribute to the mass; hence, a cluster such as the Coma Cluster might well be stabilized by material of this type.

Now I turn to a review of the two papers on radio sources in Part II. The first, by Dr. Minkowski, dealt with the problems of identification and gave some account of what we know today about the types of galaxies that are strong radio sources. A most striking feature is that these sources are very large as compared with the optical galaxies, and that, in many cases they are double; in fact, they may have more complex brightness distributions. There does not seem to be a correlation with double optical galaxies. Since the arguments are still based on about 20 identified objects with fewer than 20 red-shifts available, it is, as was stressed by Dr. Minkowski, too soon to make any statistical arguments concerning the types of source, the brightness distributions, or the frequencies of different types. The galaxies show many different features. In many cases, they are described as "peculiar" because they are seen to be peculiar on the direct plates. But, for all of the sources which are fairly distant, the peculiarity can only be a spectroscopic peculiarity. Spectroscopically, the systems can be roughly divided into two types. On the one hand, we have galaxies which show fairly normal absorption-line spectra, or absorption spectra together with weak and quite common emission features such as [O II] λ 3727. These are found, for example, in such objects as M 87 and NGC 5128. On the other hand, we have sources which certainly form a category of their own, such as the very strong emission-line objects: the Cygnus A radio source, 3C 33, and others. The only other galaxies which show strong emission lines are the very small class of galaxies known as the Seyfert objects.

Finally, we come to the paper by Dr. Biermann. He discussed our understanding of the physics of the sources. We still believe that the synchrotron theory is best able to explain the features that we observe, but the energetic requirements are still very severe. The total energy in particles and magnetic field ranges from 10^{56} to 10^{60} ergs in the various sources for which we have adequate data, i.e., red-shifts; hence, distances; hence, dimensions. These are only minimum total energies and they are not easy to explain. The duplicity of many of the sources is of very great interest and it has been proposed that these give evidence perhaps for a toroidal distribution of magnetic fields and particles. However, the distribution among the few systems that we have does not suggest that such a situation is valid in all cases. In any case, it is very hard to understand why the magnetic fields in these very large volumes should be as strong (10^{-5} gauss or greater) as they need to be if the condition of minimum total energy is to be fulfilled. Dr. Biermann suggested that such fields could be tied to the brightest, hence, perhaps the most massive, elliptical galaxies so that there was a reasonable degree of equilibrium. However, it may be that the systems, even as far as the magnetic fields are concerned, are evolving quite rapidly. The duration of a radio source and the frequency among galaxies of different type, or perhaps of some special kind, can only be estimated from the relative numbers of radio sources as compared with the total number of galaxies seen in the same volumes. No very accurate figures for these are yet available.

The source of the high-energy particles forms another problem which I shall discuss briefly. These must originate in the optical galaxies which are associated with the sources. Three different suggestions have been made. Shklovsky has proposed that these are galaxies in a fairly early stage of evolution and that, hence, massive supernovae are going off in them and ejecting large fluxes of particles. Hoyle has proposed that some kind of a galactic flare, analogous to the solar flare model proposed by Gold and Hoyle, is responsible for giving rise to the high-energy particles. Finally, I have suggested that the particles come from a catastrophic outburst in the central regions of a galaxy when one supernova which goes off normally in the course of its evolution triggers a large number of supernovae because the star density is very high. Two of these mechanisms, that of Shklovsky and my own, derive their initial energy from nuclear sources. In Hoyle's case, the energy comes from the rotational energy and the magnetic energy in the inner parts of such galaxies. It is not clear at the present time which of these mechanisms is to be preferred or whether another mechanism remains to be discovered. The energetic requirements if the synchrotron theory is to be saved are very severe. From this point of view, the presence of a small amount of antimatter is very attractive, but the disadvantages of assuming this is present on the large scale still outweigh its advantages.

This summary has been difficult because, although much valuable information has been discussed, it has certainly raised more problems than it has solved. It is much easier to propose rather speculative theories than it is to get to a deeper understanding of even one simple elementary problem. The best mode of attack in this field at the present time would seem to me to be the collection of much more observational material.

PART THREE

Galaxies as Members of the Universe

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Distribution of Galaxies on the Sphere: Observed Structures (Groups, Clusters, Clouds)

Reviews of the observational material and of previous studies of the distribution of galaxies on the sphere have been published recently by Shane [1] and by Zwicky [2]. Also, extensive new observations are due to Shane, and to Shane and Wirtanen [3,4,5], although their results for several regions will be published only shortly. Similarly, some indications have appeared [2,6,7] from the forthcoming *Catalogue of Galaxies and Clusters of Galaxies* prepared by Zwicky, Herzog, and Wild; the first volume [8] is expected momentarily. Thus, this survey article is a bit early.

However, it is of interest to present a brief review of the present understanding of the distribution of galaxies on the sphere and to indicate the noticeable dependence (to be expected, but still curious) of the theoretical thinking on the kind of observational data available. The difficulties in obtaining these observations and, thus, the effects of selection and of uncertainties in much of the material introduce complications in the discussion and even misleading conclusions unless special care is taken.

BRIEF HISTORICAL BACKGROUND

The irregularity in the apparent distribution of galaxies, their avoidance of the plane of the Milky Way and the existence of some of the nearest clusters were known to the Herschels more than a century ago from their comprehensive survey [9] of galaxies. This early catalogue was succeeded by the *New General Catalogue* of Dreyer [10] together with its two supplements. The many studies, both observational and theoretical, based on these catalogues and on plots of the galaxies listed have been rather special largely because the material is not sufficiently homogeneous.

The first modern survey is due to Shapley and Ames [11] and lists 1249 galaxies brighter than the thirteenth magnitude over the entire sky. Plots of these galaxies can reveal only some of the nearby clusters and groups, such as the Virgo Cluster and the Ursa Major Cloud. In other words, the volume of space to which this survey extends is too small to show the distribution of galaxies simply because it is of about the same order of magnitude as the structure of clusters of galaxies is now thought to be, say, 5 Mpc^3. On the other hand, a few clusters and groups were now recognized and studied for their own interest, especially by Shapley [12] and Zwicky, although many astronomers seemed to regard these clusters as oddities and to consider almost all galaxies to be field galaxies with a random (Poisson) distribution.

The fact that this attitude still prevailed after the survey of Hubble [13,14] is not surprising, since he counted galaxies in small areas widely distributed over the sky. Primarily interested in the possible change in space density with depth as a measure of the expansion of the universe, Hubble used several different limiting magnitudes to about 20^m. At the same time, the identification of clusters, especially of distant clusters, brought about the possibility of increasing the precision in the determination of the magnitude-red-shift relation in connection with the study of the expansion of the universe; this, in turn, spurred the search for clusters.

Studies of the distribution of galaxies on the sphere were started at several observatories. In 1941, Reiz [15] published a catalogue and discussion of 4666 galaxies brighter than $16^m\!.5$ and north of galactic latitude 50° which he prepared by photometering the Heidelberg plates. Between 1932 and 1951, Shapley and his coworkers [16] published extensive counts of the number of galaxies, to about 18^m, in small squares for both northern and southern hemispheres along with some details about the magnitudes. They called attention [17,18] to the clustering and suggested the presence of superclustering. These data are sufficient to demonstrate [17,19] that the observed distribution is too irregular to be Poisson. However, the Harvard Survey is not convenient for intensive study because the plates are not aligned and do not appear to be homogeneous. Extensive duplicate plates and duplicate counts are needed to clear up this question.

In the survey of Shane and Wirtanen [3,4,5], mentioned in the first section, counts were made, to about $18^{m}.4$, in contiguous $10' \times 10'$ squares in regularly spaced overlapping $6^{\circ} \times 6^{\circ}$ plates covering all the sky visible from the Lick Observatory. Special efforts were made to have the plates and the counting as homogeneous as possible. Duplicate (and some triplicate) plates of the same high quality were made of more than 21 selected fields; all of these plates and many others were counted twice or more. Here, this survey is unique in that it is now possible to estimate the reliability of the counts and then to determine appropriate corrections to the conclusions from the counts. We have found but one other appreciable system of repeat counts, that of Mayall [20], who also found that the variability is large. This problem is important and will be taken up in the next section. Whenever it is possible to check the reliability of any feature concerned with the distribution of galaxies on the sphere (not only of the counts), we find that hopes for agreement to within a few per cent are grossly optimistic.

Shane and Wirtanen have noted many striking clusters and groupings of galaxies, including apparent superclusters, in the Lick Survey. A few details about these have been published [1,3, 4,5] with their counts, but most of this material is still under study. Their equal-density contours for the galaxy counts give striking evidence of the clustering of galaxies and led Shane to suggest that a large proportion of galaxies are in clusters. Shane, together with Neyman and Scott [21,22,23], initiated a series of papers analyzing the Lick counts from the point of view that galaxies occur only in clusters of one or more members and estimating the structure of these clusters. A discussion of these and other studies is given in the next paper by Neyman. The discussion [22,23] of the Lick counts reveals that the serial correlation between counts in $1^{\circ} \times 1^{\circ}$ squares persists to about 4° and that it has very nearly the same values in the two galactic polar caps. Interpretation of the figures requires determining what part of the observed effects is due to occurrences in outer space and what part is due to the "disturbing variables": the telescope, the atmosphere, the emulsion, the observer, and so forth.

The National Geographic Society-Palomar Observatory Sky Survey and other surveys using the 18- and 48-inch Schmidt telescopes have provided a wealth of material which is still under study. An important study concerned with the distribution of galaxies on the sphere is that of Abell [24] on the distribution of rich clusters of galaxies; this subject is discussed in a separate paper by Abell in this book.

ASSIGNMENT TO CLUSTERS

The observations available for a discussion of the observed structure of the distribution of galaxies on the sphere include counts of galaxies in small regions and delineations of apparent clusters and groupings as well as characteristics of individual galaxies such as: apparent magnitude, radial velocity, color, type, and surface brightness, which are useful – perhaps unfortunately so, when the effects of selection are to be assayed – in determining membership in a particular cluster or group. The observed distributions of the characteristics have been taken up by Abell and others in this book; we shall largely confine ourselves to their use in studying the distribution of galaxies into clusters and groups.

It is clear from the literature that an appeal to the value of a galaxy's radial velocity or of its magnitude in order to determine whether the galaxy is a member of an apparent cluster is extremely common (see, e.g., [25], 109). Furthermore, we can presume that these appeals have an influence on the observed distribution of radial velocity, or of magnitude, as the case may be. This difficulty is illustrated in Figures 1 and 2, which give the observed dispersion in radial velocity and in apparent photographic magnitude, respectively, for clusters (crosses) and groups (dots) as a function of the mean radial velocity (distance) of the cluster or group.

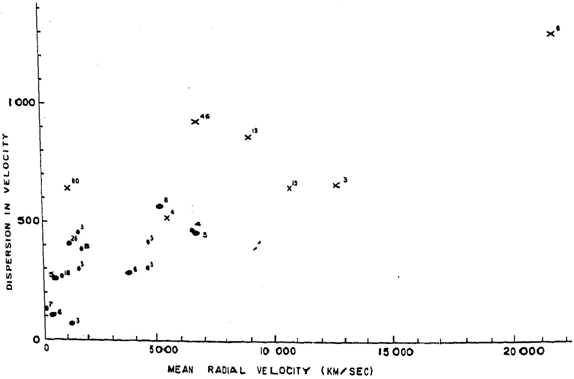


Figure 1. Increase in observed dispersion in radial velocity as a function of the distance to the cluster (cross) or group (dot). Small figures give the number of "member" galaxies for which observations are available.

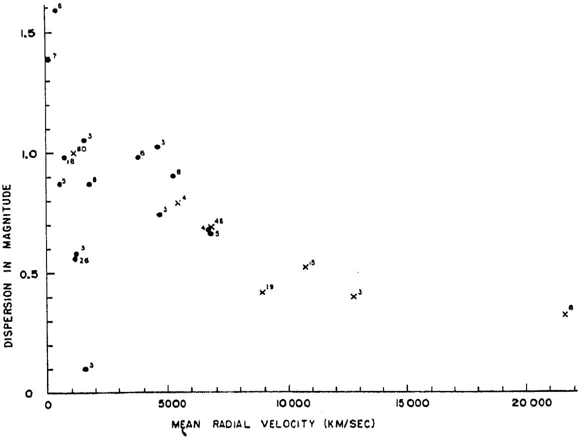


Figure 2. Decrease in observed dispersion in photographic apparent magnitude as a function of the distance to the cluster (cross) or group (dot). Small figures give the number of "member" galaxies for which observations are available.

The plots are adapted from tables in [26] which are based on the observations published in [25] with additional observations of Mayall, of Abell, and of Minkowski. We show all clusters or groups for which observations of both radial velocity and magnitude could be found for at least three so-called members; the actual number of observed members is shown by the small figure next to the symbol. Even though the number of observations is often small so that the estimate of dispersion is not reliable, general trends are apparent. The dispersion in velocity appears to increase with distance, with clusters having larger dispersion than groups. On the other hand, excepting one unreliable point, the dispersion in magnitude appears to decrease with distance, with clusters having, if anything, slightly smaller dispersion than groups. It is impossible to say how much, if any, of these trends represents real effects, reflecting intrinsic differences, evolution, and so forth, and how much results from selection of "members" so as to force agreement in magnitude and in radial velocity and, thus, decrease the dispersion artificially. Perhaps astronomers are more stringent about agreement in magnitude when classifying faint galaxies as cluster members since they realize that the proportion of foreground and of background galaxies increases rapidly with faintness. The same argument could be advanced regarding agreement in radial velocity, but it might happen that the difficulty and time required to observe the radial velocity of a faint galaxy are so great compared to the effort required for a rough estimate of the magnitude (sufficient for classification) that the astronomer is more reluctant to "discard" a galaxy on the basis of slight divergence of its radial velocity. Furthermore, in distant clusters only the brightest galaxies are observable; this selection will markedly reduce the observed dispersion in magnitude, but will have no effect on the dispersion in radial velocity unless luminosity and velocity are strongly correlated. In any case, Figures 1 and 2 refer only to types of galaxies for which both radial velocity and magnitude can be observed; thus, certain subluminous and low surface brightness objects are eliminated while emission-line objects are favored. We can conclude only that any real effects of intrinsic differences among clusters and the effects of selection of cluster "members" are highly confounded.

Similar difficulties arise when the clusters are outlined by eye on the photographic plate or when "the diameters of all the clusters are determined through the use of that specific equal-population contour line or isopleth at which the counts of the three brightest magnitude classes per unit area are equal to double the counts per unit area of galaxies in the same apparent magnitude range averaged over a large area surrounding each cluster." This is the practice of Zwicky (see [7], 169). Clearly, the probability of identifying a cluster at all, as well as the estimated number of members and the diameter, are sensitive to the ratio of the richness of the cluster in comparison to the richness of the field. They depend also on how much of the three-magnitude interval for the cluster is brighter than the plate limit. Even when these factors are held constant, there remains a selection bias in favor of richer, more compact clusters. The difficulties are illustrated schematically in Figure 3: the upper panel shows the effects of varying cluster richness and compactness when the field density, corresponding to the three-magnitude interval, is constant at a given level; the middle panel shows similar effects when the field density is fixed at a higher level by increasing the limiting magnitude and, thus, cluster density as well; the lower panel illustrates some complications introduced by variations (exaggerated) in the field density.

The diameter, γ, as defined by Zwicky [2,7], is indicated for those clusters that will not be missed by the procedure. The shaded area shows the number of "cluster galaxies" within the contour and in the three-magnitude interval. In each case, the clusters are drawn as if superimposed in the field [7]; if the field were considered as a superposition of many clusters [22,27], the details would change but the difficulties would persist. Intuitively, it seems that the chance of detecting a cluster by this procedure must become very small as the field becomes dense by increasing

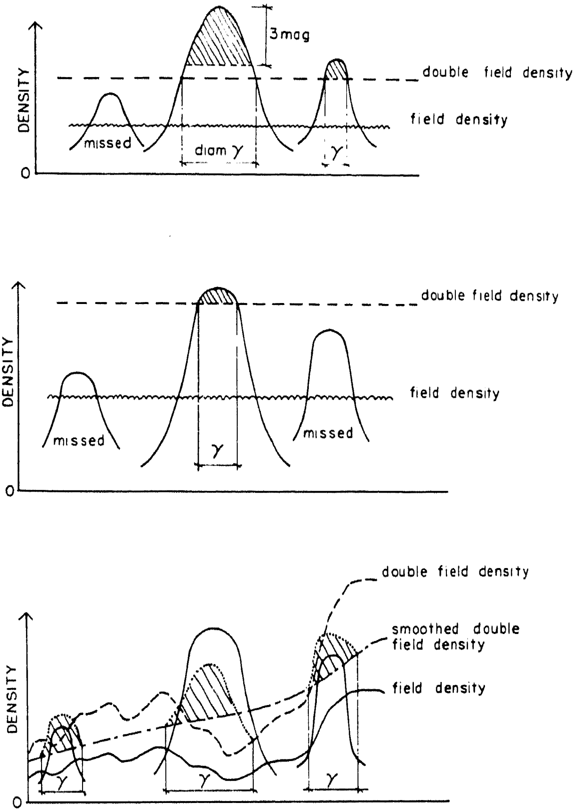


Figure 3. Effects of varying cluster richness, varying compactness, and varying field density on the delineation of the cluster contour, on the estimated diameter and on the estimated number of cluster galaxies.

the limiting magnitude. This may account, at least partly, for Zwicky's difficulties in finding clusters on 200-inch telescope plates.

Compounding the effects of selection is the lack of reproducibility of the observations. We anticipate that boundaries of a cluster as drawn independently on two different plates, or by two observers, or by the same observer at two times, would differ appreciably and might differ sharply. In order to study the reproducibility of cluster measures, systematic investigations with duplicate observations are needed. As mentioned in the previous section, the only extensive study of this sort of reproducibility is the investigation of the variability in the counts of galaxies in a small square by Shane and Wirtanen. As reported by Mayall, Scott, and Shane [28], the concept of limiting apparent magnitude is quite unrealistic. Typical counts and recounts of the same plate (shown in Table 1) and of two plates of the same field (shown in Table 2) differ considerably. We note substantial scatter about the main diagonal in both tables and see that in each table there are entries on both sides of the diagonal. Therefore, it is not possible to explain the differences in the counts as effects of a change in the limiting magnitude from one counting to the next or from one plate to another. We conclude that counting or not counting a galaxy is a random trial, and we postulate that the probability that a galaxy will be identified and counted is a function of: *(a)* its apparent magnitude, *(b)* its type or subtype, *(c)* its location on the photographic plate (a telescope effect), *(d)* the location of the field, including effects of polar distance and of zenith distance, *(e)* the average plate effect including modifications over the plate as a whole due to seeing, emulsion, etc., *(f)* the identity of the observer, and *(g)* the state of the observer at this counting.

If we assume that the above various effects are independent and additive (we realize that they may not be), and if there are extensive duplicate plates and duplicate counts, then the differences allow us to estimate the average of each of the various effects and

TABLE 1

Bivariate Frequency of Duplicate Counts in
10′ × 10′ Squares on Same Plate by Same Observer

Coordinate of center : $\alpha = 9^h\ 12^m$, $\delta = +50°\ 00'$
Abscissa : First count of plate 3340 ; total counted 1122
Ordinate : Second of plate 3340 ; total counted 1158

14															1
13															1
12									1						
11															
10									1						
9								1		1					
8					1		1	1		1					
7					2		4	3							
6			3	1	1	3	4	1							
5			1	3	6	5	1	1							
4		3	6	15	13	6	2								
3	1	10	16	31	13	4	1								
2	3	40	50	20	6	3									
1	37	100	32	7											
0	66	34	1	2	1										
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

TABLE 2

Bivariate Frequency of Counts of Images of Galaxies in 10′ × 10′ Squares on Duplicate Plates by C. D. Shane

Coordinates of center : $\alpha = 9^h\ 12^m$, $\delta = +50^\circ\ 00'$
Abscissa : Counts on plate 3340 ; total counted 1122
Ordinate : Counts on plate 3341 ; total counted 1189

14														
13														
12													1	
11													1	
10														
9						1	1		1	1				
8		1	1	1	1	1	2							
7		1		1	1		1	2	1					
6			1	2	5	2	3	3	1					
5	1	1	5	4	6	2	2	1						
4	1	3	6	13	6	4	1	1						
3	2	14	24	27	13	5	3							
2	6	42	36	20	12	3								
1	33	90	33	9	3	3								
0	64	35	3	2										
	0	1	2	3	4	6	7	8	9	10	11	12	13	14

the variances of some of them. Unfortunately, the computations must be repeated for each new observing program and whenever a new procedure or a new observable variable (counts, boundaries, diameters, etc.) is introduced. Experience in other fields, such as bioassay or ecology, shows that the reproducibility tends to decrease when more complex observations are made, especially if more subjective judgments are required. Although no evidence is available now, we can predict that the reproducibility of cluster counts, of cluster diameters, of cluster population counts, etc., must be much worse than the reproducibility of counts of galaxies in small squares. It would be very desirable to have evidence available to compute the effects of the lack of reproducibility of these determinations.

DISTRIBUTION OF APPARENT MAGNITUDE, LUMINOSITY, TYPES, ETC., ON THE SPHERE

Since the assignment of galaxies to clusters and groups is difficult and subject to both systematic and random errors, it seems advisable to investigate the distribution on the sphere without attention to the particular clusters. In their full generality, the results are complicated and the formulas are not easy to print. However, they can be evaluated numerically, although the computations are heavy. If the observational material is restricted to field galaxies, that is, if the number of galaxies from the same cluster is not overwhelming, the formulas simplify markedly and some general conclusions can be noted [29,30].

(a) Suppose that the selection of galaxies for observation of apparent magnitude depends on their apparent magnitude only. Let $\Phi(m)$ denote the probability that a galaxy of magnitude m will be selected. We require no assumptions about the selection function $\Phi(m)$ except that it tend to zero as m increases. In other words, we assume only that the probability of observing very faint galaxies is exceedingly small. The actual $\Phi(m)$ may be rather complicated and certainly will not be the same for different telescopes or different observing programs. Whatever this selection function $\Phi(m)$ may be, the distribution of apparent magnitude among the selected galaxies does not depend in any way on the luminosity function of galaxies and is given by

$$C\, 10^{.6m}\, \Phi(m) \quad \text{with} \quad C \int_{-\infty}^{+\infty} 10^{.6m}\, \Phi(m)\, dm = 1\,,$$

where C is a constant norming factor.

It follows that the observed distribution of apparent magnitude among the field galaxies may be used to estimate the selection function $\Phi(m)$, and that the observed distribution is entirely irrelevant to the problem of estimating the luminosity function.

(b) Suppose again that $\Phi(m)$ is the probability that a galaxy of magnitude m will be selected for observation. Suppose also that the observer could compute the absolute magnitude of each galaxy he selects by assuming, say, that distance is proportional to redshift. Then the absolute magnitude and the apparent magnitude of the selected galaxies would be mutually independent. Furthermore, the "catalogue" distribution of absolute magnitude for the selected galaxies would be $10^{-.6M}$ times the luminosity function times the norming constant, independent of the selection probability $\Phi(m)$. This relation between the observed "catalogue" distribution of absolute magnitude and the luminosity function has been known for a long time in the special case in which $\Phi(m) = 1$ out to a limiting magnitude m_L and is zero thereafter. The newer result quoted [29, 30] shows that $\Phi(m)$ plays no role here and, further, any attempts to "correct" the observed distribution of absolute magnitude so as to make $\Phi(m) = 1$ for $m \leq m_L$ and $\Phi(m) = 0$ for $m > m_L$, as is often done [31], are harmful and distort the luminosity function.

A special case of the relation between the catalogue distribution of absolute magnitude and the luminosity function was first found by Malmquist [32] when studying the space distribution of stars. However, he assumed that the space luminosity function and, therefore, the catalogue distribution of absolute magnitude, of stars is normal; no such assumption is necessary.

(c) Under the assumption in *(b)*, the magnitude-distance relation, that is, the catalogue regression function of the distance D on the apparent magnitude m, is simple. No matter what the selection $\Phi(m)$ on magnitude may be, and irrespective of what the luminosity function may be, the average of any k^{th} power of the distance, for all those galaxies in the catalogue that have the same apparent magnitude m, is proportional to $10^{km/5}$, independent of the luminosity function and of the selection function. Only the factor of proportionality depends on the luminosity function.

(d) If the selection function $\Phi(m)$ depends on type as well as on magnitude, then each type has to be considered separately. We then expect the $\Phi(m)$ and the various catalogue distributions to differ from type to type. Now the remarks *(a)*, *(b)*, *(c)* above apply to each type considered separately, and *not*, in general, to all galaxies.

Somewhat more complex, but still simple, general conclusions can be drawn about the abundances of the various types, the distribution of distances to the galaxies, and so forth. We refer the reader to [29,30].

For some characteristics of galaxies, the observed (catalogue) distribution of field galaxies has been used to obtain the distribution in space. An example is the determination of the luminosity function. In his pioneer work, Hubble estimated the luminosity function of galaxies and obtained a bell-shaped curve offering a reasonably good fit to the normal distribution. Since the method used by Hubble is strictly applicable to field galaxies of separate types, his calculations were repeated by Neyman and Scott [29], using the observations published by Humason, Mayall, and Sandage [25] which were vastly richer than those available to Hubble, and taking care to eliminate recognized members of clusters. The selection functions $\Phi(m)$ were estimated from the data on m for each type separately. Fresh estimates of the luminosity function were obtained, separately for the several types and with a clear limitation to field galaxies. Although errors in classification and in the assumption that a given object is a field galaxy are unavoidable, it may be presumed that the results obtained are approximately valid. All of the luminosity functions deduced are not too far from normal curves, though the fainter tail is somewhat longer, with dispersions near unity, or a little less. Thus, we have a con-

firmation of the earlier finding by Hubble. We remind the reader that these conclusions apply to types of galaxies for which red-shift and magnitude are observable.

The luminosity function of field galaxies was studied recently by Kiang [31] also using the observations in [25] but excluding: first, galaxies with observed red-shifts less than +100 km/sec where the use of the red-shift-distance relation is questionable; second, those in the rectangular area occupied by the Virgo Cluster, the galaxies of which are believed to have excessive peculiar motions; and, third, those south of declination −30°. Finally, Kiang uses some corrections for incompleteness, which are intended to produce the results which would have been obtainable if the empirical material were complete up to limiting magnitude $m_L = 15^m$. He eliminated all galaxies with apparent magnitude more than 15 and added components of double or multiple systems in Holmberg's 1937 catalogue for which the red-shift had not been observed by assuming that all components of such systems have the same red-shift. The galaxies added to the material tended to be the fainter components; 75 galaxies brighter than 15^m had their absolute magnitudes so estimated.

We cannot subscribe to Kiang's results for the following reasons. First, many known doubles, groups and clusters were included; there is the danger that these may dominate the results. All types of galaxies were treated jointly even though the selection functions Φ and, most probably, the luminosity functions differ from one type to another. Finally, for field galaxies no correction for incompleteness depending on m_L is needed. The data for multiples and clusters do need corrections, but these must be adjusted for each system separately in accordance with the selection function and with the type of galaxy considered.

The distribution of red-shift for field galaxies was investigated recently by Neyman and Scott [33]. Under the assumption that red-shift is proportional to distance, they find that the dispersion in red-shift is very small, almost surely less than 150 km/sec, thus, again confirming a pioneer result of Hubble. This result implies that, under this model, the red-shift is a good distance indicator.

We now describe an investigation which is of special interest because it allows the possibility of checking the assumptions underlying the calculations. Using the observations in [25], Neyman and Scott [29] computed the theoretical distributions of apparent magnitude as a function of red-shift for spiral field galaxies and for ellipticals separately. The selection functions $\Phi(m)$ were estimated from the observed apparent magnitudes only and are entirely independent of any assumptions regarding the luminosity function (see *(a)* above). On the other hand, each luminosity function was fitted independently of any assumptions about the selection function except that it depends only on m (see *(b)* above). Given these determinations, they computed the conditional distribution of apparent magnitude as a function of red-shift, including the regression and its confidence bounds. Both for spiral and elliptical field galaxies, the fit was excellent which tends to support not only the basic assumptions of the theory but also the special assumptions that the selection factor $\Phi(m)$ has a shape not far different from the cumulative normal and that the luminosity functions do not differ very much from the normal distribution.

We conclude this section with a brief consideration of what the distribution of characteristics of galaxies over the sphere would be if all galaxies were indeed in clusters but the individual galaxies were not, or could not be, assigned to specific clusters. Only a few features have been investigated. The series of papers by Neyman, Scott, and Shane [21,22,23] give the joint distribution of the numbers of galaxies in small squares separated by a given distance. Their work covered the expected number and the variance of the counts as well as the serial covariance. As mentioned in an earlier section, they find that the correlation between counts in 1°-squares persists for about 4°, and they use the observed distribution of the counts to study the spatial distribution of galaxies.

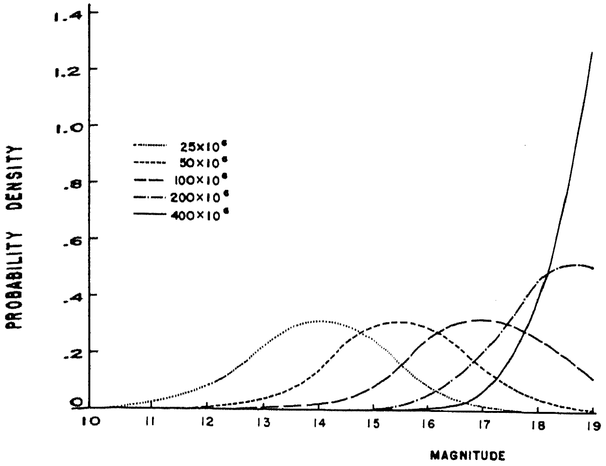
Some information about the appearance of the distribution of galaxies on the sphere is provided by the sampling experiment of Scott, Shane, and Swanson [27]. They simulated the spatial distribution of clusters and galaxies using tables of random numbers. First, cluster centers were Poisson-distributed in space. Then, a random number ν of galaxies was assigned to each cluster, and was determined separately for each cluster; each of these galaxies was assigned, again at random, a position within its cluster. Next, using the assumption that the luminosity function of galaxies is normal, a value of the absolute magnitude was determined for each galaxy. Now, the distance and the absolute magnitude are known synthetically for each galaxy, and the apparent magnitude was determined for each galaxy. All galaxies fainter than the postulated limiting magnitude of the Lick Survey, namely, $18^{m}.4$, were discarded and the rest were plotted to produce an "idealized synthetic" plate corresponding to the Lick Survey. Finally, the effects of the variability in counting (the telescopic, plate, emulsion, observer, etc., effects) were taken into account to provide a varying limiting magnitude for the synthetic plate and to allow the counting of each galaxy to be a random event. Thus was produced the plot of another, more realistic, synthetic plate which compared well with the plot of an actual plate.

The tremendous projection effects all but eliminated any noticeable clustering. In fact, the synthetic plate appeared to be slightly less clustered than the actual plate although every synthetic galaxy is known to be in some cluster, and we actually know which cluster it is in. Of course, if the underlying distributions of cluster size, of the number of galaxies per cluster, and so forth, were altered materially, the synthetic plate would be altered. In any case, we can conclude that the effects of projection are so strong that they make the detection of clusters on the photographic plate very difficult. In this connection, it is interesting to note that the synthetic plate showed many curious configurations: several galaxies arranged in a line, or in an arc, or in odd-shaped clumps. All of these configurations are, of course, due entirely to chance. Furthermore, in all the cases that were investigated, the members of these configurations, even those of about the same apparent magnitude, came from several different clusters which were more or less superimposed.

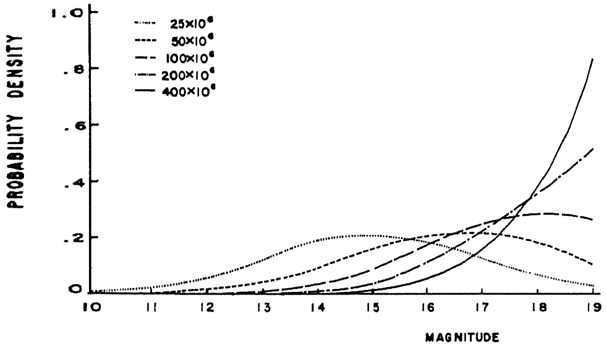
The changes in the observed distribution of apparent magnitude that would result if all galaxies were in clusters have also been investigated. The general formulas and some special cases are given in [29,30] and [34]. A numerical illustration is given in Figure 4 under the assumption that the luminosity function has normal shape. The computations show that the distribution of apparent magnitude for galaxies that are in the same cluster is the same (to at least three significant figures) for any size cluster from standard deviation $\sigma = 0$ up to $\sigma = 2 \times 10^6$ pc, with $H = 100$ km/sec/Mpc, and probably well beyond. Since $\sigma = 0$ would imply that the galaxy is located at the cluster center, we conclude that, although theoretically the clustering of galaxies affects the distribution of apparent magnitude, practically the effect is of no consequence.

The distributions in Figure 4 are drawn for the case of a fixed limiting magnitude $m_L = 19^{\mathrm{m}}$. If they were drawn for a more realistic function $\Phi(m)$, they would turn up sooner and a little less sharply. In each panel of the figure, five different curves are shown corresponding to five different distances to the cluster center. Thus, the dotted line shows the distribution of apparent magnitude for those galaxies belonging to clusters which are 25×10^6 pc distant, the smaller dashes correspond to clusters distant by 50×10^6 pc, and so on.

Figure 4a corresponds to a luminosity function which is normal with mean 36^{m} brighter than m_L and dispersion $1\overset{\mathrm{m}}{.}25$. If we could fix our attention on just those galaxies which are members of clusters 25×10^6 pc distant, even if these clusters were very large, we would have a distribution of apparent magnitude which is very nearly a displacement of the luminosity function. But, as we observe galaxies belonging to more distant clusters, the observed distribution of apparent magnitude would change its shape, the



4a



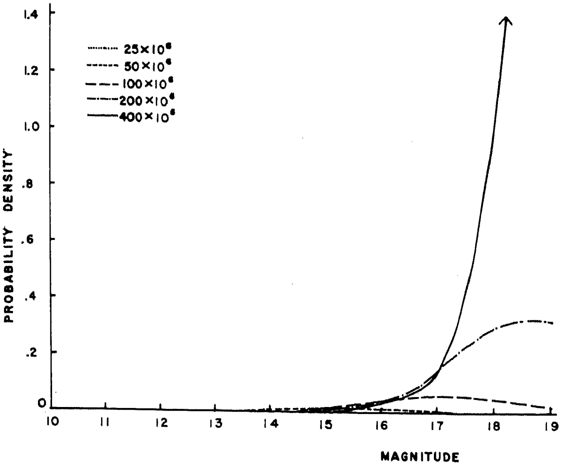
4b

Figure 4. The distribution of apparent magnitude (given less than $m_L = 19$) for galaxies belonging to clusters at each of the five distances shown.

4a. Corresponding to luminosity function normal with mean $(m_L - 36) = -17$, dispersion $1^{\mathrm{m}}.25$.

4b. Corresponding to luminosity function normal with mean $(m_L - 35) = -16$, dispersion $2^{\mathrm{m}}.00$.

4c. The distribution of apparent magnitude of galaxies on the sphere is the sum of weighted distributions, like those shown, resulting from the distributions in Figure 4a.



4c

Figure 4 (*continued*)

fainter end having much higher probability, as is well known for field objects [18].

Figure 4b corresponds to a normal density for the luminosity function with mean 35^m brighter than m_L and with a larger dispersion of $2^m.00$. If we could fix our attention on galaxies belonging to clusters at the prescribed distance, we would observe the distributions shown in Figure 4b. The situation is similar to that shown in Figure 4a except that the changes in the shape of the distribution occur earlier and are more pronounced.

If we do not consider just those galaxies belonging to clusters at a fixed known distance, but consider the entire field of galaxies on the sphere, the observed distribution will be the resultant of the individual distributions over all possible values of the distance to the cluster. As is well known, the different distances are weighted according to their squares. Figure 4c shows the weighted distributions arising from the distances used in Figure 4a. The weighted distributions for other distances could be interpolated or computed. We notice that the resultant of the weighted distributions, which is essentially the sum of the curves shown in Figure 4c, is completely dominated by the distant clusters and the corollary that the observed distribution of field galaxies is dominated by the most luminous of the galaxies. These conclusions are to be expected, of course, from the studies of the resultant distribution when the objects are Poisson-distributed.

DISTRIBUTION OF CLUSTER CHARACTERISTICS

It is convenient to separate the discussion of cluster characteristics into two steps – the estimation of the characteristic for particular clusters as individuals, and the resulting distribution of the characteristic for all clusters over the sphere – because the second step is essentially an extension of the first.

We have mentioned already the difficulties in assigning galaxies to clusters, in estimating the diameter of a cluster, and in estimating the number of cluster members. One might hope that these difficulties might be alleviated considerably by considering the characteristics of a giant, relatively isolated, individual cluster, such as the Coma Cluster. In Table 3, a comparison is shown of the estimates by Zwicky [7] and by Noonan [35] of the radius of the Coma Cluster and of the number of cluster members within this radius. Both sets of estimates are based on exactly the same data – the counts published by Zwicky – yet they are vastly different. The principal cause of difference is the difference in the estimates of the density of field galaxies; also, the methods of estimation are not the same. Noonan gives evidence to support his conclusion that Zwicky's limiting magnitude estimates are too bright. The revised estimates are more consistent with each other; in particular, the distribution of the brighter galaxies is not appreciably different from that of the fainter ones. They are more in line with the counts of Shane [1,3] and of Abell [35].

TABLE 3

Comparison of the Estimates of Zwicky and of Noonan in the Coma Cluster

Schmidt Telescope	*Estimated Radius*		*Estimated Number of Cluster Galaxies*		*Estimated Density of Field Galaxies*	
	by Z	*by N*	*by Z*	*by N*	*by Z*	*by N*
18″	160′	80′	639	557 ± 27	7.38	11 ± 2
48″	360′	100′	10724	1560 ± 40	170	275 ± 2

The average radius of a cluster, defined as: a constant times the dispersion σ in the distribution of positions about the cluster center, or defined as: the distance to the k^{th} outermost galaxy (for $k = 1, 2, \cdots$) in the cluster, etc., was estimated by Neyman, Scott, and Shane [22,23] from counts of galaxies in small squares in their investigation of the spatial distribution of galaxies. Here again, the estimates are not consistent, as they are sharply different when estimated from counts in 1°-squares and when estimated from counts in 10′-squares. The difference reflects difficulties in the underlying assumptions of their model which are not yet resolved.

Somewhat better consistency is obtained, although there is not much observational material, when we fit a distribution function to counts of the images of galaxies in an outstanding cluster, using a plate centered on the presumed center of the cluster and making counts of galaxies (cluster plus field) in small squares, strips, or concentric rings. Recently, counts have been made by Shane [1,3] for the Coma Cluster on two plates and for six other Coma-type clusters; by Abell [35] for Coma; and by Zwicky [2,36] for Coma, Corona Borealis, and six distant Coma-type clusters for each of which Humason has obtained radial velocities of about 60,000 km/sec. We should keep in mind that the counts in the outer parts of a cluster are likely to be too low due to telescopic effects, especially with 200-inch plates, and that the count in the central part of the cluster is likely to be too high due to the tendency to select that part of the cluster where the density happens to be high as in the (unknown) center of the cluster. In any case, the counts in the central part have the highest sampling variability since the average is taken over smaller areas.

Various theoretical distributions have been fitted to the observed counts, including (i) so-called isothermal gas spheres (Zwicky, Noonan), with a spatial density proportional to a constant C for distance r less than $1/C^2$ and thereafter proportional to $1/r^2$ to the edge of the cluster; (ii) empirical formulas (de Vaucouleurs, Shane), in which the number of galaxies at distance r, say n_r, is proportional to $\exp[a(r^{1/4} + b)]$ where a and b are negative constants; and (iii) Maxwellian distribution, that is, trivariate normal (Neyman, Scott, and Shane). In actual fact, the resulting theoretical distributions do not look very different and they agree well with the observations except that the immediate center of the cluster occasionally does not fit well. Taking the Coma Cluster as an example, in the central part *only*, Noonan's fit [35] by distribution (i) is somewhat too high, Shane's fit [3] by method (ii) is somewhat too low, as is Scott's fit by method (iii). In all three cases, the agreement elsewhere is good.

The situation is illustrated in Figure 5 which shows (by crosses) n_r, the average number per square degree in successive rings, for each of the five clusters with radial velocity about 60,000 km/sec that were counted by Zwicky [36] on 200-inch plates. The dots correspond to the Maxwellian distribution for which the ratio of σ over the distance D to the cluster is 4×10^{-4}.

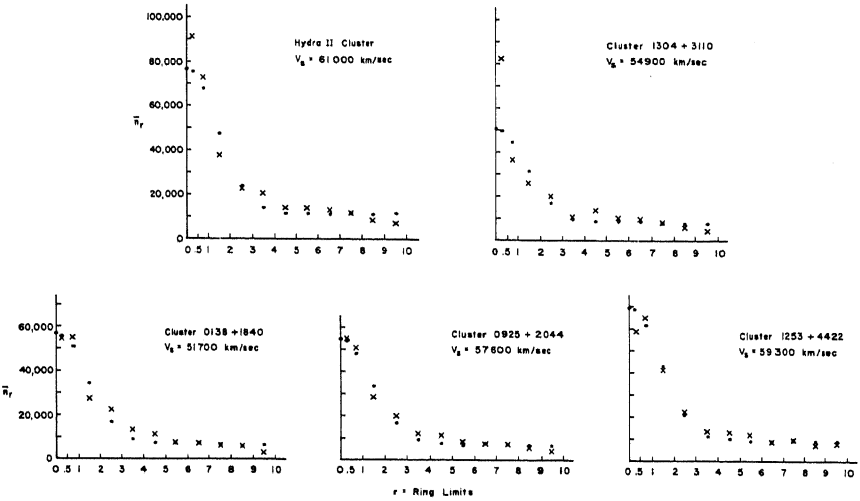


Figure 5. Comparison of observed (crosses) and theoretical (dots) values of the average number of galaxies (per square degree) in successive rings about the presumed cluster center. The observations are due to Zwicky; the theoretical distribution is trivariate normal (Maxwellian) with $\sigma/D = 4 \times 10^{-4}$.

If the distance D is 10^4 times the radial velocity (H = 100 km/sec/Mpc, then σ is between 2.1×10^5 and 2.4×10^5 pc. Any σ within this range would fit all of the clusters within the accuracy we have here. The estimated background density, or field density, varies between 7,200 and 11,300 galaxies per square degree. Inspection of Figure 5 shows that the fit is excellent throughout in the three lower plots and good in the upper plots with the exception of the count corresponding to the innermost part of the cluster. It is not clear how much of this deviation is due to selection and how much is real. This method of estimating the diameter, in fact, the structure, of a cluster appears hopeful. However, we should point out that the counts of the Coma Cluster by Shane [3] on two different plates are rather different and are fitted by different curves. At least part of the difference can be ascribed to the different depths to which the two plates penetrate. Further, the conclusions apply strictly to giant symmetrical clusters and we do not know how far they apply, if at all, to other types of clusters.

The estimation of the luminosity function of cluster galaxies has been attempted by several methods. Abell's method consists in estimating photographically the apparent magnitude of presumed cluster members, in forming a frequency polygon of the results, and in smoothing it somewhat. Ignoring the relatively minor differences in the distances of individual members of the given cluster, the smoothed frequency function appears to present an estimate of the luminosity function of the galaxies in the given cluster, apart from the zero point which depends on the unknown distance to the cluster.

There seem to be two main difficulties in this method. One difficulty, the omnipresent difficulty in all studies of individual clusters, is that we cannot be certain that a specified object really is a physical member of the given system. The other difficulty is a little more subtle. The method assumes either full completeness of the enumeration of cluster members, at least up to a given limiting magnitude, or else, if there are any omissions at all, that the frequency of omission be the same at all ranges of magnitude, again up to the limiting magnitude. As mentioned in the earlier sections, the studies of the duplicate counts by Shane and Wirtanen strongly suggest that neither of these assumptions is strictly fulfilled and that selection begins at a relatively bright magnitude and gradually increases with an increase in faintness. Thus, it appears that, in due course, Abell's work, interesting as it is, will have to be revised in order to make allowances for the effects of selection.

Kiang [31] also studies the luminosity function of cluster galaxies, using the magnitude of the r^{th} brightest galaxy, $r = 1, 3, 5, 10$, as observed by Sandage and by Shapley. Clearly, these data are subject to the same two difficulties previously mentioned; extreme observations are very sensitive both to outliers and to selection. Then, the difficulty arises that there are only four data available to estimate three or more unknowns: the unknown parameters in the luminosity function under test; plus the number of galaxies in the cluster. But the major difficulty is that the distribution of the extremes is very insensitive to the original distribution, especially if the number of galaxies in the cluster is large. In fact, there are only three possible limiting distributions for the extremes (see, for example, Fréchet [37], Gnedenko [38]) and they do not reflect the original distribution. In other words, all possible original distributions can be divided into three categories, with all the distributions in a given category having the *same* limiting distribution of the extremes.

A third method of approach to the determination of the luminosity function of cluster galaxies is due to Zwicky [2,7]. His conclusion is that, considering all types of galaxies in clusters together, the luminosity function is an increasing exponential

$$f(M) = C\, 10^{M/5}. \tag{1}$$

This particular form is treated by Zwicky as preliminary – valid for the brighter range of absolute magnitude. This function is very different from the bell-shaped luminosity functions found for field galaxies; its validity for clusters would indicate a sharp distinction between the category of field galaxies and the category of cluster galaxies.

Unfortunately, it appears that Zwicky's derivation of Eq. (1) is based on an oversight, and that his empirical findings do not imply that the luminosity function is an increasing exponential. Zwicky uses (i) the linearity of the regression of $\bar{n}_{\mathrm{Cl}}(\gamma)$, the number of cluster "members" within the isopleth and brighter than the – assumed constant – limiting magnitude m_L of the plates on the diameter γ defined earlier. He also uses (ii) a tacit assumption which cannot be correct, as we shall presently show.

The linearity of the regression is based on a substantial number of clusters selected by Zwicky for their similarity of shape and of total population. He finds that

$$\bar{n}_{\mathrm{Cl}}(\gamma) = A(\gamma - \gamma_0)\,, \tag{2}$$

where A and γ_0 are constants, provides a good fit over a considerable range of γ. Although observations of both $\bar{n}_{\mathrm{Cl}}(\gamma)$ and γ present difficulties, as we have remarked in an earlier section, for simplicity of argument we shall stipulate that Eq. (2) provides a perfect fit and that the clusters selected are identical spherical systems with the same number N of members.

We now give a heuristic discussion of Zwicky's derivation of the luminosity function, which, however, can be made rigorous [39].

Consider a cluster at distance D and write $\bar{d} = \gamma D$ where γ has the meaning defined earlier. The quantity $\bar{d}$ is described by Zwicky as the absolute diameter of the cluster and his reasoning is based on the tacit assumption that as the distance D varies, the angular diameter γ varies accordingly in such a way that the product $\bar{d}$ remains constant. In fact, using certain additional assumptions regarding the values of $M_{\max}$ and m_L he finds $\bar{d} = 1.31 \times 10^5$ pc. With another set of tentative assumptions, the value found is 5.21×10^5.

If the tacit assumption that $\bar{d}$ is constant is granted, the derivation of Eq. (1) is straightforward. In fact, to an excellent approximation one finds

$$\begin{aligned}\bar{n}_{\mathrm{Cl}}(\gamma) &= N(\bar{d})\, P\{m_{\max} < m < m_L\}\\ &= N(\bar{d})\, P\{M_{\max} < M < m_L - 5 + 5\log D\}\,,\end{aligned} \tag{3}$$

where $m_{\max}$ and $M_{\max}$ are, respectively, the apparent and the absolute magnitude of the brightest member of the cluster with the latter assumed constant, and where $N(\bar{d})$ is a constant depending on the structure of the cluster and on its population N. Now equate Eq. (3) to the right side of Eq. (2) in which y is replaced by $\bar{d}/D$ to obtain

$$N(\bar{d})\, P\left\{M_{\max} < M < m_L - 5 + 5 \log D\right\} = A\left(\frac{\bar{d}}{D} - \nu_0\right). \tag{4}$$

The next step is to differentiate (4) with respect to D, remembering that the probability on the left is an integral of the luminosity function f taken between the limits $M_{\max}$ and $m_L - 5 + 5 \log D$. The derivative of this integral yields the luminosity function f evaluated at $m_L - 5 + 5 \log D$ and this, after the substitution $m_L - 5 + 5 \log D = M$, will give $f(M)$. In the differentiation performed by Zwicky, $\bar{d}$ and $N(\bar{d})$ *are treated as independent of D* with the result

$$-a\, N(\bar{d})\, f(m_L - 5 + 5 \log D)/D = -A\, \bar{d}/D^2, \tag{5}$$

where $a = 5 \log e$. The substitution indicated, followed by easy transformations, yields Eq. (1).

However, if, contrary to the tacit assumption, $\bar{d}$ happens to be a function of D, then the two sides of Eq. (4) have derivatives which are much more complicated than in Eq. (5). They must contain terms depending upon the derivatives of $\bar{d}$ and of $N(\bar{d})$ about which no information is available; the proof of Eq. (1) breaks down and a more complicated expression would appear.

The important point is that when the angular diameter y is considered for clusters in which the total population N (or its average value) is constant, then the "absolute diameter" $\bar{d}$ must depend upon D.

In order to prove the dependence of $\bar{d}$ on D we must revert to the definition of y. For a fixed spherical cluster at distance D, consider the angular radius $y/2$ and ascribe to it an increment Δy. The angular radius $y/2$ is connected with the "absolute radius" $\bar{d}$ by the relation $\bar{d}/2 = Dy/2$ and the increment of $\bar{d}$ corresponding to Δy will be $\Delta d = D\Delta y$. Now consider the circular ring, say $R(\Delta y)$, with radii $y/2$ and $y/2 + \Delta y$. We must compute the expected number of cluster galaxies within the magnitude range $m_{\max}$ to $m_{\max} + 3$ that project themselves on $R(\Delta y)$ and then divide this expectation by the area of $R(\Delta y)$. For $m_L > m_{\max} + 3$, the result is

$$\begin{aligned}&\frac{\pi N\nu(\bar{d})\,[\bar{d}\Delta\bar{d} + (\Delta\bar{d})^2]\, P\{m_{\max} < m < m_{\max} + 3\}}{\pi[y\Delta y + (\Delta y)^2]}\\ &= N\nu(\bar{d})\, D^2\, P\{M_{\max} < M < M_{\max} + 3\},\end{aligned} \tag{6}$$

where $\nu(\bar{d})$ is the proportion of the cluster galaxies which are located appropriately so that they project themselves on the ring $R(\Delta y)$. The definition of γ requires that the quantity in Eq. (6) bear a prescribed ratio 2 to the number of field galaxies averaged per unit area of the plate. But this latter number is known to be proportional to

$$10^{3(m_{\max}+3)/5} - 10^{3m_{\max}/5} = D^3\, 10^{3(M_{\max}-5)/5}\,[10^{9/5} - 1]. \tag{7}$$

A comparison of Eq. (6) and Eq. (7) yields

$$N\,\nu(\bar{d}) = k\,D, \tag{8}$$

where k does not depend on D.

It follows that if one determines γ for several clusters located at various distances D in strict conformity with the definition of Zwicky and then computes $\bar{d} = \gamma D$ for each cluster and next finds $\nu(\bar{d})$ for each cluster, one will see that the product of the total population N by the proportion $\nu(\bar{d})$ must increase proportionately to D. If the clusters considered all have the same population N, then this would mean a change in $\bar{d}$ as a function of D. Thus, Eq. (5) does not represent the result of differentiating Eq. (4) and Zwicky's proof for an exponential luminosity function breaks down. Here we remark that Zwicky's formulas are somewhat different from those given above, but the two are equivalent.

Returning to the consideration of Eq. (8) for varying D, we expect that when D is increased, those clusters for which γ is determined tend to have increasing populations N and, therefore, the average N will increase with D. The effect of this is to make the derivative of the left side of Eq. (4) even more complicated. Whatever the case may be, the problem of estimating the luminosity function of cluster galaxies is still open.

RELATIONSHIP BETWEEN FIELD GALAXIES AND CLUSTER GALAXIES

In the 1930's, the main preoccupation of extra-galactic astronomers was individual galaxies and their general distribution over the sphere, with very broad treatment. During the 1950's, clusters of galaxies gained a firm foothold in the thinking of research workers. The present decade promises new problems concerned with comparisons of galaxies that are labeled "field galaxies," apparently not belonging to any system, and the galaxies forming clusters. Are these two categories, or the populations forming these categories, the same, or is there an essential difference between "field galaxies" as a whole and "cluster galaxies" as a whole?

Definite answers to this and similar questions would be of considerable importance in shaping our ideas about the Universe. For example, the method of estimating masses of galaxies through the study of their rotation, frequently applied to field galaxies, gives estimates of mass very considerably smaller than the estimates obtained through the use of the virial theorem applied to large clusters apparently stable and apparently not containing any appreciable amount of invisible matter. Does this mean that the clusters are expanding, or that there is a large amount of invisible matter in the cluster, or that there exist some unsuspected forces in clusters, or does it just indicate that, on the average, cluster galaxies are more massive than field galaxies?

Another area for study is the luminosity function of field galaxies versus that of cluster galaxies. Zwicky does not appear to recognize the possibility that cluster galaxies may have a luminosity function different from that of field galaxies, and he quite emphatically asserts that the earlier estimates of the luminosity function are entirely wrong. We have seen that Zwicky's luminosity function seems to be based on a misunderstanding. In any case, the problem of the similarity, or lack of it, between galaxies in clusters and field galaxies appears to be important from several points of view and we expect it to become the subject of several interesting studies.*

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Alternative Stochastic Models of the Spatial Distribution of Galaxies

1. INTRODUCTION

In the meaning of the present paper, a *mathematical model of a class of phenomena* designates a set of mathematically expressed assumptions regarding a system of hypothetical entities, designed with the hope that, after an appropriate adjustment of several intervening parameters, the consequences of the assumptions will agree with the observable characteristics of the phenomena. When it happens that such agreement is reached, the contemplation of the model provides the impression that we "understand" the phenomenon. Also, the model usually provides means of calculating certain features of phenomena that are not directly observable or those that have not been observed in the past but may be observed in the future. Frequently, these disagree with the model, thus indicating the necessity of revisions.

In order to serve the above purposes, a mathematical model must postulate the existence of entities which correspond to those real entities that are the subject of observational or experimental studies – at least to the most important of them. The ideas about the relative importance of the various objects of study vary with time. Thus, it should not be surprising to find that the mathematical models of the same class of phenomena change with time in many respects, in particular in respect to entities that are being considered. In the time of Lavoisier and for a considerable period thereafter, the corpuscular theory of matter needed no consideration of particles finer than atoms. Currently, it is just such particles, electrons, neutrons, positrons, etc., that are the subject of empirical study and any attempt at a mathematical theory of matter not involving these elements would be futile.

2. *Why a stochastic model?* As is well known, the classical cosmology is predominantly deterministic. One reason for this is probably Einstein's mastery of the classical mathematical tools and, reportedly, a purely personal dislike for indeterminism. Another probable weighty reason is that, at the time of Einstein's youth, the realm of galaxies was, so to speak, beyond the horizon of the majority of astronomers: the general attention seems to have been concentrated on what was happening in the immediate vicinity of the sun and the regions beyond the Milky Way system were relatively neglected. In particular, there were no established subjects of study in the realm of galaxies which would impose themselves on the theorist as the necessary elements of a theory. Thus, it was natural for Einstein, and for his many followers and successors, to ignore the galaxies and to treat the realm of galaxies as a problem of geometry. H. P. Robertson, whose work I admire very much, described the classical approach as follows [1]:

> *Cosmology* is, in the scientific usage of the term which has grown up within the past few decades, the study of the nature and general distribution in position and motion of the material and energetic content of the visible universe. Observations which have been coming to a head over this period of time have given rise to the notion that the distribution and motion of matter in any sufficiently large spatial region of this universe are, by and large, intrinsically much the same as those in any other similar region, regardless of its position and orientation. This presumed uniformity in the large implies a certain form of a principle of relativity, sometimes called, appropriately enough, the "cosmological principle." Clearly, any approximation to the actual universe as crude as this must be one in which the observed agglomeration of matter into stars and nebulae, and even into clusters of nebulae, is to be replaced by a smeared-out substratum which preserves only the uniformities common to all regions – and thereby robs it of most of those individual characteristics which make the skies a delight and a challenge to a poet and astronomer alike!

While the achievements of classical cosmology are most remarkable in many respects, it appears questionable whether there is a real necessity of replacing "the observed agglomerations of matter into stars and nebulae," etc., by a "smeared-out substratum." Besides, it seems to be advantageous not to do so. The point is that, due to such replacement, a great number of problems of current astronomical research, all specifically concerned with the agglomerations of matter, are placed, so to speak, outside the main body of mathematical theory and can be pulled into its folds only through the admission of a number of extraneous *ad hoc* hypotheses. Robertson's description of the process is "... the frills and furbelows required to express individuality can readily be tacked onto this basic undergarment." I am not sure that Robertson's optimism as to the ease of the solution is entirely justified. Also, I rather think that the aesthetic value of the theory will gain considerably if it is constructed so as to include at its very basis all, or at least most of, the elements necessary for an immediate and direct confrontation with the observational material involving "individualities." Can this be done? The answer is in the affirmative. The means are to abandon the deterministic approach and to replace it with the indeterministic approach.

To adopt the indeterministic approach to cosmology means simply to consider that the universe, with all of its agglomerations of matter and the varied motions, is a single realization of a stochastic process. An immediate illustration of the conceptual advantage of this point of view is provided by the precision with which one can formulate the essence of the cosmological principle described by Professor Robertson as "...that the distribution and motion of matter in any sufficiently large spatial region of this universe are, by and large, intrinsically much the same as those in any other similar region." In probabilistic terms, this admittedly vague statement is replaced by the following which has a precise meaning: the stochastic process underlying the happenings in the universe is stationary in the three spatial coordinates. Furthermore, according to the particular cosmology one wishes to consider, this postulated stationarity in three coordinates may be supplemented by that regarding stationarity in the fourth coordinate, namely, the time coordinate. This temporal stationarity would correspond to either *static* or *steady-state* universes.

Operationally, the assumption of spatial stationarity means regarding the galaxies, their masses, absolute magnitudes, velocities, etc., as being determined by a certain chance mechanism which is exactly the same at every point in space. However, while the mechanism is the same, by virtue of its being a *chance* mechanism, the outcomes of its operation in different regions are bound to be different. Similarly, even if the roulette wheels in Monte Carlo and in Reno happen to be identical, the series of games played during the same hour in the two places are bound to be different.

The answer to the question, "why a stochastic model?" is then: in order that the resulting theory directly concern the subject of the contemporary studies of observational extra-galactic astronomers, such as the variable luminosity of galaxies, their masses, their velocities, clusters of galaxies, the presence or absence of absorbing matter within clusters, and stability of clusters – the "individualities" that the classical cosmology ordinarily ignores.

The stochastic approach, as such, is not in opposition to the classical deterministic cosmology. It should be regarded as an extension of the classical work characterized by the use of the novel mathematical tool – the theory of stochastic processes.

3. Early indeterministic studies. The earliest studies in which the theory of stochastic processes was consciously applied to problems of cosmology, the pioneer works in this direction, are due to V. A. Ambartsumian and colleagues of his. The first two studies [2,3], published in 1938 and 1940, contain certain general considerations which, in 1944, led Ambartsumian [4] to the deduction of a differential equation for a Markov stochastic process meant to represent the fluctuation of brightness in the Milky Way. Later on, the same stochastic process, with certain refinements and extensions, was studied by several authors including S. Chandrasekhar and G. Münch in the United States [5], and G. E. Rusakov [6] and T. A. Agekian [7] in the U.S.S.R.

I should explain that, among the stochastic processes, the category known as the Markov process is better known than the others and is much easier to handle. For this reason, if an applied problem referring to cosmology or to any other domain can be so formulated as to lead to a Markov process, it is advantageous to do so. In order to explain the meaning of the Markov process and, at the same time, in order to outline the original Ambartsumian model of the fluctuation of brightness of the Milky Way, I refer to Figure 1.

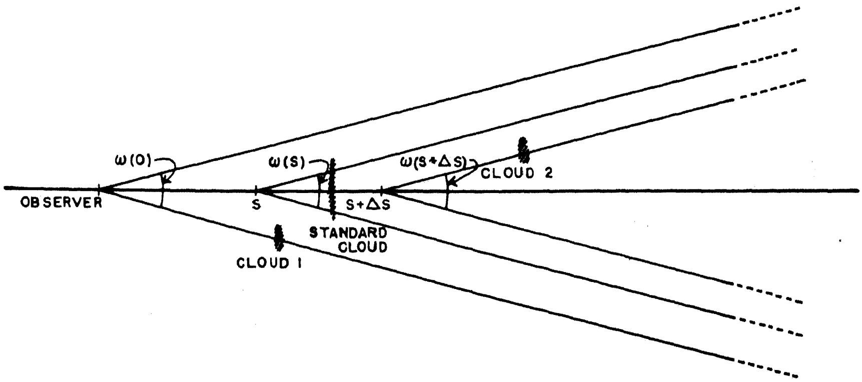

Figure 1.

Here the horizontal line represents the line of sight. Also, there are indicated three positions of the same solid angle ω, denoted by $\omega(0)$ when the vertex is at the observer. In other positions, the vertex is at a distance s and at a distance $s + \Delta s$ from the observer. Denote by $I(s)$ the amount of light reaching s from the solid angle $\omega(s)$. If the chance mechanism in governing $I(s)$ is such that the knowledge of $I(s + \Delta s)$ is sufficient to calculate the probabilities of the various possible values of $I(s)$, then the random process $I(s)$ is Markovian and there are standard methods for investigating its distribution. If this is done for an arbitrary s, then the substitution of $s = 0$ provides the distribution of the amount of light $I(0)$ at the observer. Thus, the problem faced by Ambartsumian was to admit such plausible simplifying assumptions regarding the fluctuation of brightness in the Milky Way as to endow $I(s)$ with the Markovian property. The simplification adopted includes the assumption that all the fluctuation is due to discrete absorbing clouds. If there were no such clouds, then the brightness in any direction at a fixed galactic latitude would be constant. Accordingly, the specific assumption made is that the angle $\omega(0)$ is filled with hypothetical transparent luminous matter idealizing stars (or, if one wishes to go beyond the Milky Way system, idealizing galaxies). The further assumption is that at points on the line of sight (Poisson-distributed), there are placed light-absorbing clouds, each endowed with a randomly varying transparency q. The final assumption is that if a cloud intersects any part of $\omega(s)$, then it must intersect the whole of it.

With these hypotheses, if $I(s + \Delta s)$ is given then, in order to calculate the probability of possible values of $I(s)$, it is sufficient to calculate the probabilities of the various possible happenings between the points s and $s + \Delta s$, and the process $I(s)$ is Markovian. As seen in Figure 1, this treatment is not entirely realistic and, for example, the hypothetical clouds 1 and 2 do not conform with the hypotheses made. However, any mathematical model of natural phenomena must involve simplifications and some of them prove harmless. Therefore, it is quite possible that Ambartsumian's and his followers' estimates of the average optical thickness of clouds are realistic.

On the other hand, I am somewhat less optimistic regarding certain extensions of the original model meant to study clusters of galaxies. In the model, these clusters are treated as "condensation of luminous matter." The treatment of these condensations is exactly similar to that of clouds described above, with the result that the relationship between $I(s + \Delta s)$ and $I(s)$ is postulated to be

perfectly determined by the number of "condensations" occurring between s and $s + \Delta s$. The formulas obtained using this simplification are used with reference to several systems of counts of galaxies. Only one of them could be treated to the end; it yielded an estimate of the average number of galaxies per cluster, namely $\nu_1 = 8.3$. I am not questioning this estimate. In fact, in our own studies [8] we found it impossible to indicate any single number and limited ourselves to a pair of inequalities $1 < \nu_1 < 287$. On the other hand, the method of arriving at the estimate $\nu_1 = 8.3$ does appear questionable. The reason for my uneasiness is that – if one uses the basic differential equation from which the estimate 8.3 is obtained and tries to calculate the amount of light reaching the observer from a hypothetical single "condensation" at distance x, say, composed of ten galaxies, under the assumption of no intervening absorbing clouds – one finds that, contrary to the inverse square law, this amount of light is independent of x. This circumstance suggests that, at least as an instrument of study of clustering of galaxies, the extended Ambartsumian model is, perhaps, an oversimplification. The particular point in the theory which causes the difficulty is the assumption that, in order to calculate $I(s)$ from $I(s + \Delta s)$, all that is needed is the number of "condensations of light" (and of clouds, if any) that occur between s and $s + \Delta s$. In actual fact, such calculation requires also the information about points beyond $s + \Delta s$ at which other "condensations" of light occur, and this deprives $I(s)$ of its Markovian property.

While pointing out these difficulties, I wish to reiterate my admiration for the pioneer work of Ambartsumian and his successors.

4. Stochastic process of clustering as a model of the Universe. During the last decade or so, following the briefings and friendly criticisms of our colleagues and friends, C. D. Shane and N. U. Mayall, E. L. Scott and I have been engaged in constructing a probabilistic and statistical tool for extra-galactic research. The construction had to begin with selecting the basic entities of the model, ascribing to them certain properties connected in some way with what may be observed and with postulating certain relations conforming with the general ideas of observational astronomers. There resulted the four-dimensional stochastic process of clustering [9], [10] of first order. The generalization to any order of clustering is given in [10].

The basic element of the model is a cluster of galaxies. It is characterized by coordinates $u(t) = \{u_1(t), u_2(t), u_3(t)\}$ of a hypothetical point described as the "cluster center." These coordinates are assumed to be deterministic functions of time (perhaps constants) so determined as to satisfy the cosmological principle, which requires that

$$u_i(t) = u_i(0)\, e^{\int_0^t H_1(t)dt}, \quad i = 1,2,3, \tag{1}$$

where $H_1(t)$ is an arbitrary function, perhaps determinable from the observations. The origin of time is arbitrary and may be assumed to be "the present." While the function $u(t)$ is deterministic, its value at the origin of time is supposed to be random. The first order clustering is characterized by the assumption that the three-dimensional distribution of cluster centers at time $t = 0$ (and, consequently, at any other time) is "quasi-uniform," meaning that the probability distribution of the number of the cluster centers in any given region R in space depends only upon the volume of R and that if R_1 and R_2 are disjoint, then the corresponding numbers of cluster centers are mutually independent. A particular case of the quasi-uniform distribution is the Poisson distribution.

With each cluster center C, located at $t = 0$ at $\mathbf{u} = (u_1,u_2,u_3)$, the model connects a random variable ν, described as the number of galaxies forming the cluster centered at C. The model does not impose any limitation on the distribution of ν, except that it is capable of assuming only positive integer values $1,2,\cdots$. If $\nu = 1$, then the corresponding "cluster" is composed of just one galaxy, or, in other words, we are dealing with a "field galaxy." If $\nu = 2$, then we have a "double," etc. The probability generating function of ν, left arbitrary in the model, is denoted by $G_\nu(\tau)$. The essential assumption about the variables ν is that they are mutually independent and, in conformity with the cosmological principle, independent of the location of the cluster center C.

Given that the value of ν for a particular cluster center C is $\nu = n$, the model associates with the cluster center n dimensionless (an oversimplification!) particles described as galaxies forming the cluster. The positions of the n cluster members corresponding to the time $t = 0$ are random, mutually independent, and follow the same probability law. More specifically, it is assumed that the coordinates $\mathrm{X} = (X_1,X_2,X_3)$ of any given cluster member have a probability density, called the structure of the cluster,

$$f\left\{\left[\sum_{i=1}^{3} (x_i - u_i)^2\right]^{1/2}\right\}, \tag{2}$$

depending upon the distance from the cluster center. The general theory leaves the function f unspecified but treats it as a fixed function, for all clusters large or small. In applications [11], [12], we frequently assumed that f is a spherical normal distribution with an adjustable dispersion σ. Various other functions were tried, for example, a weighted average of two normal distributions.

As was indicated to us by F. Zwicky and G. B. van Albada, the postulate that the function f does not depend upon the size of the cluster is an oversimplification. With this we readily agree, and we were, of course, aware of the fact. If independence was postulated, it was a result of the tendency to reduce the number of arbitrary functions and constants to a minimum. Furthermore, at the time of building the model, the information regarding the structure of the clusters was too scanty to introduce any specific assumption and the assumption of independence seemed the simplest. If and when either the empirical data or the dynamical theory will indicate a dependence of f on ν, the problem of generalizing the model of clustering will have to be faced.

Recently [12], the above general structure of the spatial distribution of galaxies had to be generalized by a supplementary assumption that each of the clusters contains absorbing material with the density τ magnitude per unit distance.

As far as the spatial distribution of galaxies at any given moment is concerned, the above hypotheses complete the picture of first-order clustering. In order to obtain the clustering of second order, one assumes that the cluster centers, instead of being distributed quasi-uniformly, have a first-order clustering, etc.

When contemplating the possible lack of stationarity in time, we had a period of hesitation. Suppose that the law of red-shift really represents the Doppler effect of the velocity of expansion. Accordingly, as time goes on, the mutual distances of clusters increase. This, then, is taken care of by the relations (1) with a nonvanishing function $H_1(t)$. What should one assume about the membership of single clusters? Do the clusters recede from each other, more or less, like rigid bodies, or do they expand, at the same rate as does the Universe, or at a different rate? At the time (1952-1953), there were no opinions on this point to be found in the literature. For this reason, the general model, described in [10], visualizes the possibility that, in addition to whatever random motions the galaxies of a given cluster may have, they also have a deterministic displacement characterized by equations

$$X_i(t) - u_i(t) = [X_i(0) - u_i(0)]\, e^{\int_0^t H_2(t)\,dt}, \quad i = 1,2,3, \qquad (3)$$

analogous to (1), where $H_2(t)$ is another function characterizing the rate of expansion of the clusters, which we hope may be eventually estimated from the empirical data.

This completes the description of the four-dimensional stochastic process of clustering meant to represent the spatial distribution of galaxies. In any particular study, the galaxies – members of clusters – have to be endowed with additional properties. One of them, namely the luminosity, is, in fact, indispensable because without luminosity the galaxies would not be observable. Our assumption was that the absolute magnitude of a galaxy, say $\mathfrak{M}$, is random variable with an unspecified probability density $P_{\mathfrak{M}}(M)$ independent from one galaxy to the next and independent of all variables in the system, including the size of the cluster and including the position within the cluster. Here, again, this independence may easily be an oversimplification.

The four-dimensional process of clustering is, of course, nonobservable. For this reason, the above hypotheses regarding what is happening in space had to be supplemented by several others connecting the happenings in space with what is observable, essentially, with what may be found on photographic plates. In the first place, it was necessary to introduce two new random variables μ and z, representing, respectively, the apparent magnitude and the red-shift of a galaxy, connected with the absolute magnitude $\mathfrak{M}$ and distance from the observer ξ. We adopted the relation

$$\mu = \mathfrak{M} - 5 + 5 \log_{10} \xi + k(z). \tag{4}$$

However, we were reluctant to postulate that z necessarily represents the Doppler effect. On the contrary, it was felt expedient to treat the red-shift as an observable variable affecting the apparent magnitude through the k-term in (4) and seek empirical evidence that the functions H_1 and H_2 are, in fact, nonvanishing.

While the assumption regarding the apparent magnitude does connect the events in space with those on Earth, it does not constitute a sufficient link. This is supplied by the supplementary assumption that, for any given type of galaxy and for any given type of measurements to be made, there exists a function $\Phi(m)$, called the selection factor, representing the probability that a galaxy of this type having for its apparent magnitude $\mu = m$ will be included in the observational program.

Again, the general theory leaves the selection factor unspecified with the sole restriction that, for increasing m – that is for the passage to galaxies of increasing faintness – the function $\Phi(m)$ tends to zero sufficiently fast for certain integrals to converge. The operational meaning of this assumption is that the chance of including, in a given observational program, a galaxy of extreme faintness is so small that no observational program can include an infinity of galaxies.

The meaning of the selection factor requires comment. Suppose that the observational program consists in counting images of galaxies on survey plates. One frequently hears the assertion that the limiting magnitude of a given system of counts is a stated number m_1, perhaps $m_1 = 20^m$. The exact meaning of this statement is that $\Phi(m) = 1$ for all $m \leqq m_1$ and $\Phi(m) = 0$ for $m > m_1$. The data obtained by Shane [13] indicate that the idea of the limiting apparent magnitude of counts is a dangerous misconception. If one repeats counts in the same little squares on a given plate, the results are correlated with those of the first count but no more. In fact, the second count in a given square may include fewer galaxies than the first and the recount in the next square may well give the opposite result. Instead of speaking of the limiting apparent magnitude, one might wish to characterize the depth of counting by the "median" apparent magnitude, say m^*, meaning that apparent magnitude at which the probability $\Phi(m^*)$ is equal to one-half.

Naturally, if one changes the nature of the observations, or the instruments used, the selection factor $\Phi(m)$ is expected to change too. Thus, if the observations consist in measuring the apparent magnitude and the red-shift, rather than in counting the images of galaxies, then the selection factor is likely to have its median apparent magnitude shifted to brighter values.

In many studies, the importance of this selection factor $\Phi(m)$ is supreme and, therefore, the possibility of determining it empirically is quite important. For the case of measurement of red-shift there is a convenient formula published in [14] based on the assumption that the observational data refer to field galaxies. The proof of the formula and, also, its use in estimating the relative abundances of various types of galaxies are given in [15]. For the case of counts of images of galaxies, the empirical estimation of $\Phi(m)$ is more complicated and requires duplicate counts or duplicate plates. The relevant study is in preparation.

5. *Fundamental formula of the theory of clustering of galaxies.* The general subject of study in the theory of clustering of galaxies may be described as the search for formulas connecting the various characteristics of the model described in the preceding section, that is to say, the presumed happenings in space, with the statistical characteristics of the observations. Since the latter may be symbolized by photographic plates, the problem may be restated in terms of the relation between the four-dimensional stochastic process of clusters in space-time and a two-dimensional stochastic process of clustering on the photographic plates. Many problems of this kind can be solved by an appropriate specialization of a formula [10] described as the "fundamental formula."

Consider s disjoint regions $R_1, R_2, \ldots, R_s$ in space and an equal number of characteristics $\Gamma_1, \Gamma_2, \cdots, \Gamma_s$ that a galaxy may possess. These characteristics may be the same or different. We assume that for each $i = 1, 2, \cdots, s$, and for each point in R_i with coordinates $x = (x_1, x_2, x_3)$ the definition of Γ_i implies the probability $\theta_i(x)$ that a galaxy located at x will possess the characteristic Γ_i. Denote by N_i the random variable representing the number of galaxies in R_i that possess the characteristic Γ_i. The fundamental formula referring to a four-dimensional stationary process gives then the joint probability generating function of the variables $N_1, N_2, \cdots, N_s$ in the form

$$G_{N_1, N_2, \cdots, N_s}(t_i, t_2, \cdots, t_s)$$

$$= \exp\left\{-\lambda \iiint \left[1 - G_\nu\left(1 - \sum_{i=1}^{s} (1 - t_i) p_i(u)\right)\right] du_1 du_2 du_3\right\}. \tag{5}$$

Here λ is the expected number of cluster centers per unit volume, the triple integral extends over the whole space and $p_i(u)$ stands for the probability that a galaxy from a cluster centered at u will happen to be in region R_i and that it will have the characteristic Γ_i,

$$p_i(u) = \iiint_{R_i} f\theta_i(x) dx_1 dx_2 dx_3, \tag{6}$$

where, as formerly, f stands for the structure of the clusters.

The generality of the formula, with unspecified characteristics Γ_i, makes it applicable for the solution of many problems but, at the same time, somewhat difficult to appreciate. Two illustrations must suffice.

Let $s = 2$ and let the regions R_1 and R_2 be two equal square solid angles with vertices at the observer. Further, let N_1 and N_2 mean the numbers of images of galaxies of a specified type counted on a photographic plate in the two squares subtending the solid angles R_1 and R_2. In order to obtain the joint distribution of N_1 and N_2, it is sufficient to define Γ_1 and Γ_2 in the same way as consisting in the galaxy being counted. When the selection factor $\Phi(m)$ is known, the probability $\theta(x)$ is

$$\theta(x) = \int_{-\infty}^{+\infty} \Phi(m) \int_{-\infty}^{+\infty} p_{\mathfrak{M}}\,[m + 5 - 5 \log_{10} \xi - k(z)]\, p_Z(z \mid \xi)\, dz\, dm, \tag{7}$$

where $p_Z(z \mid \xi)$ is the conditional distribution of the red-shift given the distance

$$\xi = \left\{ x_1^2 + x_2^2 + x_3^2 \right\}^{1/2}, \tag{8}$$

of the galaxy.

The distribution determined by the fundamental formula is complicated. However, the same formula determines also the moments of N_1 and N_2, including their correlation, which depends upon the size and the distance of the two squares subtending R_1 and R_2. These moments are also of some complexity, but are calculable.

As a second illustration of the use of the fundamental formula, I wish to mention a problem of some delicacy. This is concerned with the question of how realistic a picture of the universe does the stochastic process of clustering present. One possibility is that the clusters are comparable to a large number of balloons of different sizes released at a festivity. Some of these balloons may be floating near to each other, but they will be separate bodies. The alternative picture of the agglomeration of clusters of galaxies, mentioned some time ago by F. Zwicky, is that space is "paved with clusters." Which of these two pictures is closer to reality? If it is the second, then the stochastic process of clustering is no more than an interpolation formula and also a number of current studies involving the dynamics of an isolated cluster would be out of place. Otherwise, if the distribution of clusters resembles that of released balloons, the hypothetical "clusters" of the model would have real counterparts in space.

The main difficulty of resolving the problem consists in giving it a precise mathematical form. Two such forms were advanced and treated in [10] and [16]. Figure 2 represents a hypothetical cluster C_0 composed of $\nu = 5$ galaxies. These galaxies are numbered in the decreasing order of their distances from the center,

$$\eta_1 > \eta_2 > \eta_3 > \eta_4 > \eta_5 \geqq 0.$$

Denote by $S(\eta)$ a sphere of radius η centered at the center of the cluster.

Definition*. If a galaxy from a cluster C, other than C_0, lies within the sphere $S(\eta_k)$, then we say that the cluster C "penetrates" the cluster C_0 to the "depth" k.

One of the methods of treating the problem of interlocking of clusters is based on a random variable denoted by ϵ_k, defined as the number of those clusters which penetrate the given cluster C_0 (assumed to include at least k galaxies) to the depth k. The meaning of ϵ_k may be clarified by the following operational interpreta-

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*The symbol k used in this definition does not have the same meaning as the k of Eq. (4) or Eq. (7). – Ed.

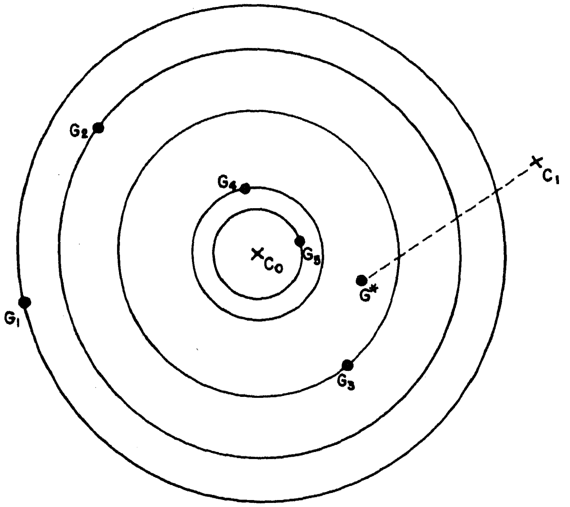

Figure 2. Penetration to the depth 3 of the cluster C_0 by the galaxy G^* of the cluster C_1.

tion of one of the relevant probabilities, say $P\{\epsilon_k = 3\}$. This probability refers to the assumption that the chance mechanism governing the distribution of galaxies is that described in the definition of the process of clustering of the first order, involving the distribution in space of "cluster centers," the assignment to each of them a value of ν, etc. The probability $P\{\epsilon_k = 3\}$ answers the question as to how frequently the mechanism described would produce a cluster C_0 penetrated to the depth k by exactly three other clusters ? Further, it will be noticed that $\epsilon_1 = 0$ means that the given cluster C_0 is isolated from the others. If the probability $P\{\epsilon_1 = 0\}$ is close to unity, this means that almost all clusters are isolated.

Perhaps unexpectedly, a particular specialization of the fundamental formula provides the probability-generating function of ϵ_k for every $k = 1,2, \cdots$. In paper [16] the tentative estimates of the various parameters obtained for the model of clustering were used to evaluate $P\{\epsilon_1 = 0\}$ and also some other similarly defined quantities. The results indicate that, if the estimates of parameters are close to reality, there are practically no isolated clusters. However, since the calculations mentioned were performed, it was found that the estimates of the various parameters used are grossly in error due to the original faulty method of treating the variability of counts of galaxies. One of the inaccuracies committed was a substantial overestimate of the average spherical volume containing a cluster. If and when this inaccuracy is corrected, it may appear that isolated clusters are reasonably frequent.

6. ***Some cosmological problems lying outside of classical cosmology which lend themselves to treatment within the theory of clustering.*** In this section, I will attempt to exemplify the problems of cosmology which elude deterministic treatment but offer a hope of solution on the ground of the theory of clustering. Strictly speaking, such problems occur whenever there is any need to compare some theoretical conclusion or hypothesis with observation. Three examples must suffice.

(a) Does the law of red-shifts reflect the expansion of the universe? An overwhelming majority of astronomers are likely to subscribe to the idea that the red-shift of galaxies reflects their velocity of recession from the observer. However, it is well to realize that this belief has a negative basis: thus far no factor has been indicated, other than velocity, that is capable of producing the observed red-shift. Thus, it is reasonable to follow the methods of Hubble and look for ways of supporting or contradicting the hypothesis that the red-shift is merely the Doppler effect of velocities of recession.

As long as one insists on using the classical theory, in which the galaxies are replaced by a smeared-out substratum, there is no way of attacking the problem for the simple reason that the galaxies, as such, have no place in the theory. On the other hand, the theory of clustering of galaxies offers a possibility [10]. Specifically, the serial correlation (really, the quasi-correlations) between counts of images of galaxies in regularly spaced squares on the photographic plates depends on the distance between the squares in a manner which itself depends upon whether the universe is expanding or not. The relevant calculations are extremely heavy but they are in progress and, in due course, we may have the solution of the problem.

(b) Scott effect. In his paper already quoted, Robertson deduced a certain formula for the luminosity-red-shift relation in which a particular coefficient is very relevant to the decision between certain hypotheses regarding the state of the universe. Sandage attempted to estimate this coefficient empirically using the data published in the fundamental paper [17], jointly with Milton Humason and N. U. Mayall. Although the problem appears subject to varying interpretations, I presume that one of the possible solutions would be provided by the regression function of the average apparent magnitude in a cluster on the average red-shift in the same cluster. However, in the more distant clusters, only the brightest galaxies afford a possibility of measuring red-shift. Thus,

at great distances the measurements of red-shift are restricted to those clusters which happen to include particularly bright galaxies. This must lead to a bias in the empirical regression line. As was shown by E. L. Scott [18], the theory of clustering provides means of evaluating the bias, but if one is limited to deterministic cosmological theory, no such evaluation is possible.

(c) Possibility that clusters and groups of galaxies expand. One of the most fascinating subjects of contemporary studies is the question of whether (or how frequently) the systems of galaxies, small groups and large clusters, form stable dynamical systems. If I am not mistaken, the possibility that at least some of such systems are far from being stable was first suggested by Ambartsumian in 1955 and, since then, there developed a substantial literature on the subject. Naturally, as long as the mathematical theory of the universe is limited to smeared-out substratum, the question of stability of clusters of galaxies will remain outside of its domain.

As already mentioned in an earlier section, the question of whether the clusters of galaxies expand as a part of the general expansion of the universe arose quite early in our studies. Once the question was formulated, it was natural to pursue it further and to ask whether it can be resolved by some sort of empirical studies. Accordingly, an optimal (specifically, a "locally most powerful test") was deduced for the hypothesis of stability of an isolated cluster of galaxies. It was published in the Richard von Mises memorial volume, 1954 [19]. The test is purely kinematic and is based on the conditional distribution of the radial velocity, given the apparent magnitude and the angular distance of a galaxy from the center of the cluster. In 1952, when this test was deduced, the richest observational material that we could find referred to the Coma Cluster and contained data for 12 objects only. The kinematic test was applied but showed no evidence of instability. Partly owing to the interest in stability aroused by the literature on the Ambartsumian hypothesis, and partly because of a considerable increase in the observational data (mostly due to N. U. Mayall, the number of available objects in the Coma Cluster grew to 50; also, there are data for a number of other clusters and groups), we returned to the empirical work some two years ago [12]. Although the available data are still insufficient for a confident conclusion, the first study of seven clusters and 15 groups of nearby galaxies seems to indicate that some of them may be in the state of expansion and some others, including the Coma Cluster, in the state of contraction. Also, there is a subsidiary equally tentative conclusion, foretold last year by Minkowski and anticipated on quite different grounds by Zwicky, that, as a rule, the clusters contain dark absorbing material.

I wish to emphasize that, thus far, the above conclusions are very tentative and that we shall not be surprised to find them contradicted by further investigations based on more data.

Incidentally, the possibility of observing both the expansion and the contraction of systems of galaxies, was predicted in an entirely independent dynamical research by van Albada [20].

7. Model of the spatial distribution of galaxies in terms of a general fluctuating density field. It must be realized that, even with the noted simplifications in the form of a fixed structure f of the cluster, independent of the cluster's size, etc., the model presented in the foregoing sections is rather complicated. As a result, the arithmetical calculations needed to obtain a theoretical value of a particular parameter of the model of clustering, frequently involving the evaluation of awkward integrals of many dimensions, are extremely heavy. While the complexity of a theory should not be surprising when its object is to present a comprehensive picture of a phenomenon as complex as is the universe of galaxies, it is tempting to see whether by giving up the comprehensiveness and by limiting the number of questions that a theory may answer, one could obtain a more manageable apparatus. Possibly, it is with such ideas in mind that a model of the spatial distribution of galaxies, alternative to the model of clustering, was proposed by Limber [21], who was inspired by the earlier paper on the fluctuation in brightness of the Milky Way by Chandrasekhar and Münch [22].

Of the various characteristics that a distribution of particles in space may have, the fluctuating density model contemplates only three and treats them without reference to any hypothetical mechanism. These selected characteristics of the distribution are: the mean density $\bar{\rho}$ of particles at a point, the dispersion β of density at a point – both supposed constant – and the correlation $\Gamma(x)$ between densities at two different points, assumed to be a function of the distance x separating these points. The object of the paper was to deduce formulas connecting the three elements of the theory with the calculable characteristics of the counts of galaxies on photographic plates. The limitation to a static universe, the neglect of the effect of red-shift on the apparent magnitude of galaxies, and also certain other simplifications lead to estimates of $\bar{\rho}$, β, and Γ involving relatively simple calculations.

The theory contemplates the evaluation of the effects of interstellar absorption as treated by Chandrasekhar and Münch [22].

In subsequent papers [23,24], Limber's formulas were applied to actual counts of images of galaxies. Because of a certain lack of precision in the definition of $\bar{\rho}$,β, and Γ the estimates resulting from these studies appear to be illusory [25]. However, the conceptual difficulties were easily resolved, leading to a redefinition of $\bar{\rho}, \beta$, and Γ and, granting a satisfactory treatment of the variability of counts of galaxies (which is itself a very serious problem) the estimates of $\bar{\rho}, \beta$, and Γ can be obtained [26,27]. A particularization of the model of fluctuations was provided by Layzer [28].

The question on which I was specifically requested to comment by the Organizing Committee is the relationship between the two models of the spatial distribution of galaxies — the model of fluctuating density and the model of clustering.

8. Comparison of the two models of the spatial distribution of galaxies. Thus far, the model of fluctuating density has not been extended to cover the possibility that the universe is expanding. However, using the approach given in [10] or some other method, perhaps following McVittie [29], such extension can be achieved. The essential point of a comparison is, then, the delineation of the domain of validity of the two models and of the problems that either model might help to solve.

As to the domain of validity, there is no question that, for the model of fluctuating density field, this domain is broader because this model assumes only that, at any given moment, the realm of galaxies is a stochastic process, stationary in space, and having two first moments finite. In particular, this is also true of the model of clustering and, in fact, the three elements of the model of fluctuations, $\bar{\rho}, \beta$, and Γ, are easily computable in terms of the various elements of the model of clustering [11]. This, then, appears to be an advantage of the model of fluctuations. However, this advantage is illusory.

The point is that, if anyone is interested in the three items, $\bar{\rho}, \beta$, and Γ, but has at his disposal the results of a study based on the model of clustering, the three characteristics, $\bar{\rho}, \beta$, and Γ, can be readily computed from the characteristics of the fitted model of clustering, *quite irrespective of whether the postulated "clusters," etc., correspond to reality or not.* On the other hand, the converse problem need not be soluble even if clusters of galaxies are a reality.

As to the range of problems available for solution by means of the two models, it should be obvious that whatever problem is soluble on the ground of the theory of fluctuations, it is also soluble on the ground of the theory of clustering, but not vice versa. In general, the theory of fluctuations as such, without any possible supplement "tacked" onto it, does not provide tools for solving any problem in which the notion of a cluster explicitly intervenes. For example, some time ago, Zwicky was interested in the formula representing the distribution of visible diameters of clusters of galaxies, both on the assumption of stationarity and of expansion of the universe. Since the cluster, as such, does not appear in the theory of fluctuations as a defined concept, without supplementing this theory by further concepts, it cannot be used to solve the problem posed. On the other hand, the relevant formula for static universe was actually obtained on the ground of theory of clustering. How realistic this formula is depends upon how realistic are the various particular assumptions, such as, for example, whether the structure f of the clusters depends upon the size of the cluster ν, etc.

The three cosmological problems, *(a), (b),* and *(c),* of section 6 may be used for further illustrations of the range of the applicability of two models.

(a) Problem of the expansion of the universe. When appropriately extended to cover the expanding universe, the theory of fluctuations can be applied to the solution of this problem. By assuming less about the nature of the spatial distribution of galaxies than does the model of clustering, the findings by means of the theory of fluctuations will be less subject to bias. On the other hand, if the assumptions of the theory of clustering are not too far from reality, its application promises a better detecting power of the tests that may be devised. In other words, the chance of detecting expansion will be better with the theory of clustering.

(b) Scott effect. This problem explicitly concerns clusters. Thus, it has no room within the theory of fluctuations.

(c) Expansion of clusters. Strictly speaking, this problem is also concerned with clusters and has no room within the theory of fluctuations. However, if and when the theory of fluctuations becomes sufficiently developed to deal with peculiar velocities of galaxies, it is not improbable that some sort of problem in a sense equivalent to the problem of the expansion of clusters can be formulated.

The general conclusion seems to be that each of the two models may be useful in an appropriate domain and each is worthy of study. Unfortunately, successful use of either of the two models depends very much on a satisfactory treatment of variability in the data caused by various circumstances here on Earth in the observatories and the laboratories. These cause very considerable trouble.

Before concluding, I wish to express my deep gratitude to the Organizing Committee for the honor of being asked to present this paper.*

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DISCUSSION

Reaves: Have you investigated dependence of Φ on the star-density, and on the galaxy-density?

E. Scott: No. Thus far, our studies have been limited to the estimation of the selection possibility for *(a)* the selection of objects for measurement of radial velocity and apparent magnitude, and *(b)* the problem of counting the images of galaxies by Shane and Wirtanen. The effect of star-density, or of galaxy-density, may well be of importance. This possibility has been suggested by several persons, including Dr. Holmberg, and an experiment to estimate the effect was suggested.

Limber: I would first like to state that Professor Neyman has been very fair in his treatment of my work. The philosophies with which his group and I have approached the problem have been somewhat different – as were our aims. I feel that the two different approaches complement each other, and that we are much indebted to Professor Neyman and Dr. Scott for their very careful analyses.

De Vaucouleurs: With reference to the selection factor Φ, it would be most important to include as its arguments not only the integrated apparent magnitude, but also the type and, especially, the surface brightness of the galaxies.

Neyman: I thank Dr. de Vaucouleurs for the suggestion. I understand it to mean that the actual chance of a galaxy being selected for study depends upon the type of galaxy and upon its surface brightness in addition to the apparent magnitude. Of course, the selection factor Φ must be determined empirically. It appears that the way to do so is easy if one deals with categories of field galaxies defined so narrowly that for galaxies *within a given category* the probability of inclusion in an observational program depends on the apparent magnitude only. Dr. de Vaucouleurs' suggestion means then that the definition of what I describe as a ''category'' must involve both the type and the surface brightness. In our own work we used only the type of galaxies and $\Phi(m)$ was estimated separately for each of the several types. Unfortunately, in addition to type, surface brightness and apparent magnitude, the selection probability appears to depend on many other factors, such as the available instruments, the observer and even the physiological state of the observer. It may be hoped that, at least for some categories of observations, some electronic machine could be devised which would eliminate the variability in the physiology of the observer. Incidentally, it is not impossible that the difficulties in reconciling the several sets of radio observations are due to the difference between the selection factors Φ that correspond to the different instruments.

Page: Is it wiser (or ''better'' in some other way) to count galaxies in squares chosen from a given field in *random order* rather than systematically by rows or columns?

E. Scott: I am not sure. The different plates were taken in a random order, and they were counted in an independent random order. Both orders were recorded. It is not clear that counting the 1296 cells in random order would increase the accuracy because it would seem to introduce blunders which would be difficult to catch. Any correlation in the errors of counting successive squares can be estimated – it is very small. We can check on the possibility of an "anticipation" effect by comparing counts in the E-W and N-S directions.

Heckmann: We got some experience in similar counts many years ago in Milky Way fields. But we improved very much the quality of the counts by having permanently a comparison image in the field of the eyepiece. The counting thus became very slow and I fully understand Dr. Neyman's wish to have a machine doing the job.

Seeger: The human plate-reader is a noisy device subject to large secular variations. If astronomers can state the judgment criteria in a language simple enough for modern machines to handle, the machine will outdo the human steadily, i.e., with a much lower, and more constant, noise level.

Neyman: I fully agree. The more so because the machine is expected not only to count images of galaxies, but also to classify them according to types. I am told by the electrical engineers that, in order to avoid difficulties with silver grains, the machine should be adjusted to "look" into the eyepiece of the telescope, rather than to scan photographic plates. Whether any one will ever finance the construction of a machine of this type is another question.

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The Angular Diameters of Radio Sources

INTRODUCTION

In a very valuable paper given at U.R.S.I. in 1960, Bolton [1] has listed 22 radio sources which have firm identifications with "abnormal galaxies." He gave angular diameters ranging from 20 to 0.5 minutes of arc. Bolton also mentioned that many extra-galactic radio sources appeared to have a complex structure quite unlike the circular Gaussian models which are frequently used in the interpretation of interferometric measurements. Amongst the identified sources Cygnus A, Hercules A, and Centaurus A all show well-marked, double structure, while Moffet and Maltby [2], and Lequeux [3] have recently emphasized that a fraction of the unidentified sources which they observed also consisted of at least two separate components.

INTERFEROMETERS OF LONG BASE-LINE

At Jodrell Bank, using the steerable telescope with smaller portable aerials, we have recently made interferometric observations with longer base-lines than were used in the work mentioned above. These transit observations were made at 160 Mc/s with base-lines which were approximately E–W. The equipment sensitivity allowed observations of fringe amplitude of about four flux units, so that fringe visibilities $y \geq 0.3$ could be measured for sources of intensity ≥ 12 flux units. About 200 such sources from the 3C catalogue [4] have been observed, and a smaller number of weaker sources have also been studied, though with reduced accuracy. Figure 1 shows as examples the fringe visibilities measured for three identified sources. 3C 295 and 298 are both identified with extra-galactic objects. The former is a very remote galactic collision, and it is interesting that the results for this source show signs of a double structure. The source 3C 48 has been provisionally identified with an F star of 16th magnitude [5]. This source does not appear to be resolved at any base-line. Its diameter is, therefore, less than one second of arc, and its surface brightness temperature greater than 10^{10}°K. We have only found, so far, two other sources (3C 119, 299) which are unresolved at all base-lines. If true radio stars all have very small diameters, they do not constitute more than 2 per cent of the sources more intense than 12 flux units.

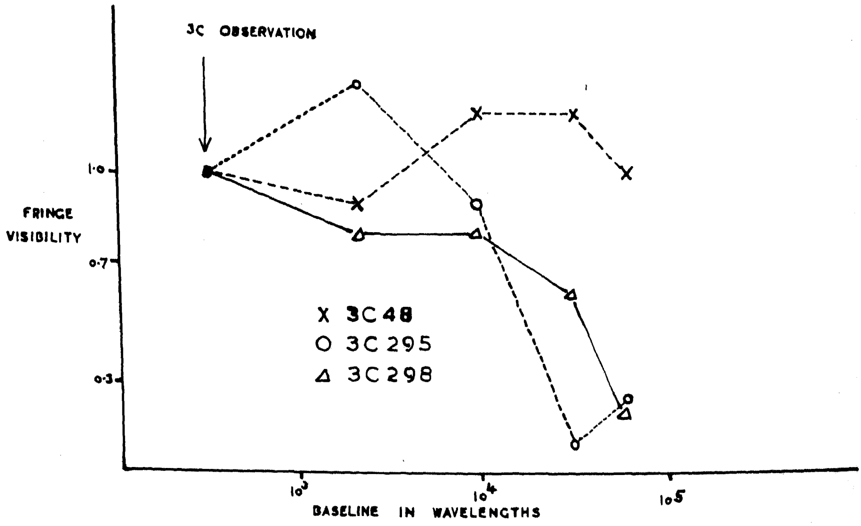

Figure 1. The values of fringe visibility measured for three identified sources.

At least 98 per cent of all the sources studied were resolved by at least one of the base-lines. The number of sources studied at each base line, and the base-lines used, are listed in Table 1.

THE ERRORS AFFECTING INTERFEROMETRIC OBSERVATIONS

The causes of errors in these observations may be divided into three groups.

TABLE 1

The Long Base-Lines Used at Jodrell Bank

Base Line	*(1)*	*(2)*	*(3)*	*(4)*
Aerial separation;	2,200λ	9,700λ	32,000λ	62,000λ
Azimuth	256°	267°	272°	085°
Diameter of Gaussian source $\gamma = 0.5$	41 secs	9.4 secs	2.8 secs	1.4 secs
No. of sources studied Flux S $\geq$ 12 flux units	222	238	96*	110†

*These sources were selected because they gave fringe patterns at Base-Line (2).
†These sources were selected because they gave fringe patterns at Base-Lines (2) or (3).

1. The fringe amplitude due to the source may be affected by confusion with weaker sources also present in the aerial beam. The area of sky observed by an interferometer is limited by the bandwidth of the instrument. For some instruments of high resolution, this area may be smaller than the aerial polar diagram, and the area observed thus varies as (Base-line)$^{-2}$. Errors due to confusion will, therefore, be reduced at longer base-lines [6] even if the confusing sources are not resolved.

2. Errors will arise from the fluctuation noise on the record, and from gain and sensitivity variations of the receivers and radio link equipment. These errors probably become larger as the base-line is increased and the radio link system becomes more complicated.

3. Calibration errors may arise in converting the fringe amplitudes observed at one base-line to a common scale of source intensity. The base-lines 1 and 2 were calibrated by reference to three sources which appeared to be unresolved also at base-line 3. Base-line 3 was calibrated on the assumption that 3C 147 was unresolved, an assumption since justified by observations at higher resolution. Base-line 4 was calibrated by assuming that 3C 48 was unresolved, and comparing transit observations with others made about four hours before transit, when the effective resolution was the same as that of base-line 3 at transit.

The combined standard error of the fringe amplitudes due to the above three causes is probably much the same at all four base-lines, and equal to about four flux units. The values of fringe visibility have been calculated using the flux densities given in the 3C catalogue, so that any flux errors in that catalogue will affect the values of γ quoted in this paper. Serious errors in the declinations quoted in the catalogue would be reported here as sources which are resolved at all base-lines, and such cases have been excluded unless an independent measurement of the declination is available.

RESULTS

Figure 2 shows histograms of the results at base lines 1, 2, and 3 for sources with galactic latitude ≥ 20 ° and intensity ≥ 12 flux units. The histograms for 2,200λ and 9,700λ refer to the same set of sources, while the third histogram shows the results for those sources in the same set which, having given fringe patterns at 9,700λ, were observed at 32,000λ. Because of technical difficulties, the remaining sources were not observed at 32,000λ. The observations at the longest base-line of 62,000λ (70 miles) were only made 3 to 6 weeks ago, and they have not yet been analysed fully. For sources with galactic latitude ≥ 20°, preliminary results show only five with $\gamma_{62} \geq 0.5$ (3C 48, 147, 225, 256, 286). A fringe pattern was observed in six other cases (3C 123, 196, 295, 298, 380, 459), but for these sources $\gamma_{62} < 0.5$. In 70 cases, the source was observed on at least two occasions, but no fringe pattern was recorded.

DISCUSSION

It is obvious from Figure 2 that the average value of γ decreases at the longer base-lines. The results from any one base-line, taken alone, can be converted into a distribution of source diameters, using Gaussian models for the sources. The corresponding scales of angular diameters are also shown in Figure 2. However, when the histograms are considered together, the observations cannot be interpreted in terms of a family of Gaussian sources. For in that case the 60 sources having $\gamma_{2.2} < 0.8$ should all give $\gamma_{9.7} < 0.1$. In fact, the histograms show 43 upper limits which could all be < 0.1, and 6 other sources with $\gamma_{9.7} \leq 0.1$. Therefore, 11 sources have too high a value of $\gamma_{9.7}$.

The complex structure of the sources becomes more obvious as soon as a comparison is made of the values of γ for individual sources at two or more base-lines. The diagrams of Figure 3 show the value of γ for individual sources at successive base-lines. On such diagrams, the effects of flux errors in the 3C catalogue are reduced. For a Gaussian source, the points would lie on the dotted line. Even if the errors of individual values are larger than has been estimated, it is clear that the points are not distributed symmetrically about the Gaussian line, but are widely scattered, in both coordinates, about the mean value of γ. There is no known systematic error which could produce these results, and they are, therefore, interpreted as clear evidence that the individual sources have complex structure. Furthermore, in 11 cases out of 89 in the top diagram in Figure 3, the value of γ is actually higher at the longer base-line (the points lie above the line at 45° to the axes).

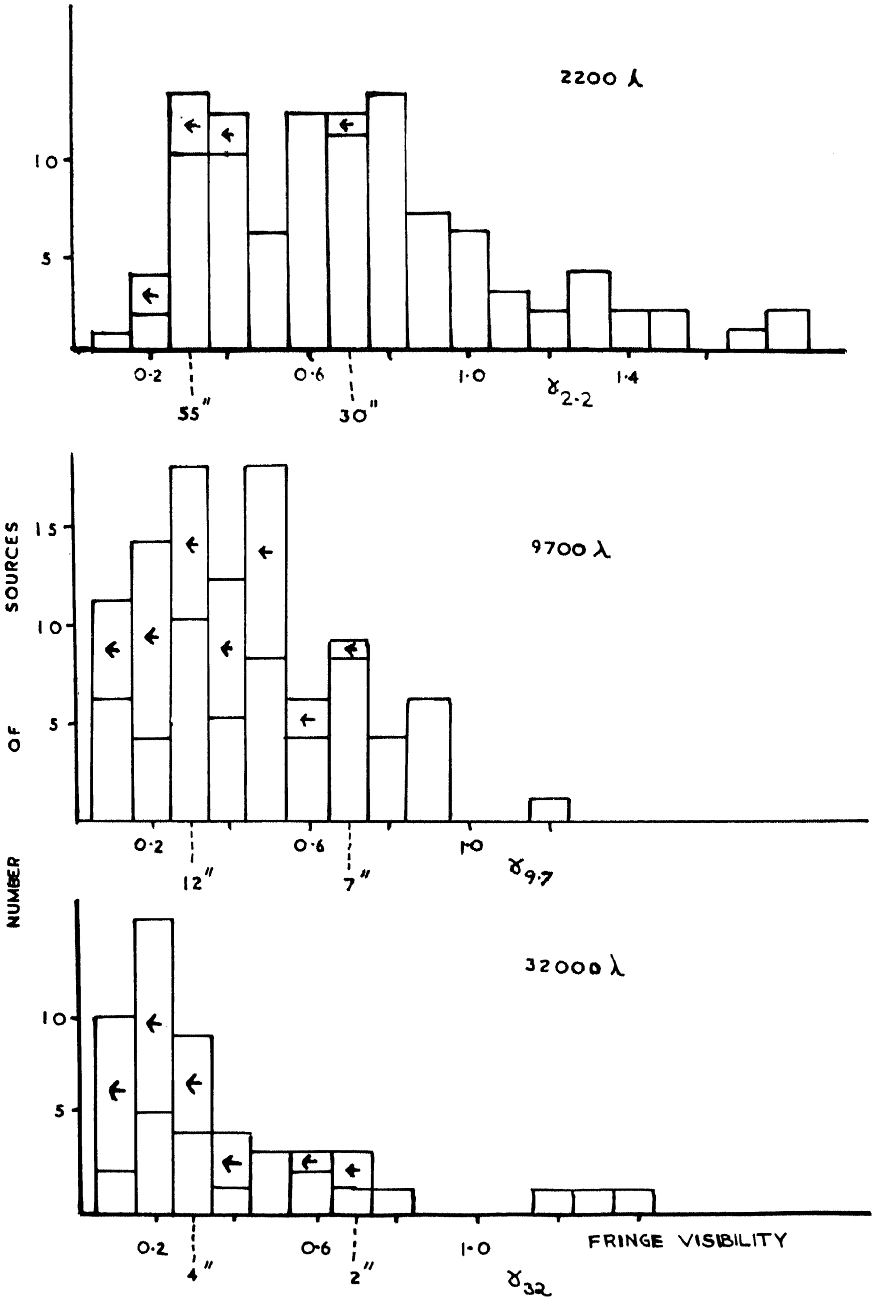

Figure 2. Histograms of the values of the fringe visibility observed at 2,200 λ, 9,700 λ and 32,000 λ for sources with galactic latitude ≥ 20° and flux density ≥ 12 flux units.

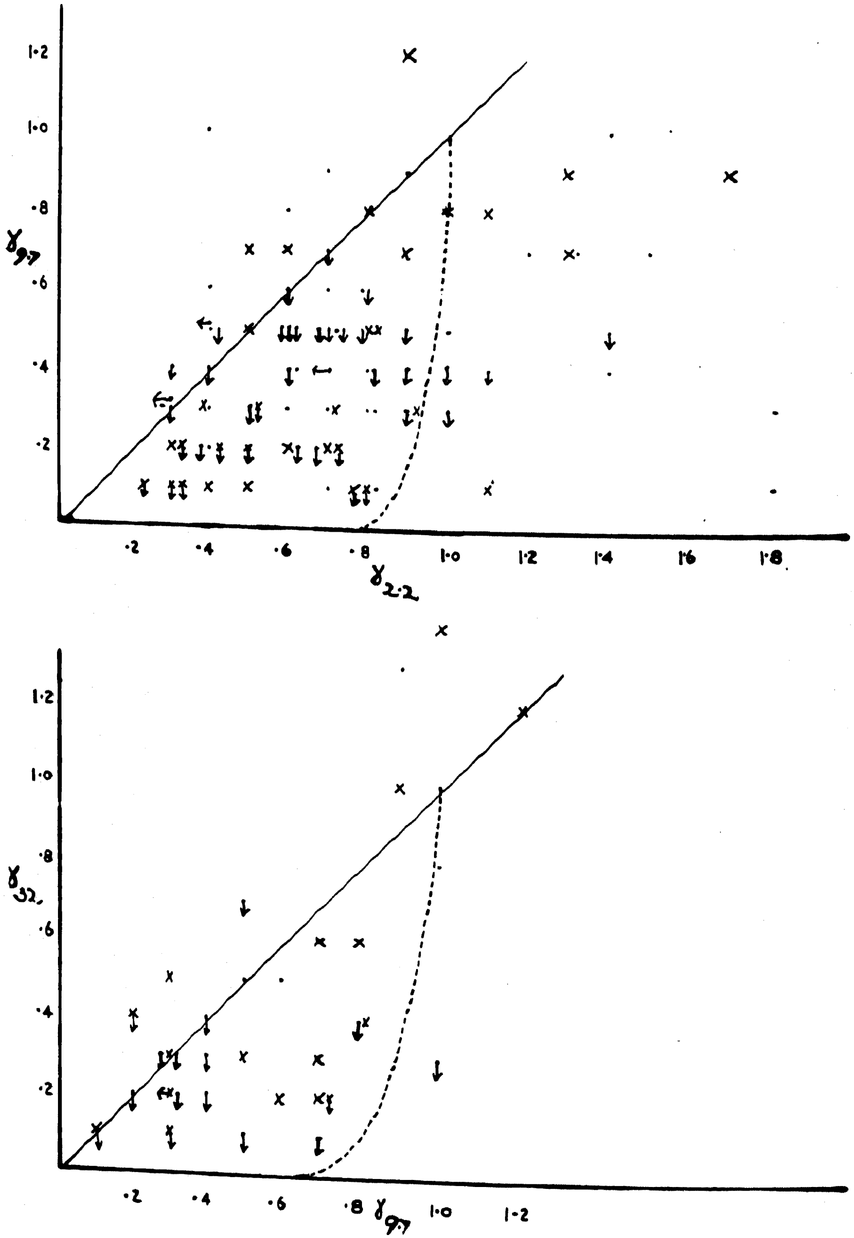

Figure 3. The values of γ measured for individual sources at different base-lines.

· Flux $S < 20$ f.u.

x Flux $S \geq 20$ f.u.

In these cases, the sources appear to consist of two components separated in Right Ascension by more than their diameter. It seems reasonable to assume that the sources are randomly orientated in space, hence, at least half the sources have a double structure, though the components may not be equally intense.

Further evidence about the structure of the sources is given by the average value of γ for all sources of intensity ≥ 12 flux units, and galactic latitude $\geq 20°$. The results described above, supplemented by some figures derived from the work of Moffet [7] are summarized in Table 2. Values of $\bar{\gamma}$ for 389λ, 779λ, 1557λ were extracted from Moffet's data. It will be seen that $\bar{\gamma}$ is less than 1 at all base-lines. At the base-lines out to 2,200 λ, this is attributed to the double nature of the sources, giving an oscillatory character to the individual transforms. The characteristic spacing of the doubles is about 1′of arc. It is thought that at longer base-lines the component parts of the sources are resolved, and the value of $\bar{\gamma}$ gives, approximately, the transform of these components. Table 2 shows that their characteristic size is 15″ to 30″, but they are not Gaussian, and some contain smaller hot regions with sizes going down to 1″ of arc.

TABLE 2

Mean Value of $\bar{\gamma}$ for 3C Sources of Intensity ≥ 12 Flux Units and Galactic Latitude $\geq 20°$

Base Line	*389λ*	*779λ*	*1557λ*	*2,200λ*	*9,700λ*	*32,000λ*	*62,000λ*
Flux limit	20	20	20	12	12	12	12
No. of sources studied	54	59	55	108	105	47	90
Number with $\gamma > 0.5$	47	43	33	78	45	8	5
Mean value of fringe visibility $\bar{\gamma}$	0.78	0.71	0.66	0.67	0.51	< 0.2*	< 0.1*

*Sources for which no fringe pattern was recorded, assumed to have $\gamma < 0.01$.

BRIGHTNESS TEMPERATURES

These results can also be replotted as a histogram of surface brightness temperature, as shown in Figure 4. To calculate T_b in this histogram, a Gaussian model has been adopted for each source, its size being derived from the value of γ at the longest base-line at which a fringe pattern was observed of $\gamma \geq 0.2$. The temperature plotted is the central temperature of the Gaussian model. The letters on the histogram indicate which base-line was used to calculate the temperatures of individual sources.

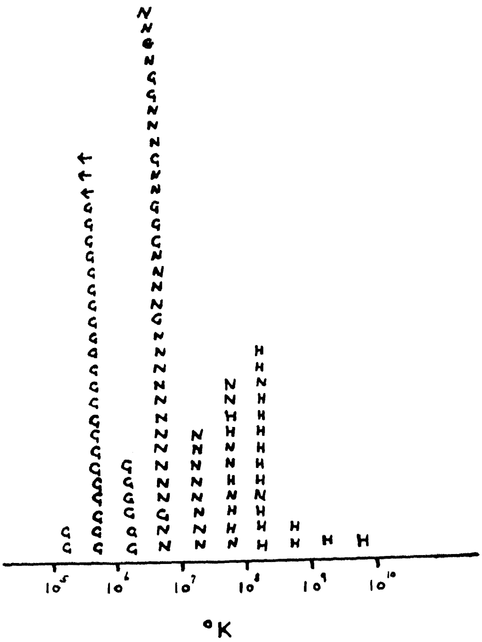

Figure 4. Histogram of maximum brightness temperatures for sources of flux density ≥ 12 flux units, and galactic latitude ≥ 20°. The source diameters were calculated from the value of γ at the longest base-line at which a fringe of $\gamma \geq 0.2$ was recorded. The letters indicate which γ was used for individual sources:

G, using the value γ observed at 2,200λ;
N, using the value γ observed at 9,700λ;
H, using the value γ observed at 32,000λ.

The true values of maximum brightness temperature could only be calculated if the detailed structure of each source were known. The values in the histogram of Figure 4 will tend to be lower limits which may be increased because of the complex structure of the sources by factors of 2 or 4. A similar calculation for Cygnus A gives a brightness temperature of 3×10^{8}°K. Most sources appear to be cooler than this by factors of 30 to 100, and these factors seem to be too large to be explained by the complexity of the sources. We conclude that most sources in this flux range have maximum brightness temperatures which are less than one tenth of that of Cygnus A, and their average brightness temperature is of the order of 1/300 of that of Cygnus A.

CONCLUSION

The histogram of temperatures could be converted into a luminosity function if the linear sizes or the distances of the sources

were known. Bolton [1] lists the identified sources for which red-shifts have been measured. The radio data on the angular size of these sources can be used to show that the components have a mean linear size of 21 kpc and the double sources a separation of up to 100 kpc. These figures are comparable with the known value for Cygnus A (35 kpc and 80 kpc, respectively).

For a typical source of $m_r = 8.9$, the absolute radio magnitude M_r is therefore between –26 and –28. The distance modulus $m_r - M_r$ is very similar to that of Cygnus A, and cosmological effects are, therefore, likely to be negligible in this flux range.

The results discussed in this paper may conveniently be summarized by the tentative and approximate model of the typical source of flux density 12 flux units shown in Figure 5. This double source with centre-brightened components is thought to occur randomly orientated in space. The intensities of the two components are not necessarily equal. The average surface temperature of the components is about 10^{6}°K, but the temperatures of the small bright regions may rise above 2×10^{7}°K. This is a remarkably complicated source model to deduce from transit observations with a few base-lines of almost constant direction but having widely different resolving power. No simpler model appears to fit all the "facts."

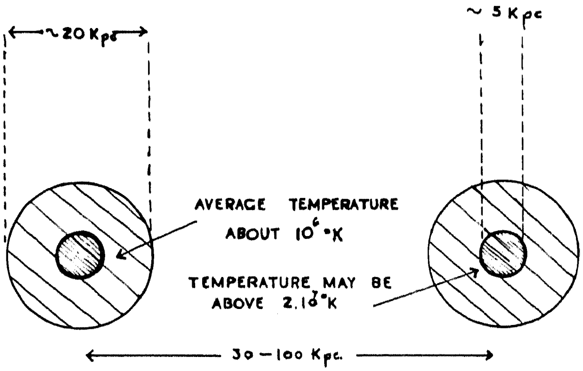
~20 Kpc
~ 5 Kpc
AVERAGE TEMPERATURE ABOUT 10^{6} °K
TEMPERATURE MAY BE ABOVE 2.10^{7} °K
30 — 100 Kpc.

Figure 5. A tentative model for a typical extra-galactic radio source observed in the flux range down to $m_r = 8.9$. The components may be of unequal radio intensity.

ACKNOWLEDGEMENTS

The work at Jodrell Bank described above was suggested by Professor Hanbury Brown, who has taken a keen and very helpful interest in its progress. It was made possible by the active encouragement of Professor Sir Bernard Lovell. Many colleagues and students have contributed to this work, including Dr. Rowson and Mr. Allen, whose efforts were essential to the observations at every one of these base lines. It is a pleasure to acknowledge their unstinting cooperation.

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DISCUSSION

Moffet: P. Maltby and I have recently completed the analysis of our observations of 174 presumably extra-galactic sources using the North-South and East-West base lines of the Caltech interferometers at 31.3 cm. The maximum base-line length is 1557 wavelengths in each direction. Ninety-one sources were observed with both base-lines; the remaining 83 were observed with one base-line or the other. Off-transit measurements have also given us information about the brightness distribution of many of these sources in directions other than North-South or East-West.

Because we have sampled the visibility function at more frequent intervals and along more axes than did Palmer, and because we have in most cases measured both the amplitude and phase of the complex visibility function, we are able to reconstruct more detailed pictures of those sources having angular extents of between $1'.5$ and $15'$. Our results indicate that a model consisting of two well-separated components is applicable to most extra-galactic sources.

More specifically, of the 174 sources studied, 76 had dimensions within the above range. We have classified these as follows:

E	U	H	S	Total
14	42	7	13	76

Here, the members of Class E are equal double sources in which the components have intensity ratios of 1.4 : 1 or less. Class U contains unequal doubles or sources with a higher degree of complexity, generally having an axial ratio $\overline{>}$ 2 : 1. Class H contains sources in which a bright core is superposed on a fainter halo. Only 13 of 76 sources appear to have simple, symmetrical shapes, and some of these may be sources of greater complexity which appear simple in projection. The high axial ratio which we generally observe would seem to exclude the possibility that the typical brightness distribution is toroidal.

Lequeux: I would like to describe some new results obtained at Nancay on Cygnus A near 1420 Mcs with the two-antenna interferometer. The two components of Cygnus A are separated by 100″ and both seem to be elongated along the major axis of the source. Their half-intensity dimensions are 25″ by 18″. Both have a rather sharp front in the external part of the radio source: this suggests that we are dealing with the propagation of two shock waves in the intergalactic medium. The known fact that the angular separation of the components increases with frequency might indicate that relativistic electrons are accelerated in the shock wave and lose their energy as they diffuse behind the front.

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The Radio Luminosity Function and the Number Versus Flux-Density Relationship for the Discrete Sources

I. INTRODUCTION

A number of possible cosmological tests based on radio astronomical observations have been proposed, the most promising of which are, perhaps, the following:

1. ***Measurement of the intensity of the extra-galactic contribution to the background radiation.*** This method, suggested independently by Shakeshaft [1] and Priester [2], is based on the relationship, on different cosmological models, between the integrated emission and the average emissivity of extra-galactic space; owing to the relatively flat spectrum of radio emission, most of the radiation originates at great distances, and the method is, in principle, a sensitive one. Its use, however, requires the recognition of all types of extra-galactic sources and measurements to distinguish between the integrated radiation due to the extra-galactic space, and radiation from the outer regions of the Galaxy.

2. ***Measurements to determine the density of inter-galactic matter by observation of the 21-cm. absorption in the emission from a distant source.*** If it were possible to detect the 21-cm. absorp-

tion in the continuum emission from a distant source by observing the reduction in intensity at frequencies between the rest frequency of the hydrogen line and that appropriate to the distant source, it would be possible to determine the density of neutral hydrogen in inter-galactic space [3]. The intensity of the absorption band is, however, extremely weak even with very large telescopes observing very intense sources, and its detection seems likely to be very difficult. In addition, it is possible that the inter-galactic material would be largely ionized [4].

3. *Measurements of the angular diameters of radio sources.* The relationship between the angular size of a source of given proper dimensions, and its red-shift has been examined in relation to radio sources by Hoyle [5] and Davidson [6], and it is concluded that certain cosmological models should be readily distinguishable. In practice, however, it will be necessary to make use of flux-density in place of red-shift, by making a separate estimate of the radio luminosity function, and the adoption of a proper dimension for the sources is likely to be difficult; there appears to be a wide divergence between the dimensions of the optical and radio sources for those that have been identified, and some are highly elongated while others appear to be double sources [7,8,9,10]. The possibility of an entirely new class of source, with perhaps little optical emission, must not be ignored.

Uncertainties in the proper dimensions of the sources, or in their red-shift (due to a wide dispersion in the absolute radio luminosity), could be largely overcome if there were a sufficient number of sources at small red-shifts to allow the observations of distant sources to be normalized; a suitable plot of angular diameter against flux-density, if made over a sufficient range of flux-density, might allow distinctions to be made between certain of the cosmological models. With a large dispersion in the radio luminosity, and the rather small number of sources of large flux-density for which the effects of the red-shift may be assumed small, the uncertainties in the distance scale may, however, make it difficult to reach a definite conclusion by this method.

4. *Determination of the "Number versus flux-density" relationship.* If the relative numbers of radio sources having different red-shifts were known, it should become possible to distinguish between a number of different cosmological models [11,12,13]. Unfortunately, apart from the small number of sources for which definite optical identifications have been made, no direct measure of red-shift is possible, and it is again necessary to determine the scale of distance indirectly from the flux-density and from some knowledge of the radio luminosity function.

In this case, however, there is no additional uncertainty arising from the need to know the proper dimensions of the source, and the measurements involve simply a measurement of the flux-density, and not of angular size. It is, therefore, possible to extend the observations over a very much larger range of flux-density, hence, of red-shift, than is possible in the case of diameter measurements. The difficulties arising from uncertainties in the luminosity function are, consequently, likely to be of less importance.

In view of these considerations and of the unexpected number versus flux-density relationship already found in earlier observations [14], it was decided that the most promising approach to the cosmological problem lay in an extension of the final method (4). A more powerful instrument was, therefore, constructed during 1957, and brought into use in 1958, which allowed observations to be made of considerably weaker sources than hitherto [15,16]. The results of the first observations with this instrument have been described in a series of papers [17,18,19,20,21,22,23].

These results have been used to derive limits on the radio luminosity function (section II) and to investigate a number of possible selection effects which might arise in the determination of the number versus flux-density relationship (section III). Finally, all the results have been assembled to derive the best estimate of the number versus flux-density relationship for areas of sky having $|b| > 20°$, and in section IV these results have been compared with those predicted by one particular cosmological model – the steady-state model – proposed by Bondi and Gold [24] and Hoyle [25].

II. THE RADIO LUMINOSITY FUNCTION

Although considerable progress has been made in the identification of radio sources with optical objects [26,27], the number of sources definitely associated with optical objects still represents only a small fraction of the total radio sources. Some results reported recently by Mills [28] and Harris and Roberts [29] in which 15 to 30 per cent of the sources were related to external galaxies is not supported by a recent analysis at Cambridge, in which the positions of all sources in an area of 1.1 steradian and having $S > 5 \times 10^{-26}$ W m^{-2}(c/s)$^{-1}$ were examined down to the limit of the prints of the Palomar Sky Atlas; the number of objects found in this search, in which the radio positions had errors of about 20 square minutes of arc, did not differ greatly from that using an identical search procedure with randomly selected positions. This result suggests that less than about 10 per cent of the sources in this range of flux-density have an associated photographic magnitide of less than 18.

If it becomes possible to identify an appreciable fraction of all sources having flux-densities greater than a given value, it is clearly possible to use the optical distances to establish a radio luminosity function which will allow predictions to be made on the number versus flux-density relationship expected on a given cosmological model. So long as the identified sources represent only a small percentage of the total, it is evident that such predictions may be in serious error. It is conceivable that sources of an unknown type, either within, or outside, the Galaxy, which have little optical emission, could seriously dilute the effects of powerful sources on the number versus flux-density relationship.

Similar difficulties apply to the derivation of the distances, and hence, the radio luminosities of sources by means of measurements of their angular diameters.

An analysis has, therefore, been made which allows limits to be set to the luminosity function independently of assumptions about the nature of the sources. It is based on an earlier analysis [30] which has been extended in the light of the new observational material [31]. In this analysis, the number of sources observed in a given range of flux-density is related to the average volume emissivity associated with different components of the background emission and to the observed isotropic distribution found for sources away from the galactic plane.

*1. **The number versus flux-density relationship.*** It is convenient to consider the distribution $n(P)$ which gives the number of sources of radio luminosity P falling in a given range of flux-density. It is supposed that the sources are distributed uniformly, but randomly, in space and that the dispersion of the luminosity has the form

$$n(P) \propto \exp\left[-(\log_{10} P/P_0)^2/2\right].$$

This relation can be rewritten to give the true space density

$$\rho(P) \propto P^{-3/2} \exp\left[-(\log_{10} P/P_0)^2/2\right].$$

Strictly $n(P)$ should be considered as the sum of two terms, corresponding to sources within, and outside, the Galaxy, but in the analysis which follows it is clear that the contribution to $n(P)$ from galactic sources may be neglected at $|b| > 20°$.

The number of sources per unit solid angle which produce a flux-density greater than S may now be written:

$$N = \frac{1}{3}\,\rho_0 {P_0}^{3/2}\, S^{-3/2},$$

where ρ_0 represents a weighted mean density of the sources. The relationship between N and S may be determined from observations and, although it is found not to obey the expression for a uniform distribution of sources given above, a numerical value for $\rho_0 P_0^{3/2}$ may be established at a sufficiently large value of N for the statistical errors to be small without approaching the limits of observation. If ρ_0 is expressed in sources per cubic pc, and

$$P_0 \text{ in } \mathrm{W}(\mathrm{c/s})^{-1}\ \mathrm{ster}^{-1}$$

at a frequency of 178 Mc/s, it is found that $\rho_0 P_0^{3/2} = 1.4 \times 10^{14}$. This result is shown as a plot of log P_0 against log ρ_0 in Figure 1; any point on the line $\rho_0 P_0^{3/2} = 1.4 \times 10^{14}$ will satisfy the general observations of the number of sources in a given range of flux-density, although not, of course, the nonuniformity in the observed relationship.

2. *Isotropy*. An isotropic distribution of the sources is to be expected if most are extra-galactic, but if an appreciable fraction were galactic, then departures from isotropy should be detectable if the average space density $\rho < 3N/r^3$ where N is the number of sources per unit solid angle at the limit of the observations, and r represents the smallest distance to the boundaries of the Galaxy. In the previous analysis [30], it was shown that few radio sources could be situated in the spiral arms, but a population throughout the galactic halo would have been permissible.

A special investigation was, therefore, made to detect anisotropy associated with such a distribution, and observations in two areas centered on $l^{\mathrm{II}} = 160°$, $b^{\mathrm{II}} = -55°$ and $l^{\mathrm{II}} = 355°$, $b^{\mathrm{II}} = 45°$, (corresponding to near-minimum and near-maximum distances through the galactic halo) were compared. The statistical method of record analysis introduced by Scheuer [32] and recently extended by Hewish [33] was used to improve the sensitivity of the comparison, and showed no differences in the number of sources in the two regions at a level of $N = 2 \times 10^4$ sources steradian^{-1}. This result shows that, if most of the sources were in the galactic halo, then their effective space density ρ_0 must exceed 3×10^{-3} pc^{-3}. [Figure 1].

3. *The integrated emission from the sources*. Observations of the distribution of the background radiation have allowed a model of the galactic emission to be derived, hence, the average volume emissivity for different regions of the Galaxy and for the extra-galactic space [34]. Spectral observations by Costain [35], Chapman and Molozzi [36], Seeger [37], and Smith [38] have enabled the volume emissivity at different frequencies to be established.

Without understanding the mechanism of emission, it is clear that these results allow an upper limit to be set to $\Sigma\rho P$, the contribution made to the integrated emission by the sources; an upper

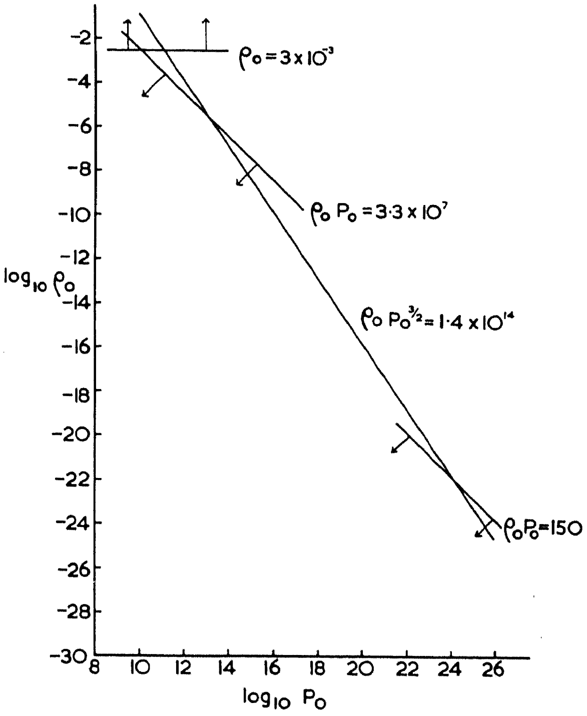

Figure 1. Diagram relating the values of the effective radio luminosity P_0 and space density ρ_0 which are compatible with: (a) counts of sources falling in a given range of flux-density; (b) the observed isotropy of the sources; and (c) the maximum contribution of the sources to the background emission.

limit for $\rho_0 P_0$ may hence be derived which for sources situated in the galactic halo has a value of about 10^7 W(c/s)$^{-1}$ ster^{-1} pc^{-3}.

The discrepancy between the intersection of the lines $\rho_0 P_0 < 10^7$ and $\rho_0 > 3 \times 10^{-3}$ in Figure 1 with the line $\rho_0 P^{3/2} = 1.4 \times 10^{14}$ indicates that an insignificant proportion of the observed sources are within the Galaxy.

A similar analysis may now be made for the case in which most of the sources are extra-galactic; the relation between the volume emissivity and integrated emission of the extra-galactic space now depends on the cosmological model, but for a particular model it is again possible to derive an upper limit to $\rho_0 P_0$. Although it is difficult to distinguish, observationally, between an isotropic extra-galactic component, and one due to outer shells of the galactic halo, by making such observations over a range of wave lengths, it is again possible to determine the upper limit to the radiation having a spectral distribution similar to that of the sources;

Costain's observations [35] lead to an upper limit of 25°K for the sky brightness at 178 Mc/s due to sources having a spectral distribution $S \propto f^{0.8}$. For the steady-state model and the adopted model of dispersion, this result leads to an upper limit of

$$\rho_0 P_0 = 150 \text{ W}(\text{c/s})^{-1} \text{ ster}^{-1} \text{ pc}^{-3}.$$

From Figure 1, the minimum value of P_0 is then 10^{24} W(c/s)$^{-1}$ ster^{-1}, for the assumed dispersion in the luminosity; any greater dispersion would require a larger limiting value for P_0.

4. Evidence from optical observations. The analysis given in the previous sections has shown that a negligible fraction of the observed sources are within the Galaxy, and that most must have radio luminosities at least 1000 times greater than that of the Galaxy; this conclusion does not depend on any knowledge of the nature of the sources.

If, now, it is supposed that the sources represent galaxies having optical luminosities comparable with those of normal galaxies (and those with positive identifications lead to a mean value of $M_{pg} = -20.6 \pm 1.0$, which is in close agreement with the figure derived by Kiang [39] for field galaxies), then the limiting brightness which can be associated with a given radio source can be used to give a minimum distance, and hence, radio luminosity. An analysis of this type has been carried out by Long and Marks [21] using the first set of observations from the new Cambridge instrument. In their analysis, they ascribed an optical magnitude, or limiting magnitude, to each source on the basis that it was associated with the brightest object falling within the permissible error rectangle. Such a procedure may include a large number of galaxies which are not related to the radio sources, and as has already been mentioned, a similar search has recently been made in positions chosen at random in order to arrive at a more reliable determination of $m_r - m_{pg}$. By relating the numbers of objects found in the two searches for each range of $m_r - m_{pg}$, and taking account of the statistical uncertainty, it was possible to derive a histogram showing the maximum number (for a 95 per cent probability) of genuine identifications. If, now, it is supposed that the sources have an absolute optical magnitude of $M_{pg} \cong -20.6$, this distribution may be converted into a limiting radio luminosity function, which is shown in Figure 2(c), where it may be compared with the lower limit derived entirely from radio observations [Figure 2(a)].

A number of authors [27,28,40] have derived the radio luminosity function on the basis of those sources with reasonably certain identifications and not on the basis of all sources falling in a given range of S; while such a procedure must involve selection effects favoring the weaker radio sources, the results are entirely

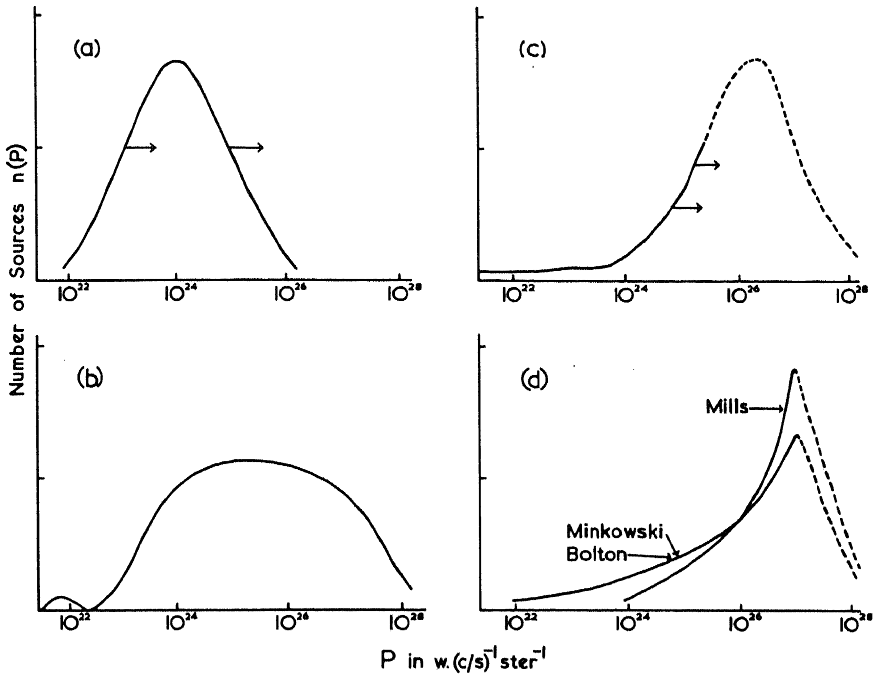

Figure 2. The distribution $n(P)$ in radio luminosity P, of the sources lying in a given range of flux-density. (a) Lower limit derived from radio observations without assumptions concerning the physical size or optical emission from the sources. (b) Model derived from consideration of the radio surface brightness of the sources. (c) Model derived from limiting optical magnitudes on the supposition that $M_{pg} \simeq -20.6$. (d) Models derived on basis of identified sources.

compatible with those already derived, and are shown in Figure 2(d) replotted to give the distribution of the sources falling in a given range of flux-density.

5. *Evidence provided by measurements of radio "surface brightness."* If it is assumed that the radio sources have physical dimensions comparable with that of the Galaxy, then measurements of angular size in relation to flux-density, or radio "surface brightness," may be used to determine the radio luminosity of a source.

Observations have been made by Miss Leslie [17] of some 900 sources having $S > 6 \times 10^{-26}$ W m^{-2}(c/s)$^{-1}$, in which sources having surface brightness temperatures in excess of 100°K could be measured. In most cases, it was not possible to resolve the sources, and only lower limits of the brightness temperature (of about 10^5°K for sources having $S > 10 \times 10^{-26}$ W m^{-2}(c/s)$^{-1}$) could be determined. By considering these observations with those of a smaller sample of sources using larger resolving powers [8,41], it is possible to derive a composite curve showing the distribution of surface brightness without the selection effects which might be present in the latter two sets of observations. On the supposition that the sources are of galactic dimensions, this curve may also be expressed as a luminosity function [Figure 2(b)].*

All the available data are, therefore, consistent with a distribution of radio luminosity similar to that of the identified sources, with most of the observed sources having P lying between 10^{26} and 10^{27} W(c/s)$^{-1}$ ster^{-1}, a figure comparable with that for the Cygnus A source. It is also possible, without any assumptions concerning the physical size or optical emission of the sources, to set a lower limit of $P_0 \simeq 10^{24}$ W(c/s)$^{-1}$ ster^{-1} from radio observations alone.

Even with this conservative limit, the region within which sources may be studied individually ($S \geq 2 \times 10^{-26}$ W m^{-2}(c/s)$^{-1}$) extends to distances of at least 10^9 pc.

III. THE OBSERVED NUMBER VERSUS FLUX-DENSITY RELATIONSHIP

Two sets of observations were used to derive the number versus flux-density relationship.

1. A survey covering most of the observable sky ($\delta = -05°$ to $+60°$) with an instrument which could be used both as a fan-beam $14' \times 4\overset{\circ}{.}6$ and as an East-West interferometer of 465λ aperture having the same fan-beam response [17].

This system was used to observe all sources having

$$S > 6 \times 10^{-26} \text{ W m}^{-2} (\text{c/s})^{-1},$$

and to provide information on their angular extent; a total of 910 sources were observed of which 629 were situated at galactic latitudes greater than 20°. At the limit of detection there was one source per 20 beam areas. As well as providing numbers of good statistical weight for the intense sources, the distribution of surface brightness derived from these sources could be used to establish the magnitudes of any errors due to partial resolution of the sources in the deep survey, which was made exclusively with an interferometer.

2. The second set of observations used the method of aperture synthesis to obtain a 465λ interferometric pattern of considerably higher primary resolution (25′ arc in α and 35′ arc in δ). The pres-

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*The most recent observations of surface brightness at Jodrell Bank reported by Dr. Palmer are consistent with a model in which most of the sources observed have values of P in the neighbourhood of 10^{26} W(c/s)$^{-1}$ ster^{-1}.

ent analysis is based on observations of four areas of sky each covering nearly 24^h in α and $\delta = 50°$ to $54°$, $\delta = 48°$ to $52°$, $\delta = 40°$ to $44°$, and $\delta = 16° 30'$ to $21° 30'$ [18]. The total area with $|b| > 20°$ was 0.6 steradian, and contains a sufficiently large number of sources for the statistical errors to be small compared with those of the "whole-sky" counts of the more intense sources.

The positions and flux-densities of all sources having

$$S > 2 \times 10^{-26} \text{ W m}^{-2} (\text{c/s})^{-1}$$

were obtained; at this level, there is approximately one source per 30 beam areas, and the signal-to-noise ratio is about 25 : 1.

The flux-density scales of all the observations were normalized from a "standard sequence" of sources observed in a separate experiment [42]. A comparison of the counts obtained in the four high-resolution surveys showed no significant differences for areas having $|b| > 20°$, and the results were, therefore, combined for relating to the "whole-sky" counts of the more intense sources.

1. *THE EFFECTS OF ANGULAR DIAMETER AND CLUSTERING OF THE SOURCES AND OF SIDE-LOBES OF THE AERIAL SYSTEM*

(a) The presence of extended sources. Sources having

$$S > 6 \times 10^{-26} \text{ W m}^{-2} (\text{c/s})^{-1}$$

were observed with a "total-power" system and provide the true integrated flux-density, except for sources of such low surface brightness ($T_b \sim 100$ °K) that they are resolved in the fan-beam observations. Sources with $T_b < 200°$ were, therefore, excluded from the analysis since they will be unobservable with the interferometer until they are at such a distance that

$$S < 0.15 \times 10^{-26} \text{ w m}^{-2} (\text{c/s})^{-1},$$

a figure below the limit of the present analysis. Miss Leslie's observations [17] show that they, in any case, only represent about 2 per cent of the total number of sources. An analysis of the distribution of surface brightness also allows corrections to be made for partial resolution of extended sources in the deep survey made with a 465λ interferometer. The corrections are negligible for $0.15 < S < 3 \times 10^{-26}$ W m^{-2} (c/s)$^{-1}$, and reach about 10 per cent in the apparent number of sources at $S = 6 \times 10^{-26}$; at large fluxes the total-power observations are used.

(b) Source-clustering. If a significant proportion of the sources occurred in close double, or multiple, systems they might be observed individually when nearby, but a similar system at a great distance might be mistaken for a single and possibly extended source. The effects of source-clustering on observations made both with total power and interferometric systems must, therefore, be examined.

A special analysis was made by Miss Leslie [23] to investigate the presence of clustering with angular separations in the range 3'.5 to 200', a range sufficient to determine the effects on all the observations. No definite evidence for clustering was found, and upper limits to the fraction of sources occurring in clusters of different sizes were derived; these limits were used to determine the maximum corrections which might be required at various points in the number versus flux-density relationship.

(c) The effects of aerial side-lobes. The observed number versus flux-density relationship would be modified if intense sources received in subsidiary responses of the aerial system were mistaken for weak sources. A detailed analysis was, therefore, made [22] which showed that if areas 2° × 4° were omitted from the analysis near each source having $S > 25 \times 10^{-26}$ W m^{-2}(c/s)$^{-1}$, and, also, larger strips near the α of the four intense sources having $S > 400 \times 10^{-26}$ W m^{-2}(c/s)$^{-1}$, then the errors were negligible.

(d) The effects of receiver noise level and confusion with weak sources. The effects of receiver noise and the presence of weak, unresolved sources in the beam may be determined from an analysis of the histogram of the recorded deflection [43]. The noise level during most of the observations was 0.08×10^{-26} W m^{-2}(c/s)$^{-1}$ and was negligible. The effects of noise and confusion introduce an error of about 5 per cent in the number of sources at the limit of the observations of individual sources, but, as will be discussed in the next section, the distribution at the smallest flux-densities may best be investigated by a different method.

The results of combining all the observations of individual sources are shown in Figure 3. The observed counts have been corrected for the effects of source diameter, clustering, receiver noise, and confusion. The errors shown include the uncertainty in these effects, the statistical errors, and the uncertainty between the scales of flux-density used in the two sets of observation.

Over the range $2 < S < 100 \times 10^{-26}$ W m^{-2}(c/s)$^{-1}$ the observations are best fitted by a straight line of slope −1.80.

2. THE EXTENSION OF THE NUMBER VERSUS FLUX-DENSITY RELATIONSHIP TO SMALLER VALUES OF FLUX-DENSITY

A statistical examination of the record deflections may, with a sufficiently low noise level, be used to examine the N-S relationship to smaller values of S [32]. The method has recently been

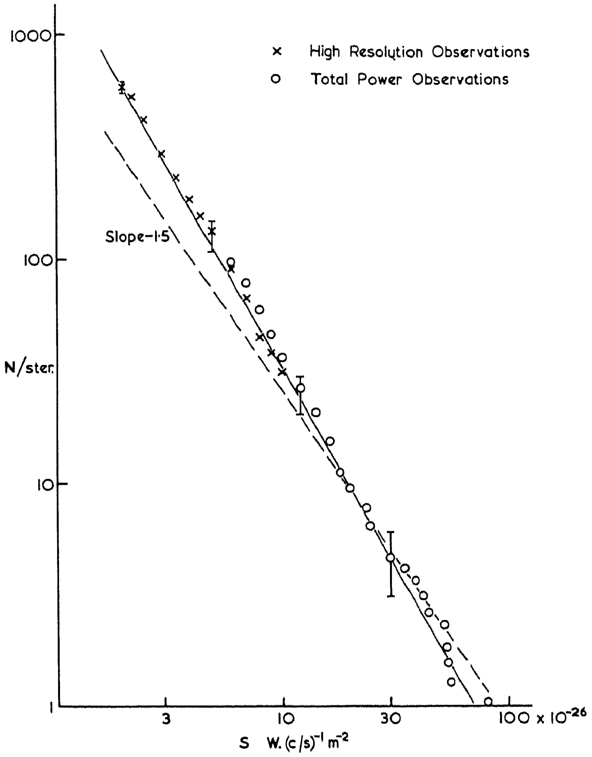

Figure 3. The corrected counts of sources (N per steradian) having a flux-density greater than S plotted against flux-density for $2 < S < 100 \times 10^{-26}$ W m^{-2}(c/s)$^{-1}$.

extended by Hewish [33] who has used a "Monte Carlo" method to derive the statistical properties of the record which would be obtained with a given interferometric reception pattern when observing sources having a given N-S relationship. By computing these curves for a large number of different N-S models, he has been able to show that the method may provide a very sensitive distinction between models which only differ at levels of S far below that at which useful information on individual sources could be obtained.

A number of N-S models are shown in Figure 4(*a*), and the corresponding histograms $P(D)$ of record deflection are shown in Figure 4(*b*). It can be seen that differences in the models which occur only below a level of 10 sources per beam area produce marked changes in the $P(D)$ curve.

By comparing the observed histogram with those of a number of models of the N-S distribution, it has been possible to determine limiting cases of the N-S distribution for

$$5 \times 10^{-28} < S < 2 \times 10^{-26} \text{ W m}^{-2}(\text{c/s})^{-1} \text{ [33].}$$

The results of the counts of individual sources are combined with these data in Figure 5 in which the results are plotted in terms of N/N_0 where N is the number of sources observed with a flux-density greater than S, and N_0 is the number which would have been observed on a static Euclidean Universe. The points repre-

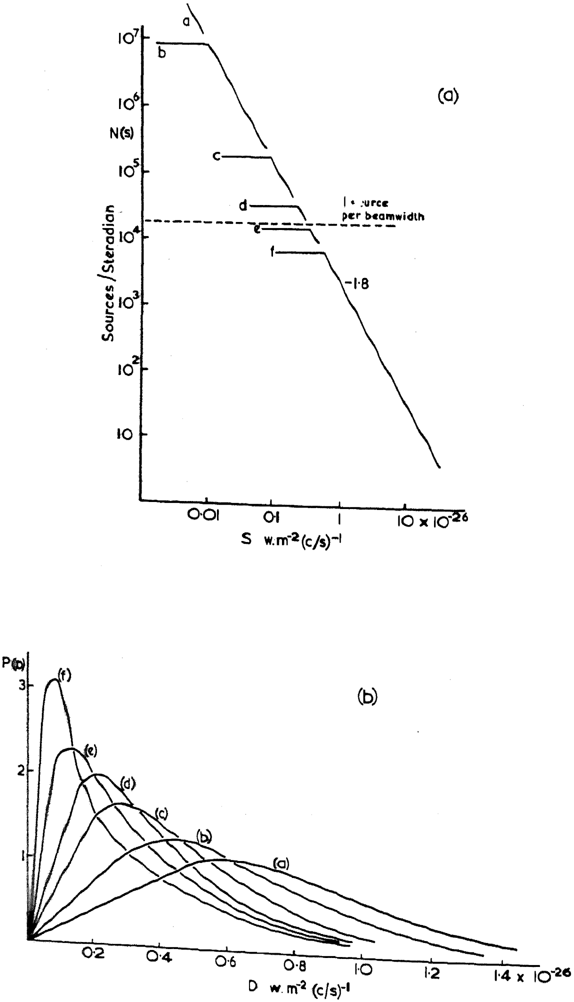

Figure 4. (a) Models of N–S. (b) Histograms of the record deflection (D) associated with the models of source distribution.

sent the information provided by the counts of individual sources, with errors which include all the uncertainties mentioned in the discussion of Figure 4; the extensions to smaller S represent three limiting cases of models which are compatible with the observed histogram of the record amplitude. These results demonstrate the existence of convergence at small S, and this is confirmed by a consideration of the contribution which the sources would make to the total emission from the sky.

IV. THE PREDICTED NUMBER VERSUS FLUX-DENSITY RELATIONSHIP

Although it is interesting to compare the results described in the previous section with the predictions of a number of cosmological models, it is impossible to reach definite conclusions in most cases owing to the uncertainties caused by possible evolutionary effects, and whether or not the radio sources are due to encounters between galaxies; in this case, their space density will show a stronger variation with time than that of galaxies [12,13].

In the case of the steady-state model [24,25], however, an explicit N-S relationship is available as soon as *(a)* the source luminosity function and *(b)* the spectral distribution of the radio emission are known. It has been shown [31] that if the emission obeys the law $S \alpha f^{-x}$, then variations of x between 0.5 and 1.0 involve an uncertainty in N/N_0 of only about 20 per cent over a range of 1000:1 in S; observations of a large number of sources have shown a variation of x of 0.8 ± 0.2 [29] and little uncertainty is, therefore, introduced.

The N-S relationship has been computed for the steady-state model assuming: *(a)* that the luminosity function corresponds to the lower limit derived from the radio observations alone with the adopted dispersion [Figure 2(a)], and *(b)* that the luminosity function corresponds to that of the sources having reasonably certain optical identifications [Figure 2(d)].

The results are shown, plotted as the variation of N/N_0, in Figure 5; all the curves have been normalized at $S = 30 \times 10^{-26} \mathrm{Wm}^{-2}(\mathrm{c/s})^{-1}$, which represents the largest value of S for which the statistical errors permit a useful value of N. It can be seen that there is a serious discrepancy between the observed and predicted curves; even when the smallest permissible value of P_0 is adopted, the number of sources in the range $0.5 < S < 2 \times 10^{-26}$ W m^{-2}(c/s)$^{-1}$ is 3 ± 0.8 that predicted. If a luminosity function similar to that of the identified sources is adopted, the discrepancy is 11 ± 2.

Although it does not seem possible to account for the discrep-

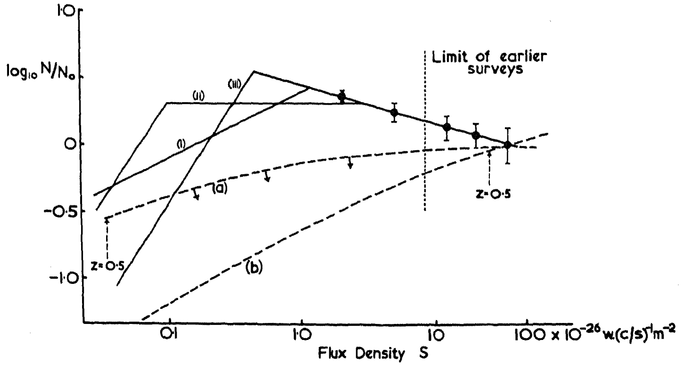

Figure 5. Curves showing variation of N/N_0 with S, where N represents the number of sources with a flux-density greater than S, and N_0 represents the number on a static Euclidean Universe. Full lines—Models compatible with the counts of individual sources and the histogram of record deflections, $P(D)$. Dashed lines: curves predicted by the steady-state model for values of (a) $P_0 = 10^{24}$, and (b) $P_0 = 10^{27}$ W(c/s)$^{-1}$ ster^{-1} ($x = 0.8$).

ancy between the observations and the curve predicted by the steady-state model as it was originally formulated, Hoyle and Narlikar [44] have recently proposed a possible explanation of the results still within the general framework of the steady-state model. They consider the possibilities (a) that galaxies become radio galaxies at extreme age ($\sim H^{-1}$), and (b) that there is a very large scale nonuniformity in the distribution of galaxies with clusters containing $\sim 10^5$ galaxies. Under these circumstances, they show that there can be a local deficit of radio sources.

One difficulty with this interpretation is related to the problem of providing the source of energy required to maintain the radio emission [45]; existing theories are not capable of maintaining a source having P lying between 10^{26} and 10^{27} W(c/s)$^{-1}$ ster^{-1} for periods greater than about 10^7 years [46,47]. In order to account for the observations in this way, it seems necessary to suppose that the radio emission continues for times of the order of H^{-1}. Although the energy problem would be less serious if a value of P_0 near the lower limit of 10^{24} W(c/s)$^{-1}$ ster^{-1} were adopted, the optical evidence then provides a serious objection, since most of the sources would then be relatively bright galaxies.

The alternative suggestion has been made [48,49,50] that radio galaxies should be associated with an *early* stage in galactic evolution when intense supernova activity may provide an adequate source of energy to account for the radio emission. If, on the other hand, the sources are due to collisions between galaxies, those occurring at earlier epochs may involve larger masses of gas, hence they would provide a greater energy of interaction. Both of these mechanisms would produce sources for which the value of P_0 was greater at an earlier epoch. In the Einstein-de Sitter model, for example, a decrease of P_0 from about 2×10^{26} W(c/s)$^{-1}$ ster^{-1} to 10^{25} W(c/s)$^{-1}$ ster^{-1} over the past 5×10^9 years would produce curves similar to that observed.

I should like to acknowledge the collaboration of Mr. R. W. Clarke in this work.

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DISCUSSION

Palmer: Professor Ryle has interpreted his results to show an increase in the space-density of radio sources at great distances. On the other hand, I should like to point out that Professor Hanbury Brown has shown in a study of the Cambridge, Sydney, Jodrell Bank, and Owens Valley observations that the curvature of the log N/log S curve can also be interpreted in terms of a local deficiency of radio sources.

Briefly, he points out that optical observations suggest that our galaxy is situated in a supercluster with a diameter of the order of 50 Mpc. In this supercluster, the nearby galaxies and the nearby radio sources are concentrated into a rough ellipsoid, as suggested by de Vaucouleurs. Away from the plane of this ellipsoid, there will be a deficiency of galaxies in the first 30 to 50 Mpc. If now we neglect the clustered nearby sources, we shall reach the conclusion that the distribution of sources is roughly isotropic and that the log N/log S curve has a slope exceeding 1.5.

The analysis made by Hanbury Brown is based on a luminosity function derived from the measured red-shifts and the Owens Valley and Jodrell Bank diameter measures. The most frequent absolute radio magnitude lies between M_r equal to -26 and -28.

It is implied in this interpretation that, in directions away from the plane of the supercluster, there should be a cutoff in the surface brightness of radio sources below about 10^5 °K for surveys carried out to a limit of about 10 flux units; this effect was observed by Miss Leslie. There should also be a slight anisotropy in the number of radio stars in the northern and southern galactic hemispheres; this anisotropy should be in the reverse direction to that found in the counts of bright galaxies. The surveys by Mills suggest that such an anisotropy may well exist.

Finally, it is clear that this interpretation cannot stand if all radio sources are associated with objects of absolute photographic magnitude $M_p = -20.5$ as has been assumed in recent papers. However, the luminosity function of galaxies strongly favors the discovery of objects of this absolute magnitude, and it is an assumption which should be put to more stringent observational tests before it is accepted.

Ryle: Professor Hanbury Brown has, I think, neglected the effect of the red-shift in interpreting the measures of surface brightness. Since the intensity of a source decreases faster than the inverse-square law, and since the solid angle subtended by a source decreases less quickly, the apparent surface brightness shows a fairly marked change with distance. If, for example, the Cygnus A source were placed at a distance at which its intensity was the same as that of most of the sources observed in the recent Jodrell Bank observations, its apparent surface brightness would be reduced by a factor of about 20. The most frequent absolute radio magnitude would, therefore, be about -31. ($P \simeq 10^{26}$ W(c/s)$^{-1}$ ster^{-1}), and most of the sources must be at very large distances.

Even if we ignore the angular diameter and optical evidence, and use the limit set by the radio data alone, the "local deficiency" of sources must extend to distances of atleast 2×10^8 pc.

Mills: There is really a tremendous amount which could be said about this most interesting and provocative paper by Professor Ryle, but since the discussion must necessarily be limited, I would like to make just a few remarks about the N-S relation. First, it is very gratifying to see that the Cambridge *observational* results have now approached the Sydney ones so closely. In 1955, when the 2C-catalogue gave a source count slope of -3.0 and the Sydney catalogue a slope of -1.8, it seemed that any hope of reconciliation was impossible. It now appears to me to be distinctly possible. Secondly, while it is likely that the present Cambridge results, as far as they go, are better in many ways than the Sydney ones – since their instrument has about twice the resolution and, consequently, better sensitivity – I think that there is some point in investigating the remaining small differences. The different philosophy of observation underlying the two surveys may be the cause of difference rather than errors in actual measurement.

It is perhaps rather surprising that, over the upper part of the range covered in the Cambridge counts, our final results gave a slope of −1.85 (i.e., even steeper than Cambridge). However, our best estimate of the slope was about −1.5, with considerable uncertainty. The reduction occurred because of (i) an allowance for instrumental effects, and (ii) because we extended our counts to brighter galaxies, for which the slope was considerably less than −1.5. (The starting point of the Cambridge counts represents the bottom of a downward kink in our curve.)

Finally, I would comment on the inclusion of multiple sources as "extended" sources in our counts. This has been criticized by several people, but in my estimation this procedure – to overcome the effects of counting multiple radio sources singly when they are close, and in groups when they are distant – is superior to the correction method necessary in the Cambridge analysis. As we discussed in my initial paper on the subject, effects of such clustering, either chance or physical, can only serve to *increase* the observed slope, whatever counting procedure is adopted.

I rather suspect that the Cambridge catalogue still omits some large or multiple sources and, as evidence, I would like to show a slide showing the extent to which our catalogue is confirmed by 3C and by the California Institute of Technology results. The 3C-survey appears very incomplete.

Ryle: I think I must point out that the large slope of the N-S curve obtained for individual sources in the 2C-catalogue had already been corrected in 1959 by the 3C-survey, the results of which are entirely consistent with those now presented (the slope was −2.0 before correcting for the presence of extended sources). However, the statistical method of analysis which was first used in the 2C-survey was not subject to these errors of source-counting.

As regards Dr. Mills' second point, our new analysis has been restricted to $S < 100 \times 10^{-26}$ W m^{-2} (c/s)$^{-1}$, because even when observations are available for a large part of the sky, the statistical errors become large for more intense sources.

Dr. Mills' third remark on the effect of multiple sources does not seem to me correct. The analysis made by Miss Leslie showed there to be very little nonstatistical clustering; her observations set an upper limit of 4.5 per cent to the possible correction in source counts at any point of the N-S curve. If nonstatistical associations of sources are counted as single sources, it is evident that weak sources are removed, and intense sources are added, and the apparent slope of the log N-log S curve is, therefore, decreased.

The possibility of errors due to the presence of sources of low surface brightness has been avoided in the new observations because we are able to detect sources with $T_b \geq 100°K$; sources of lower surface brightness would only cause errors in the counts at $S < 0.1 \times 10^{-26}$ W m^{-2} (c/s)$^{-1}$.

Priester: I would like to emphasize that we face great difficulties when we try to explain the steep slope of –1.8 in the number-flux diagram from the viewpoint of relativistic cosmology. It is clear in advance that we cannot explain the data if we assume that we have a constant percentage of radio galaxies among the galaxies during the entire past. What we need in order to explain the observations is an evolutionary effect which gives the galaxies in the past (or, let me say, the young galaxies) a much greater tendency to become radio galaxies. Some years ago we believed that the collisions hypothesis could provide such a tendency because the number-density of *radio* galaxies would, of course, increase proportionately to the square of the number-density of galaxies, if we go into the past. In 1958, I calculated a large number of such cosmological models, but only an *extreme* model was able to fit the steep slope of –1.8. Unfortunately, this model has a mean density of some 10^{-27} gr/cm^3. That is too much by a factor of 100 or even more. So this explanation has to be ruled out. But still we have to find a process which provides a stronger increase of the frequency of radio galaxies in the past, an increase more rapid that is provided by proportionality to the square of the number-density of galaxies. In my opinion, this would be due to an evolutionary process which would make it highly probable that a young galaxy would be a radio source.

Ryle: We also have tried unsuccessfully to explain the observations in terms of the extra percentage of radio galaxies which would occur if they were due to collisions. I would agree with Dr. Priester that one must suppose either that radio galaxies were more numerous, or, alternatively, more powerful, in the past.

Von Hoerner: I think there is no doubt that a *reliable* survey would be extremely important, but, from what we have just heard, the agreement between the existing catalogues is not too good. Out of this situation I would like to suggest for future surveys three points:

1. At the resolution limit, the number of beam areas per source, μ, is connected with the signal-to-noise ratio with respect to the fluctuation of the faint background sources, q, by $\mu = 3q^2$. Some experiments with random numbers have shown that one should not take q smaller than 5. This means that one should not have less than 75 beam areas per source. If the present catalogues are cut down to this limit, then the agreement, indeed, is very good, but only 1/3 of the brighter sources remain.

2. Each survey should be made in, say, three frequencies. The positions of side-lobes depend on frequency; those of sources do not.

3. Each survey should be imitated, as closely as possible, by a large-scale Monte Carlo experiment.

Ryle: I think that Dr. von Hoerner's remarks on the agreement between existing catalogues refer to the 3C, the Mills, and the Owens Valley observations. The new observations which I have been reporting were made with a radio telescope of considerably greater power than that of any of those instruments.

As regards the relation between the number of sources per beam area and the signal-to-noise ratio which should be adopted, this depends on *(a)* the form of the log N - log S curve, and *(b)* whether it is intended to interpret the records in terms of individual sources which are subsequently counted, or whether the statistical properties of the record are used directly.

In our recent observations, we analyzed individual sources with $S \geq 2 \times 10^{-26}$ W m^{-2} (c/s)$^{-1}$ at this level there is about 1 source per 30 beam areas. The uncertainty due to noise fluctuations has a root-mean-square value of 0.08×10^{-26} W m^{-2} (c/s)$^{-1}$, and the uncertainty due to weak confusing sources introduces an uncertainty of 0.3×10^{-26} W m^{-2}(c/s)$^{-1}$ in the flux-density of any particular source. The errors discussed by Dr. von Hoerner do not, of course, arise with the statistical method of record analysis used by Hewish, in which the observations are compared with records obtained by a Monte Carlo method.

An analysis of the effects of side-lobes in our observations has been given in references [18,22] in my communication.

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New Observations of Importance to Cosmology

PURPOSE AND PLANS FOR THE OBSERVATIONAL PROGRAM

On September 5, 1936, the first exposures were made with the 18-inch Schmidt telescope on Palomar Mountain. The writer has used this telescope since, and, subsequently, the 48-inch Schmidt and the 200-inch Hale telescope with the following major goals in mind:

a) To fill the lacunae in our knowledge of the material content of the universe. In this effort, I concentrated my attention on the discovery of supernovae, novae, and unusual types of stars such as the Humason-Zwicky (HZ) stars, as well as the discovery of dwarf galaxies.

b) To investigate the large scale distribution of matter in the universe, with particular attention to the distribution of galaxies and of clusters of galaxies.

c) To prove the existence of inter-galactic matter and to show that it is composed of gases, dust, stars, groups of stars, and pygmy and dwarf galaxies, which previously had escaped attention.

d) To gain more knowledge of the laws which govern cosmic matter and radiation and to elucidate in particular: (i) the nature of the universal red-shift; (ii) the problem of the range of validity of Newton's law of gravitation and the question of whether or not the general theory of relativity must be replaced by a more adequate theory; (iii) the possible existence of terrene and contraterrene matter (recently renamed matter and antimatter).

Several of these programs were in part supported by funds from the Office of Naval Research, by the National Science Foundation, and by various observatories outside of the United States. On the supernova search and on the studies of the distribution, the luminosities, and the spectra of galaxies, Dr. E. Herzog, Dr. M. L. Humason, Dr. A. M. Gomes and Messrs. H. S. Gates, and P. Wild have collaborated with me for several years.

THE LARGE SCALE DISTRIBUTION OF CLUSTERS OF GALAXIES

In Figures 1 to 5, the distribution of clusters of galaxies in the area of the north galactic cap from Decl. $-3°$ to $+15°$ is shown. Near clusters (N) are those with symbolic velocities of recession V_S in the range $0 < V_S < 15{,}000$ km/sec. Medium distant (MD), distant (D), very distant (VD), and extremely distant (ED), clusters of galaxies have values of V_S in the respective ranges from 15,000 to 30,000 km/sec, 30,000 to 45,000 km/sec, 45,000 to 60,000 km/sec, and

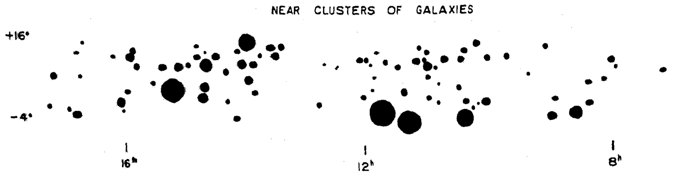
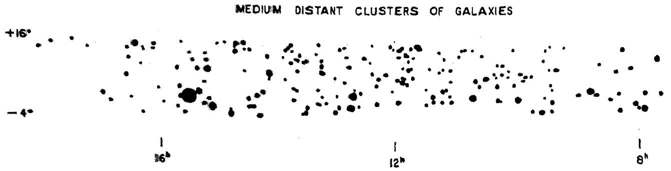


Figure 1.



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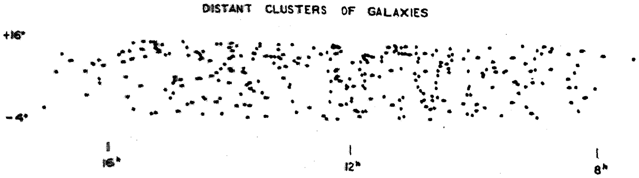

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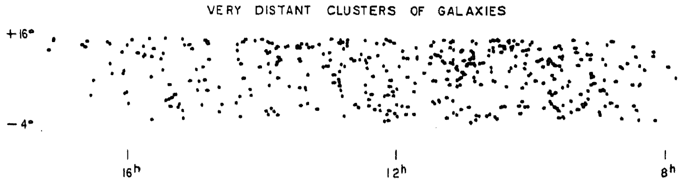

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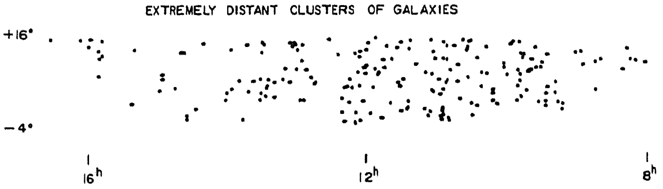

Figure 5.

Figures 1 to 5. Distribution on the celestial sphere of the various types of clusters in Dec. $-3°$ to $+15°$ and R.A. 8^h to 17^h.

greater than 60,000 km/sec. Some of the important conclusions to to be drawn from these diagrams are as follows.

1. In extended regions where the interstellar and the inter-galactic effects of obscuration are not spotty, the distribution of the ED and the VD clusters is remarkably uniform and random. The region from R. A. 10^h to 12^h clearly illustrates the validity of this statement with respect to the distribution of the ED clusters. Likewise, the distribution of the VD clusters shows great uniformity and randomness in the region of R. A. 9^h to 12^h, except, of course, for the systematic effect due to increasing interstellar absorption with decreasing galactic latitude. We conclude from these observations that there exists no clustering of clusters of galaxies. We further conclude that the range of the gravitational forces is of the order of 20 million pc. The quantum mechanical "particle picture" of gravitation consequently leads to a rest mass of the gravitons of about 5.7×10^{-64} gr., as well as to the two major additional conclusions that the general theory of relativity must be replaced by a more adequate theory and that the universal red-shift is due to some phenomenon other than the Doppler effect.

2. Wherever there are rich and compact nearby clusters of galaxies, the numbers of observable VD and ED clusters are materially depleted. This indicates that inter-galactic dust exists in observable amounts and that this dust is partially concentrated in rich clusters. The regions in which the Virgo Cluster, the Coma Cluster

and the Ursa Major Cloud of galaxies are located provide a good illustration for our statement. The apparent distribution of MD and D clusters is not materially affected by the presence of nearby clusters.

3. The richest field of galaxies and of clusters of galaxies which I have so far found is around R.A. $11^h17^m10^s$ and Dec. +35° 28!9 (Epoch 1950). This field, which covers an area of 36 square degrees, contains about 110 clusters of galaxies and 120,000 galaxies as observable on limiting 103 a-E plates obtained with the 48-inch Schmidt telescope. The distribution of compact, and of medium compact clusters over distances L which correspond to symbolic velocities of recession V_S in the range from zero to 60,000 km/sec in the above field is proportional to L^2. This field, in contradistinction to most others is, therefore, not noticeably affected by inter-galactic obscuration within the distance range mentioned. Also, the distribution in depth as well as the distribution of the apparent angular sizes of the clusters corresponds to that expected for a uniform distribution in a nonexpanding universe.

4. In Figure 6, an illustration is given of how the presence of two staggered rich, compact clusters affects the distribution of VD and ED clusters. The principal condensations of the two near clusters are respectively at R.A. $11^h08^m\!.2$, Dec. + 28°57′ and R.A. $11^h13^m\!.8$, Dec. + 29°32′ (Epoch 1950). All of the other nearby clusters shown on the diagram are of the open or medium compact type, which do not seem to contain as much inter-galactic dust as the compact clusters and which, therefore, eliminate fewer of the VD and ED clusters.

SPECTRA AND VELOCITY DISTRIBUTION OF GALAXIES IN GROUPS AND IN CLUSTERS

The spectra of about 200 galaxies in groups and in clusters have been photographed with the prime-focus spectograph of the Hale telescope at dispersions of 185 A/mm and 400 A/mm respectively. A series of articles will be sent jointly with Dr. M. L. Humason to the *Astrophysical Journal,* of which the first two have already been published [2, 3].

We give here only a selection of unpublished results which we derived for the nearby cluster of galaxies designated as Cl. 0123-0138. This cluster was chosen for a thorough investigation because: *(a)* it is of medium size and population and it, therefore, serves well for a check on the mass-luminosity ratio of a cluster which lies halfway between small groups of galaxies on the one hand, and the giant clusters, such as the Coma Cluster, on the other hand; *(b)* because it seems to stand alone, which enabled me to pick its mem-

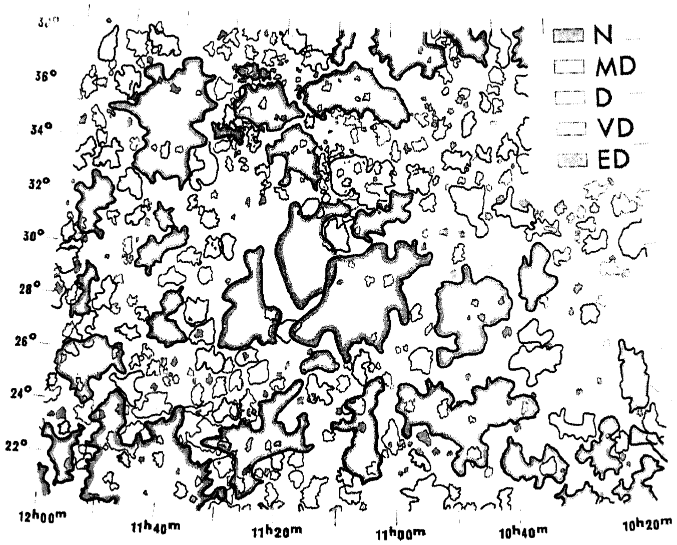

Figure 6. Map of clusters of galaxies between the limits of Dec. and R.A. shown. Original is in color. Generally, N, MD, D, VD, and ED clusters are successively of smaller size.

bers with great assurance by direct inspection; *(c)* because its members appear to be uniform in type, that is, elliptical or Sa with very few irregular galaxies or open spirals; and *(d)* because intergalactic luminous clouds are clearly visible in the center of the cluster, which also contains two medium radio sources.

About 300 members of Cl. 0123-0138 can be clearly detached from the background population on limiting red plates obtained with the 48-inch Schmidt. The cluster, however, contains about one thousand members in the range of apparent photographic magnitudes $13 < m_p < 19.5$.

Fifty-two of the brighter member galaxies have been investigated so far for their distribution (see Figure 7) and for their diameters, apparent photographic magnitudes, and spectra. Figures 8 and 9 show the distribution of the values of m_p for the 52 objects chosen, as well as the symbolic velocities of recession of 43 of them.

Actually, in Figure 8, as more and more member galaxies are included, the luminosity function in the range $13 < m_p \leq 16$ increases monotonely and smoothly. Some of the weird humps which other authors have recently claimed to have found for the luminosity functions of galaxies in the clusters in Coma, Corona Borealis, etc., have, therefore, no universal significance. According to our own investigations of these clusters, these humps are only obtained if groups of galaxies which are in the line of sight of the clusters

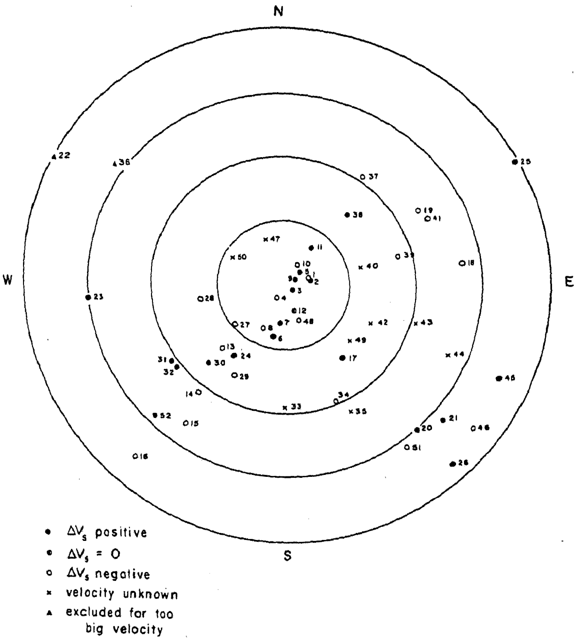

Figure 7. Distribution and velocity dispersions of 52 galaxies in Cl. 0123-0138.

investigated are mistakenly included in the counts. The mistakes of this kind can always be traced either by checking on the values of V_S or, more laboriously, by counting in a range of magnitudes least affected by the superposed groups. As I have maintained for the past 25 years, our general conclusion remains that the general luminosity function which includes all types of galaxies in a large volume of space is a monotonely rising one and that there is neither a single, nor any multiple, hump maximum as in Hubble's original luminosity function or in those of some more recent investigations.

Figure 7 represents the symbolic velocities of recession and shows the following significant features:

1. Except for the velocities of the galaxies Numbers 22 and 36,

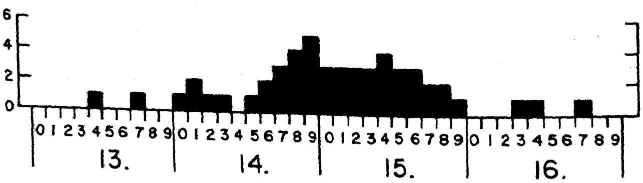

Figure 8. Histogram of numbers of galaxies (ordinates) against apparent photographic magnitudes in Cl. 0123-0138.

the velocity distribution is Maxwellian, around an average

$$\overline{V}_S = 5321 \text{ km/sec} , \qquad (1)$$

and a velocity dispersion

$$(\overline{\Delta V_S{}^2})^{1/2} = 406 \text{ km/sec} , \qquad (2)$$

2. We exclude, for the time being, Numbers 22 and 36 from the cluster because of their locations and their velocities. For the remaining 41 galaxies we have the following significant facts:

The average velocities and the velocity dispersions for the brighter ($m_p < 14.9$) and the fainter ($m_p \geq 15.0$) galaxies are respectively

$$\overline{V}_S = 5254 \text{ km/sec}, \qquad (\overline{\Delta V_S{}^2})^{1/2} = 360 \text{ km/sec}, \qquad (3)$$

and

$$\overline{V}_S = 5392 \text{ km/sec}, \qquad (\overline{\Delta V_S{}^2})^{1/2} = 439 \text{ km/sec}. \qquad (4)$$

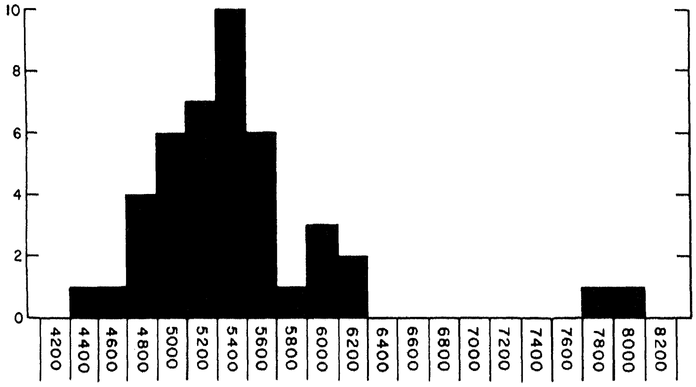

Figure 9. Histogram of numbers of galaxies (ordinates) against velocities of recession in km/sec (abscissae) in Cl. 0123–0138.

As expected for a Maxwell-Boltzmann distribution, equipartition of energy is indicated by this result, which is, actually, the first result of this type found as applying to galaxies of different mass in a statistically stable cluster. In such a cluster, there occurs also segregation of masses, the heavier galaxies being concentrated more toward the center. In accordance with this, the symbolic velocities of recession and their dispersions in a central circle and three circular rings of equal widths are found as follows:

Central Circle: $V_S = 5280$ km/sec, $(\overline{\Delta V_S{}^2})^{1/2} = 237$ km/sec . (5)

First Ring: $V_S = 5305$ km/sec, $(\overline{\Delta V_S{}^2})^{1/2} = 396$ km/sec. (6)

Second Ring: $V_S = 5250$ km/sec, $(\overline{\Delta V_S{}^2})^{1/2} = 482$ km/sec. (7)

Third Ring: $V_S = 5448$ km/sec $(\overline{\Delta V_S{}^2})^{1/2} = 533$ km/sec. (8)

These results confirm that the fainter galaxies, which become more numerous toward the outskirts of the cluster, have the higher velocity dispersion and, presumably, about the same average kinetic energy as the brighter galaxies. Why the velocity dispersion for still fainter galaxies will not grow any more need not be discussed here. The second conclusion we may draw from our analysis is that our cluster is not expanding, but is stationary, as is already evident from the various facts presented above. Indeed, if the cluster were expanding, it could not only not have any concentration in the center, but the dispersion of the symbolic velocities of recession would tend toward zero at its periphery.

Since the cluster 0123-0138 appears to be elliptical in outline, with the major axis running from SW to NE, I made a test for rotation, dissecting the cluster by a straight line running through from NW to SE through its center. The average symbolic velocities of recession and their dispersion to the NE and to the SW of this line are respectively

NE: $\overline{V}_S = 5359$ km/sec, $(\overline{\Delta V_S{}^2})^{1/2} = 448$ km/sec, (9)

and

SW: $\overline{V}_S = 5285$ km/sec, $(\overline{\Delta V_S{}^2})^{1/2} = 357$ km/sec. (10)

There is, therefore, no indication for any rotation of our cluster around the said axis.

From the velocity dispersion, as observed, and applying the virial theorem for a stationary cluster, we derive an indicative mass-luminosity ratio of the order

$$\mathfrak{M}_{cl}/L_{cl} \cong 400\mathfrak{M}_{\odot}/L_{\odot} = R\mathfrak{M}_{\odot}/L_{\odot}\,. \tag{11}$$

As defined in the above quoted papers by Zwicky and Humason, indicative quantities refer to an arbitrarily adopted red-shift constant of 100 km/sec/Mpc. For the summed-up apparent luminosity of the 52 brightest galaxies in the cluster, we find $m_p = +10.7$ corresponding to an indicative absolute luminosity $M_p = -22.93$ corresponding to $L = 2.2 \times 10^{11}\, L_{\odot}$.

The value of R for our medium rich cluster, therefore, lies between those of small groups which are in the range from 10 to 40 and those of very large, compact clusters which are of the order of 1000. The larger the cluster, the larger is the mass in it which remains unaccounted for by the individually distinctly observable galaxies. By including all of the member galaxies in the apparent magnitude range $13 < m_p < 19.5$ the ratio R for our cluster could possibly be reduced to 200. The unaccounted-for mass may be of four types: *(a)* inter-galactic dust, which can contribute very little unless there were much mass in particle sizes greater than one micron; *(b)* protons and electrons of fairly high energies; *(c)* neutral hydrogen molecules; *(d)* inter-galactic stars and small groups of stars, or even pigmy and dwarf galaxies.

For the time being, I favor the case *(d)*. Inter-galactic luminous matter is, indeed, present in the center with a surface brightness greater than about 23 magnitudes per square second of arc. Luminous matter at the 25th magnitude per square second of arc could not possibly be seen by present means. Spread over the whole area of the cluster which covers several square degrees, matter of this surface brightness would, nevertheless, integrate up to a total apparent magnitude $m_p \cong +7.0$ or 3.5 magnitudes brighter than the sum of the individual galaxies. The final value for R could, therefore, be reduced in this way by a factor 25, bringing it down to $R \sim 10$, in accord with the values found for individual galaxies and small groups.

SUPERNOVAE

The renewed search for supernovae, as organized by the writer with the aid of funds from the National Science Foundation and various foreign governments, is now in full swing and some preliminary results of importance have been achieved. The observatories at Palomar, Mount Wilson, Tucson, Kitt Peak, Tonantzintla, Cordoba, Meudon, Berne, and Asiago are cooperating either in the direct search or in observations of spectra and of magnitudes. In general, the results reported upon in the reviewing article by F. Zwicky [4] have been confirmed. Among the new results gained the following may be briefly mentioned.

a) To the Types I and II of supernovae, which were found in the original systematic search from 1936 to 1941, at least two more types must be added, which are distinguished by their spectra and their light curves.

b) The new data generally confirm the observations originally made on supernovae of the Type I, except that new features have made their appearance. Some of these follow.

The light curves are more varied than was originally thought. Their "straight-line" sections are hardly straight lines, especially when plotted in different color ranges, and their rates of decline may vary by a factor of almost two. There is no basis in fact for associating these declines with the decay of any radioactive isotopes.

Some rather rapid changes in the spectra have been observed. These refer, first, to rapid changes of the intensities of the broad emission bands, and, second, to sharp features and patterns within the broad bands. These sharp features which were looked for systematically may last from one to several hours and they are thought of as caused by local shock waves and explosive actions in localized spots of the gas clouds which are being ejected. There is hope that these sharp and temporarily recurring features will enable us to identify the origin of the broad bands in the spectra of supernovae of Type I which are now completely mysterious.

Supernovae of Type I have appeared in all types of galaxies and have been found particularly at very great distances from the central parts of elliptical galaxies.

Preliminary investigations indicate that the absolute magnitudes of supernovae of Type I have a dispersion of less than a magnitude. If true, this fact will enable us to establish a relative, as well as an absolute, cosmic distance scale which is more reliable than any of the scales currently in use.

c) Supernovae of Type II have been found which hover around maximum brightness for several weeks and which have an absolute magnitude similar to that of supernovae of Type I. According to Dr. J. L. Greenstein, the spectra exhibit features which make it doubtful that this type of supernova originates in a star–a conclusion which is strengthened by the fact that the energy radiated in the first hundred days seems to be larger than could be accounted for by the explosion of a star. As the author has suggested [1], some supernovae may, thus, originate in giant collapsing gas clouds.

d) Two spectra obtained for the supernova 1961 in NGC 3003 show features never before found in any other supernova. This suggests the existence of a Type III, to which a Type IV must be added because of the low brightness and the light curves of the objects in M 101 and NGC 5236.

e) The over-all frequency of supernovae of all types in the brighter galaxies remains at the original estimate of one supernova per galaxy per 360 years. The frequency, however, seems to be smaller for members of the large compact clusters of galaxies such as the Coma Cluster. On the other hand, there are individual galaxies with a high rate of supernovae. For instance, three supernovae appeared during the past fifty years in each of the following: NGC

3184, 4321 and 6946; while two supernovae were found in the same period in each of the following: NGC 2841, 4303, and 5236.

f) A comparison of the magnitudes of supernovae of Type I in nearby galaxies and in five clusters of galaxies leads to a preliminary value of the red-shift constant of 175 km/sec per million parsecs. This determination is free from most of the almost insuperable difficulties which plague all of the other methods of the determination of the Lundmark-Hubble constant.

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DISCUSSION

Minkowski: Baade took a plate of the supernova in IC 4182 a few years back and could not see it. If it is now of 23rd magnitude, do you think it may have brightened? Such a behavior may have been observed by Simon Marius, who claims he saw Tycho's nova of 1572 again at the time of Kepler's nova of 1604. The time interval would be just right.

Zwicky: I think you have an idea there.

Abell: How many clusters have you in each of your distance groups, i.e., N, MD, D, VD, and ED?

Zwicky: See our coming paper on the subject. In all groups, 1400 clusters.

Abell: May I, tonight, have a copy of each of your plots showing the cluster distribution at various distances? Or if not, may I borrow your figures for tonight? Or may I have information on your statistical study to inspect?

Zwicky: The reductions used on the figures will appear in the volumes accompanying each volume of the catalogue. The catalogue will be available at the California Institute of Technology bookstore.

De Vaucouleurs: You have indicated that the diameters of the largest clusters are of the order of 20 Mpc on a distance scale for which $H = 100$ km/sec/Mpc; this result does not conflict with present estimates of the maximum sizes of superclusters on the same distance scale. The disagreement is merely one of terminology, which is trivial.

Regarding absorption effects in nearby clusters, there is a negative correlation between the surface density of Shapley-Ames galaxies ($m < 13$) and the Hubble counts ($m < 20$) at galactic latitudes $b > +40°$ which tends to support your findings from clusters.

Tifft: What criteria are used to assign any given cluster to a given distance group?

Zwicky: Estimated apparent luminosities and diameters of the 50 brightest galaxies, combined with estimated absorption by interstellar or inter-galactic dust.

Thackeray: May I suggest that the supernovae with flat maxima may resemble Eta Carinae which, as we know, remained bright for some centuries before reaching its maximum at about −13 to −14? For some time, I have suggested looking for such supernovae on plates of extra-galactic nebulae separated by intervals of some decades. Perhaps this is being done?

Zwicky: I think that we have had checks of this sort of thing since 1936.

Ambartsumian: I should like to emphasize the importance of the new data on supernovae, obtained by Zwicky and his group. The Humason supernova of 1960 in the Coma Cluster was found on plates of the Burakan Observatory, taken during the three months before the discovery and it was found that, during these months, its brightness was changing very slowly (flat maximum).

Zwicky: We should like to get these data from Dr. Ambartsumian.

Oort: Why doesn't the empty hole in the region of the Virgo Cluster which you found in the extremely distant clusters show up also in the groups of less distant clusters?

Zwicky: Because 0.3 to 0.4 magnitudes less would not make MD or D clusters unidentifiable, while it excludes the ED and some VD clusters.

Arp: Do the three supernovae which have stayed at maximum for a long time have Type I spectra?

Zwicky: They have Type II spectra.

Arp: Yes. In that case we should be careful about accepting them as a new kind of supernova. Type II supernovae show a considerable range in decay time. These three could well be long decay-time members of this group.

Zwicky: The presence of the same bands, with different relative intensities and different light curves would suggest that we classify these supernovae as different types.

V. Rubin: The increase in velocity dispersion with increasing distance from the center of the cluster would imply that the fainter galaxies are observed at greater distance from the center. Is this effect observed in the other five clusters you observed in clear areas?

Zwicky: Segregation is observed in many clusters but we do not have enough spectra in any of them except in Cl. 0123-0138.

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The Distance Scale

I. INTRODUCTION

This paper describes recent progress in the determination of distances to galaxies and the consequent evaluation of the Hubble expansion rate. Significant advances have recently been made in the problem due primarily to (1) the isolation and semitheoretical understanding of the region of instability for Cepheid variable stars in the H-R diagram; (2) a rather precise calibration of the period-luminosity-color relation for Cepheids in the solar neighborhood by using those few variables which occur in galactic clusters; (3) extensive photoelectric measurements of very faint stars in, and near, many galaxies in the Local Group; (4) precision photometric studies of the Cepheids in the LMC, SMC, M 31, and IC 1613; and (5) results of modern measurements of other distance indicators such as novae, brightest stars, RR Lyrae variables, and the sizes of the H II regions.

II. THE CEPHEID VARIABLES

The most significant progress in the understanding of the Cepheid phenomenon has come from the isolation of a narrow region of instability in the H-R diagram of width $\Delta(B\text{-}V) \simeq 0^{\mathrm{m}}.2$, which slants upward and to the red in the diagram starting at $M_V \simeq +4$, $B\text{-}V \simeq 0.0$, and extending to $M_V \simeq -6$ at $B\text{-}V \simeq 0.9$, It is now almost certain that all Cepheid variables are found in this region, including the classical Cepheids, the W Vir "Type II" Cepheids, the RR Lyrae stars, and the very short-period, low-luminosity Cepheids such as AI Vel and CY Aqr. But because this region is of finite width in B-V, there *cannot* be a dispersionless relation between period and luminosity. It can be shown [1] that a width of the instability strip of $\Delta(B\text{-}V) = 0^{\mathrm{m}}.2$ requires that the period-luminosity (P-L) relation show a scatter of about 1^{m}. The brightest Cepheids of a given period should be bluer than average, while the faintest should be redder. Because the predicted dispersion in the P-L relation was so great, the Cepheids were rejected [1] as a primary distance indicator in a discussion of the problem in 1958. This rejection was considered provisional until some parameter such as amplitude of the light variation or the intrinsic color could be used to correct for the scatter in the P-L relation and, as a result, give a calibrated luminosity value for a given period. Recently, the observational data and discussion by Arp [2] and by Kraft [3] for Cepheids in the SMC and in the Galaxy have indeed shown that those variables with the maximum amplitude of light variation for a given period lie in the middle of the instability strip and, therefore, in the middle of the scatter of the P-L relation. Furthermore, the intrinsically bluest Cepheids of a given period are observed to be brighter than the intrinsically reddest, as required by the model. Consequently, there does exist now a method of finding distances to galaxies by the observed P-L relation, *with its scatter,* by taking into account the intrinsic color and amplitude relations within the scatter. The procedure to be followed in this paper is to adopt the classical Cepheids as the primary distance indicator, and then to see how consistently the other distance indicators agree from independently determined data.

Arp [2] has given complete data for 64 Cepheids in the SMC whose light curves in B and V were obtained from photoelectrically calibrated sequences. The slope of the P-L relation in both B and V magnitudes was very well determined, but no direct information on the zero point in absolute magnitude of the P-L relation can be found from these data. Kraft [3] has discussed the zero-point calibration using the absolute magnitudes of five Cepheids which occur in galactic clusters (EV Sct, CF Cas, U Sgr, DL Cas, S Nor). He re-examined Arp's slope determination from the SMC data, used color and period data for a number of Cepheids in the solar neighborhood, and used the theory of the instability strip mentioned previously. Kraft's resulting relations, valid for the center of the strip and, therefore, valid for the center of the scatter in the P-L relations, are:

$$\langle M_V \rangle = -1.67 - 2.54 \log P \tag{1}$$

and

$$\langle M_B \rangle = -1.33 - 2.25 \log P \tag{2}$$

where the magnitude averages are taken as the mean intensity converted to a magnitude scale. We now adopt the calibration given by these equations at $\log P = 0.8$, but adopt, with Arp, a slope of 2.47 for the M_V relation. Distance moduli to galaxies in the Local Group are now obtained for those systems where Cepheids have been studied by modern methods. For the moment, we assume that the Cepheids in one galaxy are similar to those in another.

III. CEPHEIDS IN LOCAL GROUP GALAXIES

Cepheids have now been observed with photoelectric magnitude scales in the SMC (Arp), the LMC (Herstmonceux group comprised of Woolley, Alexander, and Eggen), M 31 (Baade and Swope with photoelectric calibration by Arp), and IC 1613 (Baade).

a) M 31. Baade and Swope [4] have worked four fields of M 31 for Cepheids. The first three fields are embedded in dust regions and have large amounts of internal absorption. The fourth field lies 96′ from the center along the south preceding part of the major axis and appears to be quite free of internal obscuration. Light curves for twenty Cepheids have been obtained in B and in V by Miss H. Swope and form the basis of the distance determination. She has very kindly made these extensive and hard-won data available before publication. The observed P-L relation, uncorrected for reddening and absorption in $\langle B \rangle$ and $\langle V \rangle$ is shown in Figures 1 and 2. The equations given in the diagrams have been fitted to the data using Arp's slope coefficients.

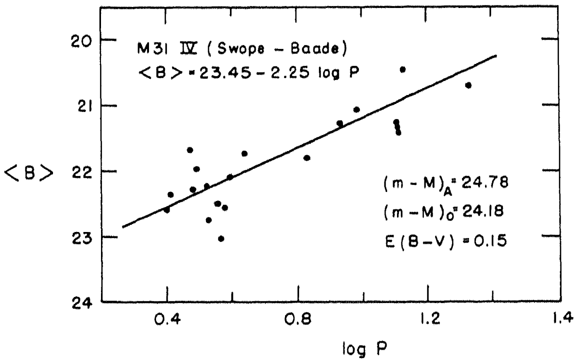

Figure 1. The period-luminosity relation in $\langle B \rangle$ mags. for Field IV in M 31. These data were furnished by Henrietta Swope before publication. The magnitudes $\langle B \rangle$ are intensity means taken over the light curves and reduced to the magnitude scale.

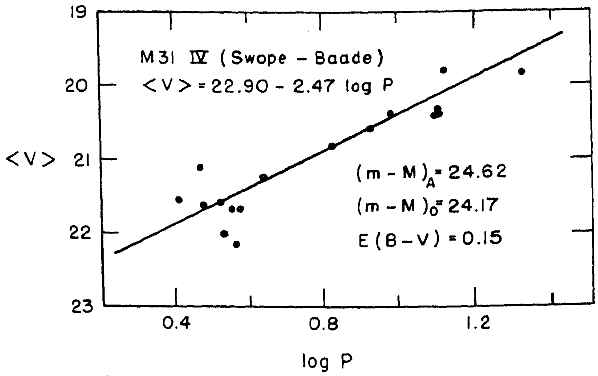

Figure 2. Same as Figure 1, but for $<V>$ magnitudes.

Equations (1) and (2) require that $<M_B> = -3.13$ and $<M_V> = -3.70$ at $\log P = 0.8$, which, when applied to the M 31 data, give apparent distance moduli of $(m\text{-}M)_{A\,B} = 24.78$ and $(m\text{-}M)_{A\,V} = 24.62$. The subscript A signifies "apparent." The subscripts B and V specify the type of magnitude used. The true modulus follows from these values once the reddening is known. Miss Swope [5] adopts $E(B\text{-}V) = 0\overset{\mathrm{m}}{.}15$, which is estimated in two ways. (1) The measured $B\text{-}V$ colors for the Cepheids at mean light are, on the average, $0\overset{\mathrm{m}}{.}15$ redder than reddening-corrected Cepheids in the Galaxy. (2) The color-magnitude diagram shows that the bluest main-sequence stars in field IV are $0\overset{\mathrm{m}}{.}15$ redder in $B\text{-}V$ than stars in the Galaxy. Adopting $E(B\text{-}V) = 0\overset{\mathrm{m}}{.}15$ gives $(m\text{-}M)_{OB} = 24.18$ for the true modulus from the B data, and $(m\text{-}M)_{OV} = 24.17$ for the true modulus from the V data.

Three features of Figures 1 and 2 should be noted. (1) The P-L slopes of 2.25 and 2.47, which apply to the SMC data, fit the M 31 data extremely well. (2) The scatter in the P-L relation is about 1^{m}, as required by theory. (3) The scatter is real because Miss Swope's detailed data show that the stars which are brighter than average are bluer than average, and vice versa. Furthermore, stars near the solid line have larger amplitudes than the stars which have large deviations from the line. Both of these features are required by the theory of the instability strip.

b) SMC. Arp's data [2] for the SMC give

$$<B>_0 = 17.45 - 2.25 \log P \tag{3}$$

$$<V>_0 = 17.05 - 2.47 \log P \tag{4}$$

which have been corrected for absorption and reddening. Arp gives $E(B\text{-}V) = 0\overset{\mathrm{m}}{.}06$, which agrees very well with an independent value of $E(B\text{-}V) = 0\overset{\mathrm{m}}{.}07 \pm 0\overset{\mathrm{m}}{.}01$ by Feast, Thackeray, and Wesselink [6].

These numbers then require that $(m\text{-}M)_{AB} = 19.02$; $(m\text{-}M)_{AV} = 18.95$; $(m\text{-}M)_{0B} = 18.78$; $(m\text{-}M)_{0V} = 18.77$. Equations (3) and (4) are plotted in Figures 3 and 4 as broken lines so that a comparison with the LMC data can be made.

c) LMC. The data reported here are first results of the Herstmonceux-DSIR study of Cepheids in two fields of the LMC. The data were very kindly furnished before publication by Woolley and Alexander and represent the reduction of only a small fraction of the final material. The photometry was based on a photoelectric sequence [7] obtained with the Radcliffe 74-inch telescope. The individual LMC stars are plotted in Figures 3 *and* 4, where no

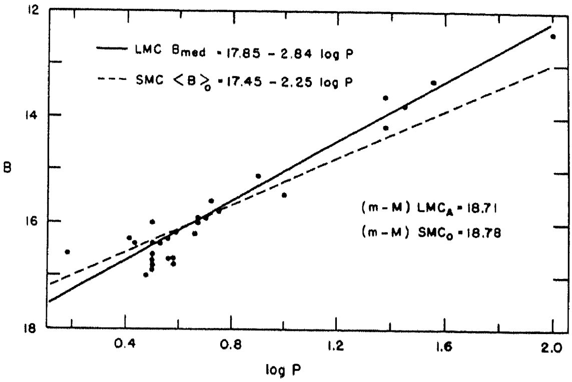

Figure 3. Preliminary data for 29 Cepheids in Field I near NGC 1783 in the LMC furnished by Woolley and Alexander before publication. Individual Cepheids in the LMC are shown by dots. The magnitudes are for median light. No correction for absorption has been made to the LMC data. The dotted line represents the absorption-free data from the SMC by Arp. The quoted modulus for LMC is apparent, determined at $\log P = 0.8$. The quoted modulus for SMC is true.

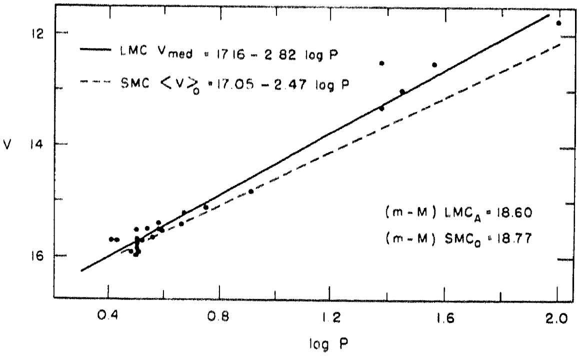

Figure 4. Same as Figure 3, but for V magnitudes.

correction for absorption has been made. (Remember that the broken lines representing Arp's SMC data in Figures 3 and 4 are corrected for absorption.) It is evident that the slope of the P-L relation for both B and V data is steeper for the LMC than for the SMC and M 31. Errors in the photometry can probably be excluded as an explanation. Eggen has checked and has extended the photoelectric sequences in a second season of observation. Gascoigne has checked the sequence near NGC 1783 with the result that, if anything, the stars at $V \simeq 16$, $B \simeq 17$ should be even fainter than obtained by Woolley and Alexander by about $0^{\mathrm{m}}.1$ to $0^{\mathrm{m}}.2$ relative to the bright stars. Such a correction, if real, increases the discrepancy in the slopes between the SMC and LMC. This discouraging state of affairs is not understood theoretically. The model based on the instability strip will produce a change of slope of the P-L relation if either the slope of the lines of constant period, or the slope of the strip itself, change in the H-R diagram. Sufficient data are not available at this writing to check these possibilities, and the reasons for such changes, if they exist, can only be guessed.

In any case, a distance determination to the LMC based on equations (1) and (2) cannot be made with certainty. If we apply equations (1) and (2) to the observed data at $\log P = 0.8$, then the following values result. The apparent modulus from the B data is $(m\text{-}M)_{AB} = 18.71$, and from the V data $(m\text{-}M)_{AV} = 18.60$. We adopt a reddening of $E(B\text{-}V) = 0.07$ from Feast, Thackeray, and Wesselink [6] and obtain true modulus values of $(m\text{-}M)_{OB} = 18.43$ and $(m\text{-}M)_{OV} = 18.39$. On the other hand, if we wish to apply equations (1) and (2) at $\log P = 1.6$, then $(m\text{-}M)_{AB} = 18.24$, $(m\text{-}M)_{AV} = 18.38$, $(m\text{-}M)_{OB} = 17.96$, and $(m\text{-}M)_{OV} = 18.16$. The first set of values ($\log P = 0.8$) is preferred because equations (1) and (2) are calibrated with the five galactic Cepheids whose mean $\log P$ is 0.78. We have little information about the calibration of equations (1) and (2) beyond $\log P \simeq 1.0$ in our Galaxy. Consequently, in what follows, we adopt $(m\text{-}M)_0 = 18.41$ for LMC as a provisional working value.

d) IC 1613. Baade began an extensive study of IC 1613 in 1936 with the 100-inch reflector. More than 60 Cepheids were found over the years. Eighteen of these were worked up completely in one color (m_{pg}). The photometric standards were determined from photographic transfers from SA 68. Baade worked for many observing seasons improving the sequences near each variable, and he continued the sequence transfers with the 200-inch telescope, using Baum's unpublished photoelectric data in SA 68. Baade was still not satisfied with the results in 1958 and requested that photoelectric calibration be made directly in IC 1613 itself. This has now been done to $B = 21.0$ and the scales are being extended to $B = 22.5$

in the 1961 observing season. However, the scale now available to $B = 21$ permits the correction of Baade's data at maximum light for the 18 Cepheids and the data are shown in Figure 5. We can now use Arp's data in the SMC for $B_{\max}$ to obtain the distance to IC 1613. Arp's data, uncorrected for absorption, requires for SMC

$$B_{\max} = 17.30 - 2.25 \log P. \tag{5}$$

The best fit to Baade's data corrected to the photoelectric scale in IC 1613 gives

$$B_{\max} = 22.61 - 2.25 \log P. \tag{6}$$

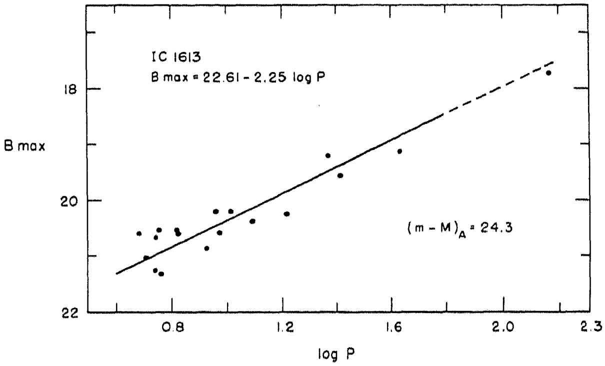

Figure 5. The P-L relation for IC 1613 from unpublished data by Baade. The magnitudes are B at maximum light and are based on a photoelectric sequence to $B = 21$.

Consequently, the difference in the apparent modulus between these galaxies, assuming equality of the Cepheids, is $\Delta(m\text{-}M)_{AB} = 5\overset{\mathrm{m}}{.}31$. With $(m\text{-}M)_{AB}$ for SMC of 19.02, we derive $(m\text{-}M)_{AB} = 24.33$ for IC 1613. Study of the U,B,V data for stars near IC 1613 on the plane of the sky gives $E(B\text{-}V) = 0\overset{\mathrm{m}}{.}03$, which gives $(m\text{-}M)_{OB} = 24.2$ for the true modulus of IC 1613. Note the remarkable agreement of the slope of the P-L relation of Figure 5 for IC 1613 with the slope of the SMC and M 31.

e) M 33. Studies of the Cepheids in M 33 have not yet been completed although work is in progress using a strong photoelectric sequence by Johnson and Sandage extending to $B = 22$. For the moment we take Hubble's result that the Cepheids in M 33 appear $0\overset{\mathrm{m}}{.}1$ brighter than in M 31 and adopt the Baade-Swope data for M 31 given earlier. The result is $(m\text{-}M)_{AB} = 24.68$ for M 33. Again, U,B,V photometry of many field stars in our Galaxy in the direction of M 33 shows that $E(B\text{-}V) = 0\overset{\mathrm{m}}{.}09$, which requires $(m\text{-}M)_{OB} = 24.3$ for M 33. Table 1 summarizes the data of this section.

TABLE 1

Distance Moduli to Members of the Local Group

Galaxy	$(m-M)_{AB}$	$(m-M)_{AV}$	$E(B-V)$	$\overline{(m-M)_0}$
M 31	24.78	24.62	0.15	24.18
M 33	24.68	.....	0.09	24.3
LMC	18.71	18.60	0.07	18.4
SMC	19.02	18.95	0.06	18.78
IC 1613	24.33	.....	0.03	24.2

We now adopt the $(m\text{-}M)_0$ values of Table 1 to compare with answers given by other distance indicators.

IV. THE BRIGHTEST STARS

Extensive photometric data have recently become available on the brightest stars in the Magellanic Clouds from the work of Feast, Thackeray, and Wesselink [6]. Less complete new data exist for IC 1613, M 31, and M 33, and for two members of outlying groups, NGC 2403 and M 101. The data are summarized in Table 2. The adopted moduli for NGC 2403 and M 101 are derived in section VIII from the sizes of the H II regions. The columns of Table 2 give (1) the galaxy name, (2) the identification of the brightest star, (3) the apparent B magnitude of the star (denoted by B_S), (4) the adopted apparent modulus $(m\text{-}M)_{AB}$ from Table 1, (5) the resulting absolute blue magnitude M_{BS}, (6) the integrated B magnitude of the galaxy B_G as taken from Holmberg [8] uncorrected for tilt effects, and, finally (7), the resulting absolute magnitude M_{BG} of the galaxy, again uncorrected for tilt effects. The data for B_S are from Feast, Thackeray, and Wesselink [6] for the LMC and SMC; from Baade (unpublished) for IC 1613; and from Hubble and Sandage [9] for M 31

TABLE 2

Brightest Stars in External Galaxies

(1) Galaxy	(2) Brightest Star	(3) B_s	(4) $(m-M)_{AB}$	(5) M_{BS}	(6) B_G	(7) M_{BG}
M 31	Var No. 19	15.7	24.8	−9.1	4.33	−20.5
M 33	Var B	15.2	24.7	−9.5	6.19	−18.5
LMC	HDE 33579	9.3	18.7	−9.4	0.63	−18.1
SMC	HDE 7583	10.3	19.0	−8.7	2.79	−16.2
IC 1613	Anon	17.1	24.3	−7.2	10.00	−14.3
NGC 2403	Anon	18.6	27.6	−9.0	8.80	−18.8
M 101	Anon	18.2	27.7	−9.5	8.20	−19.5

and M 33. The average M_{BS} for the five galaxies with M_{BG} brighter than –18 is $\overline{M_{BS}} = -9^{m}.3 \pm 0^{m}.2$ (m.e.). If we include the SMC in the sample, then $\overline{M_{BS}} = -9^{m}.2 \pm 0^{m}.3$ (m.e.). The dispersion is remarkably small and suggests that the brightest stars are good distance indicators if they can be isolated uniquely in systems beyond the Local Group. (However, the difficulties of this isolation are well known [1]). It is instructive to note that Hubble's calibration of $M_{pg,s}$ in 1936 gave –6.11, which is three magnitudes fainter than the present value.

Is a value of $M_{BS} = -9.3$ reasonable from what is known of supergiants in our Galaxy? Most O and B stars in young clusters and associations average only $M_B \simeq -7$. However, there are at least a half dozen stars in the Galaxy which are candidates for superluminosity of the order of $M_B = -9$. The data are given in Table 3.

TABLE 3

Some Superluminous Stars in the Galaxy

Name	*Spectra*	M_B	M_V	*Ref.**
No. 12 VI Cyg	B5 Ia^+	–9.7:	–9.5:	(1)
ζ^1 Sco	B1.5 Ia^+	–9.2	–9.0	(2)
β Ori	B8 Ia	–8.5	–8.5	(3)
ϕ Cas	F0 Ia	–8.6	–8.8	(4)
89 Her	F2 Ia	–9 to –10	–9 to –10	(5)
HD 161796	F3 Ia	–9 to –10	–9 to –10	(5)
ι^1 Sco	F2 Ia	–9 to –10	–9 to –10	(5)

*The references are as follows:

(1) Sharpless, S., *P.A.S.P.*, *69*, 239, 1957. Schulte, D. H., *Ap. J.*, 128, 41, 1958.
(2) Code, A. D., and Houck, T. E., *P.A.S.P.*, 70, 261, 1958.
(3) Morgan, W. W., Strömgren, B., and Johnson, H. M., *A.J.*, 59, 188, 1954.
(4) Pesch, P., *Ap. J.*, 130, 764, 1959.
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The highest luminosity is ascribed by Abt to the high latitude supergiants 89 Her and HD 161796 on the basis of his determination of electron pressures and an extrapolation of the correlation of log P_e with M_V. This correlation rests very heavily on ϕ Cas as the last calibrated point and, therefore, it depends on Pesch's work [10] which identifies ϕ Cas as a member of the cluster NGC 457. The M_V of star No. 12 in VI Cyg is uncertain because of the enormous absorption correction of $10^{m}.2$ which is required. The best-determined star is probably ζ^1 Sco by Houck (unpublished thesis) where there appears to be little doubt of the high absolute magnitude. Therefore, the conclusion is that the data of Table 2 for stars in other galaxies are in good agreement with present knowledge of high luminosity stars in our Galaxy.

V. NORMAL NOVAE

Arp's extensive photometry [11] of the normal novae in M 31, together with Th. Schmidt-Kaler's rediscussion [12] of the absolute magnitudes, is the basis of this section. We consider, first, the very brightest of the novae observed by Arp. Novae 1 and 2 of $5^{d}_{\cdot}2$ duration reached $m_{pg} = 15.7$ ($B = 15.8$). If $(m\text{-}M)_{AB} = 24.78$ given by the Cepheids, then $M_B = -9.0$ for both novae 1 and 2 at maximum. The absolute magnitudes of other novae in Arp's material of longer duration are fainter according to a life-luminosity relation (see Arp's Figures 31 and 32 in [11]).

Schmidt-Kaler's rediscussion [12] of the expansion parallaxes and the galactic absorption of eleven well-determined novae in the Galaxy gives an absolute calibration of the life-luminosity relation and, also, gives an independent check on the form of this correlation. The brightest galactic nova was Nova Pup 1942, which, according to Schmidt-Kaler, reached $M_{pg} = -9.4$ ($M_B = -9.3$). The next brightest was Nova Aql 1918 with $M_B = -8.7$ at maximum. The average of these two brightest is $M_B = -9.0$, in exact agreement with novae 1 and 2 in M 31 if we adopt the Cepheid distance modulus. If all 30 of Arp's novae are considered, then equally impressive agreement is reached. Schmidt-Kaler fitted Arp's life-luminosity relation to the eleven calibrated galactic novae and obtained the apparent modulus of M 31 from novae alone as $(m\text{-}M)_{AB} = 24.6 \pm 0.3$. This should be compared with $(m\text{-}M)_{AB} = 24.78$ from the Cepheid data alone. The conclusion is that the Cepheid data and the nova data agree to within the accuracy of each determination. And here it is important to realize that the steps for calibration of the Cepheids and of the novae are completely independent and, therefore, that this agreement is quite significant.

VI. RR LYRAE STARS

Outside our Galaxy, RR Lyrae stars are known only in the Magellanic Clouds. They are important not only as a check on the distance to the Clouds, but they also are the prime distance indicator for globular clusters in the Galaxy, and hence, are important in determining the integrated luminosities of these clusters. The "traditional" value of $M_V = 0.0$ for the RR Lyrae is based on statistical parallax determinations by R. E. Wilson and others, or on Shapley's first calibration using longer period Cepheids as an intermediate step.

Two new methods with potentially greater weight have been developed in recent years which give $M_V \simeq +0.5$ to $M_V = +1.0$ as the most probable value. Evidence is also accumulating which indicates that all RR Lyrae stars are not of the same M_V in different clusters.

One of the methods was developed by O. J. Eggen with his discovery of moving kinematic groups of stars. Most significant was his discovery that the subdwarf Groombridge 1830 is moving in space with a space vector parallel to RR Lyrae itself [13]. Eggen isolated other subdwarf members of the Gmb 1830 group and the skeleton of the color-magnitude diagram for the group was found. Distances were determined by the classical moving-group method and $M_V = +0.65$ was obtained for RR Lyrae. Eggen tentatively isolated four other moving groups containing subdwarfs and RR Lyrae stars with the result that $M_V = +0.4$ for TU UMa, $M_V = +0.8$ for SU Dra, $M_V = +0.8$ for X Ari, and $M_V = +0.2$ for W CVn.

The second method consists of observations of main sequences of globular clusters to determine distance moduli. The main-sequence stars, corrected for line-blanketing, are fitted to the Hyades age-zero main sequence and the modulus determined. Questions of the validity of the fit in view of the large difference in the metal abundance between globular cluster stars and Hyades stars are discussed in detail elsewhere, and the conclusion is that the method is valid to better than $\pm 0^m.2$. The available data for M_V of the RR Lyrae stars contained in clusters investigated to date are given in Table 4. The probable error of each determination is about $\pm 0^m.2$.

The grand average of the values in Table 4 and of the individual values for the five field variables previously mentioned is $\overline{M_V} = 0.75 \pm 0.23$ (m.e.). Although the calibration problem is certainly not solved, it appears that (1) $\overline{M_V}$ for RR Lyrae stars is

TABLE 4

Absolute Magnitude of RR Lyrae Stars in Globular Clusters

Cluster	*M_V RR Lyrae*	*Method*	*Reference*
M 13	+0.3	Main seq. fit	[13], [14]
M 5	+0.8	Main seq. fit	[15]
M 3	+1.0:	Main seq. fit	[16]
M 2	+1.0:	Main seq. fit	[17]
NGC 6397	+1.1	Main seq. fit	Eggen (unpub.)
ω Cen	+0.7:	Fit to M 13 upper	Eggen (unpub.)
47 Tuc	+1.0	Fit to γ Leo group	Eggen (unpub.)
M 15	+1.0	Main seq. fit	Sandage (unpub.)

fainter by more than $0^{m}.5$ from the "traditional" value of $M_V = 0.0$, and (2) there may be a real spread in M_V among the stars of this class.

It is now of interest to apply the above $\overline{M_V}$ to the RR Lyrae stars in the Magellanic Clouds. Tifft has recently completed photometry of the globular cluster NGC 121 associated with SMC but located far from the central regions. He reports the apparent magnitude of the horizontal branch to be at $V = 19.6$. If we adopt $\overline{M_V} = +0.7$, then $(m\text{-}M)_{AV} = 18.9$ for NGC 121. If the reddening is zero, then 18.9 is the true modulus. If $E(B\text{-}V) = 0.06$ as in Table 1, then $(m\text{-}M)_0 = 18.7$. Both values are in good agreement with the numbers given in Table 1 for the SMC.

VII. THE GLOBULAR CLUSTERS

Among the brightest objects which can be identified in external galaxies are the globular clusters. However, the spread in absolute magnitude among the sample of clusters in our Galaxy is about 4^{m}, a circumstance which makes their use as distance indicators difficult. An extensive study of globular clusters in our Galaxy and in M 31 has recently been completed by Kron and Mayall [18]. On the assumption that $M_P = 0.0$ for the RR Lyrae stars, these authors obtain absolute integrated magnitudes corrected for absorption for 42 globular clusters in the Galaxy. Photometry of 70 star clusters in M 31 was also obtained. These data permit, in principle, an independent determination of the distance to M 31 by matching luminosity functions.

The brightest clusters in our Galaxy are ω Cen with $M_P = -10.4$ and 47 Tuc with $M_P = -10.0$, if $M_P = 0.0$ for the RR Lyrae variables. However, results of the last section show that $M_V \simeq +0.7$. Furthermore, it is known that $P\text{-}V = 0.2$ to $P\text{-}V = 0.4$ for the cluster-type variables which makes $\overline{M_P} \simeq +1.0$. Consequently, the two brightest clusters in our Galaxy probably have $M_P = -9.0$ and $M_P = -9.4$, respectively. The brightest clusters in M 31 have apparent P magnitudes of about 14.5. Assuming $\overline{M_P} = -9.2$ for these clusters gives $(m\text{-}M)_{AP} = 23.7$ [or $(m\text{-}M)_{AB} = 23.85$ because $B\text{-}P \simeq 0.15$]. This modulus is brighter by about 1^{m} from the modulus of M 31 given by the Cepheids, the novae, and the brightest stars. The discrepancy seems to be real. Kron and Mayall show that nearly the same difference results if the entire histogram of the distributions of M_V or M_P for galactic globular clusters is compared with the histogram of the P or V distribution of clusters in M 31. Furthermore, the results of Hiltner for the clusters in M 33 show an even larger difference which amounts to nearly 3^{m}. We are, therefore, forced to the conclusion that globular clusters are very poor distance indicators because, at least with the present data, the absolute magnitudes apparently vary from galaxy to galaxy.

VIII. SIZES OF H II REGIONS

It has often been suggested that angular sizes of H II regions might be a statistical distance indicator. An observational program was started several years ago using the 200-inch telescope to obtain photographs of nearby Irr, Sc, and Sb galaxies to test this suggestion. In 1959, Sersic inspected the plates in the Mount Wilson and Palomar Observatories files, made an important discussion of the results [19], and derived a value of the Hubble constant from a calibration of the linear sizes of H II regions using the Local Group galaxies.

One of the calibration galaxies was M 33, for which an extensive series of photographs in 3 colors (*B, V,* and Hα) exists. Sersic identified all H II regions on these plates by blinking the Hα plate against the visual plate. He found 369 H II regions, most of which were smaller than 5 sec. of arc. Figure 6 is a histogram of the distribution of angular diameters of these 369 emission patches. The two largest are the well-known objects NGC 604 and NGC 595.

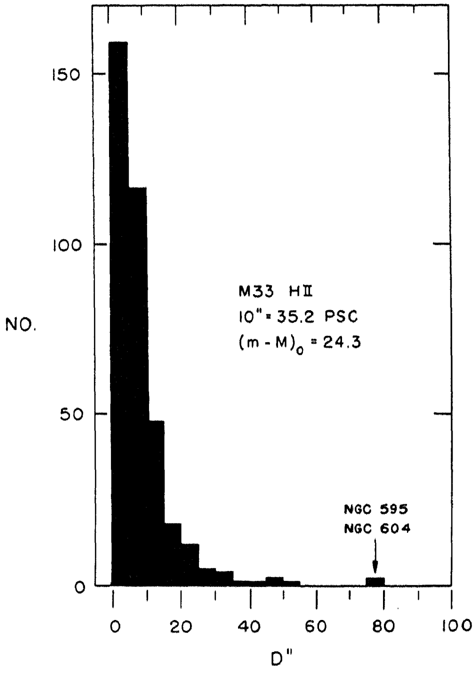

Figure 6. The distribution of angular sizes of 369 H II regions in M 33 determined by Sersic from plates taken with the 200-inch telescope.

M 33 and the LMC are the calibrating galaxies to obtain linear sizes of the largest and the mean of the five largest H II regions in galaxies of type Irr to late Sc. At the distance of M 33, 10 sec. of arc corresponds to 35.2 pc. Thus, most of the patches in M 33 are smaller than 20 pc in diameter, which is in agreement with the situation in the Galaxy.

The writer independently inspected all the plates in the files taken in Hα light (RG2 filter plus Eastman 103aE plates) and estimated angular sizes for a number of H II regions. The adopted calibration data on the writer's system of visually estimated diameters is given in Table 5 for M 33 and the LMC. Column 2 gives the angular diameter of the largest H II region (NGC 604 in M 33, 30 Dor in LMC); Column 3 gives the linear dimension based on $(m\text{-}M)_0 = 24.3$ for M 33 and $(m\text{-}M)_0 = 18.4$ for the LMC; Column 4 gives the angular diameter of the mean of the five largest H II regions; and Column 5 gives the corresponding linear diameter. We adopt $\overline{D}$ (largest) = 245 pc, and $\overline{D(5)}$ = 175 pc.

TABLE 5

Preliminary Calibration of Largest H II Regions

(1) Galaxy	(2) $d(L)''$	(3) $D(L)$ psc	(4) $d(5)''$	(5) $D(5)$
M 33	75	265	60	211
LMC	985	228	624	145
Adopt		245		175

These values were then applied to four distant galaxies beyond the Local Group given in Table 6. NGC 925 and NGC 4321 are distant enough so significant red-shifts occur. The power of the H II method is evident here because NGC 4321 in the Virgo Cluster has H II regions whose size of about 3″ is still well above the resolution of the plates. The method, therefore, promises to be

TABLE 6

Diameters of H II Regions in Four External Galaxies

(1) Galaxy	(2) $d(L)''$	(3) $d(5)''$	(4) $(m-M)_L$	(5) $(m-M)_5$
NGC 2403	15.6	11.5	27.6	27.6
M 101	14.5	10.1	27.7	27.7
NGC 925	6.1	3.7	29.6	30.0
NGC 4321	3.8	2.5	30.6	30.8

powerful in finding the value of the Hubble constant when enough data are collected.

The *m-M* values of Table 6 show excellent agreement in Columns 4 and 5, which represent the moduli determined from the largest H II region (assumed to be 245 pc in diameter) and from the mean of the first five (assumed to be 175 pc in diameter). Furthermore, the modulus of NGC 2403 of 27.6 agrees well with an early preliminary value given in 1954 by Sandage [20] based on novae, Cepheids, and brightest stars in M 81 when corrections are applied for the new value of the modulus to M 31. Furthermore, the modulus of 30.7 for NGC 4321 is in excellent agreement with the value for the Virgo Cluster obtained by Sandage in 1958 by correcting Hubble's 1936 data [1].

Finally, from the red-shift values for (1) NGC 925 itself, (2) for the mean of the NGC 1023 group of which NGC 925 is a member, (3) for NGC 4321 itself, and (4) for the mean of the Virgo Cluster members, the moduli quoted in Table 6 give a Hubble constant of $H = 82 \pm 18$ (m.e.) km/sec/Mpc. This will be discussed with other values in the next section.

IX. THE HUBBLE CONSTANT

The previous eight sections have been principally concerned with the distances to galaxies in the Local Group. Although these tell us nothing directly about the Hubble constant, several recent methods of determining H are based on properties of the Local Group members. Four independent determinations of H are in the literature since 1958. They are (1) Sersic's use of H II region diameters [19] based on blue plates taken with the 200-inch telescope, calibrated with M 31, M 33, LMC, SMC, and IC 1613; (2) van den Bergh's [21] luminosity classification of galaxies, calibrated from his classification of the Local Group galaxies M 31, M 33, SMC, LMC, and NGC 6822; (3) Holmberg's correlation [8] between average surface brightness and absolute magnitude, calibrated with M 31, M 33, and M 81; and (4) Sandage's correction to Hubble's data of 1936 [1].

Sersic, van den Bergh, and Holmberg all used slightly different distance moduli for the Local Group members than the values given in Table 1. Corrections to their values of H have been made on the basis of comparison of their adopted values of $(m-M)$ for individual galaxies with the values of Table 1. We assume that the apparent modulus of NGC 6822 is 24.1 to correct van den Bergh's discussion and that $(m-M)_{AB} = 27.6$ for M 81 (the same as for NGC 2403) to correct Holmberg's data. The factor by which each value of H is reduced to the present distance scale of Local Group members is 1.11 for Sersic, 1.13 for van den Bergh, and 1.20 for Holmberg.

Table 7 summarizes the five available values of H reduced to the scale of Table 1. The values are listed in order of excellence of the method, with the first three values probably having about equal weight.

TABLE 7

Recent Values of H

Method		*H km/sec/Mpc*
Sersic	(H II regions)	113 ± 5
Van den Bergh	(luminosity class)	106 ± 20
Holmberg	(surface brightness)	112 ± 10
Sandage	(H II regions)	82 ± 18
Sandage	(correction to Hubble 1936)	75 ± 25
	Average	98 ± 15 (m.e.)

It is significant that all of the methods except the fourth are completely independent of each other. Although objections can be raised to all five procedures, it is encouraging to see the close agreement of the present data. Programs now in progress at several observatories promise to give what might be called a "first definitive answer" to the value of H soon.

It is obvious that the data quoted in this paper represent the work of many people. This summary could not have been written without the very generous permission of Miss Henrietta Swope to present her Cepheid data for Field IV, M 31. Similarly, thanks go to Dr. Woolley and John Alexander for permitting their important results on the Cepheids in the LMC to be shown before publication.

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DISCUSSION

Gascoigne: I was surprised to see the difference in apparent brightness between the longer-period Cepheids in the SMC and LMC. According to photoelectric observations of Kron and myself for Cepheids with periods greater than about 15 days, this difference does not exceed $0\stackrel{\rm m}{.}1$, the LMC being nearer. I think also that the very limited real evidence available does not suggest a difference in moduli between the Clouds as large as $0\stackrel{\rm m}{.}4$. Could you comment on this?

Sandage: Woolley, Eggen, and I were also surprised to find the long-period Cepheids in the LMC to be so bright. The data shown for the LMC Cepheids rest entirely on the Greenwich (Herstmonceux) expedition to the Radcliffe Observatory in 1958. We used photoelectric calibration of all the material. It is interesting to note that if the LMC and SMC Cepheids are of the same apparent magnitudes for Cepheids for $P > 15^{\rm d}$, as your data indicate, then the slope discrepancy in the period-luminosity relation disappears (see Figures 3 and 4). For this reason, I hope our data are incorrect but, as of now, we have no other reason to suspect our data.

As regards the second point, I do not believe there is evidence which is accurate enough to rule out a difference in apparent distance modulus of $0\stackrel{\rm m}{.}3$ to $0\stackrel{\rm m}{.}4$ as required by Table 1.

Gascoigne: SMC Cepheids are well known to differ from those in the Galaxy, especially in period-frequency and period-amplitude relations. How do they compare in these respects with those in other galaxies, especially IC 1613?

Sandage: I have not compared the period-frequency or period-amplitude relations of the SMC data with Miss Swope's data in M 31. In the LMC, our data are biased toward variables with amplitudes greater than $0\stackrel{\rm m}{.}5$ because this was the limit we could reliably detect in our blink survey. Furthermore, we have measured only 41

Cepheids in our Field I near NGC 1783, so the statistics are poor. Baade's material in IC 1613 is similarly unsuited for a comparison because the data can be corrected to the proper Pogson magnitude scale only to $B = 21.0$ which reaches the Cepheids at maximum light only. As of now, no corrected data on the amplitudes of Baade's 18 Cepheids in IC 1613 are available.

Tifft: What uncertainty is present in moduli derived from period-luminosity fits for normal Cepheids?

Sandage: At least two uncertainties exist:

1. The calibration of the absolute magnitudes of the period-luminosity relation rests on the five Cepheids known in galactic clusters (Ev Sct, CF Cas, U Sgr, DL Cas, and S Nor) whose absolute magnitudes depend on fitting the cluster H-R diagrams to the age-zero-main sequence. The age-zero sequence depends on the Hyades whose distance is found by the moving-cluster method. The distance to the Hyades depends on the accuracy of the proper motions and radial velocities of the Hyades' members. We probably know the distance to the Hyades to an accuracy of better than $0\overset{\mathrm{m}}{.}05$. The fit of the five galactic clusters to the age-zero sequence is probably known to better than $0\overset{\mathrm{m}}{.}2$.

2. The photometric data for the period-luminosity relation in M 31, the LMC and SMC and IC 1613 have uncertainties of perhaps $0\overset{\mathrm{m}}{.}1$ at best. This means that the total over-all uncertainty in the apparent distance moduli in Table 1 may be as high as $0\overset{\mathrm{m}}{.}2$ to $0\overset{\mathrm{m}}{.}3$, assuming that the Cepheids are similar from galaxy to galaxy.

Evans: Would Dr. Sandage like to comment further on his table of absolute magnitudes of brightest stars in galaxies and integrated absolute magnitudes of the galaxies in which they occur? Is the apparent correlation a real one?

Sandage: Seven galaxies is too small a sample from which to draw a definite conclusion. However, Table 2 plus data from other galaxies such as the *Sextans* dwarf, the *W-L-M* system, NGC 6822, IC 2574, NGC 2366, and others show that when M_G is fainter than about −17, the M_{BS} is always fainter than −9. But for galaxies brighter than −17 or −18, M_{BS} seems to reach an upper limit of about −9 and then does not appear to be correlated with M_G.

Feast: As regards the difference in absolute magnitude of the brightest stars in the LMC and the SMC, I think that this can be satisfactorily explained in terms of the smaller size of the SMC and does not imply any difference in luminosity function (at the bright end) between the two galaxies.

Whitrow: What is the general position today as regards the determination of the distances of the more remote galaxies?

Sandage: For galaxies whose red-shifts are greater than about 5000 km/sec, relative distances can presumably be determined with great precision from the red-shift-distance relation. Such a relation is predicted by most theories, and all observational evidence to date fully confirms this expectation. However, distances in parsecs can only be determined once the Hubble expansion rate is known. Most, or all, individual precision distance indicators have faded below the limit of detection for red-shifts of 5000 km/sec and greater. For very large distances (red-shifts > 50,000 km/sec), corrections for deceleration of the expansion must, of course, be applied. Furthermore, at these large red-shifts, the definition of distances is a matter of choice.

Abell: We can probably obtain good relative distances to distant clusters by fitting together their luminosity functions. If we can then obtain the distance to one of these clusters, say the Virgo Cluster (at least the elliptical galaxies in it), by means of stars, novae, globular clusters, and so on, we have direct luminosity distances to the remote clusters. I do not think that we have done this adequately yet, but sufficient data should soon be available.

De Vaucouleurs: I wish to stress the extreme importance of securing radial velocities in the south galactic hemisphere, where we reach out into the general field and where the slope of the $\log V$ vs. m relation is much steeper than in the north galactic hemisphere.

Baum: In addition to fitting the luminosity function of the Coma Cluster to that of the Virgo Cluster, one can also fit the luminosity profiles of individual galaxies in Coma to those of galaxies in Virgo. The advantage of matching profiles is that the angular diameters (from profiles) of galaxies have less intrinsic spread than the magnitudes do. Some work of this kind was reported in 1952.

Abell: To what extent do the values of the Hubble constant which you have listed depend upon velocities in the Virgo Cluster or other objects in the Local Supergalaxy? Certainly, one must base the Hubble constant upon velocities of remote objects, such as clusters that are at least as distant as the Coma Cluster. The distances of these objects must, of course, depend upon calibration with nearby galaxies.

Sandage: Van den Bergh's method uses all the galaxies in the red-shift catalogue for which his luminosity classification applies. A number of these galaxies are beyond the Virgo Cluster. Sersic and Holmberg's data apply primarily to galaxies at or closer than the Virgo Cluster distance. However, if there was great difficulty in using the Virgo Cluster data to determine H, then this cluster would not fit onto the red-shift-magnitude relation. But the Virgo Cluster does fit this relation to within $0^{\mathrm{m}}.3$, a divergence which amounts to only a factor of 1.15 in the distance. At present, we do not know the Hubble constant this accurately for other reasons.

Hoyle: I would like to comment on the applicability of the value of H to the more distant regions of the universe. If one doubts the applicability, on the grounds that the rate of expansion of the local supercluster might be different from that of the general field (say, beyond 50 Mpc), then it seems reasonable that H for the general field should be taken larger than the local value. Otherwise, the local irregularity of the supercluster would have vanished into the general field.

Sandage: I should again like to state that the observational data show that the Virgo Cluster fits well on the $[m,z]$ relation defined by galaxies with red-shifts greater than 5000 km/sec and that, therefore, there cannot be a large peculiar expansion rate for this cluster.

Oort: I do not think that the Virgo Cluster should be discredited for the determination of H. There is a considerable dispersion of velocities in the Virgo Cluster, but this can be averaged out.

After considering Hoyle's remarks, I agree that there should be some reserve in interpreting the result derived from the Virgo Cluster, as the Galaxy might have a "random" motion relative to the Virgo Cluster. But it seems unlikely that this would be a large fraction of the expansion velocity.

Page: Your reply to Hoyle seemed to me the strongest argument *against* a local supercluster or metagalaxy. If the $[m,z]$ relation shows no lower slope for $z \leq 0.01$ than for higher values, then there can scarcely be a local supercluster.

Zwicky: I fail to see how the red-shift constant can be accurately determined unless one ties the data in the Local Group directly to red-shifts in clusters for which there is little dispersion of internal distance and of velocities. H from supernovae in the Cancer Cluster, the Coma Cluster, and the Local Group gives 175 km/sec.

Sandage: It is certainly clear that the local and random effects in the velocity field will be minimized if H can be determined from galaxies with red-shifts greater than 1000 km/sec. Perhaps, supernovae can eventually be used. However, it is by no means clear that you have produced data for supernovae which give either the dispersion in absolute magnitude about the mean, or an absolute calibration of this mean. We know so little about the group characteristics of supernovae that a determination of H with the present data appears to me to be completely out of the question.

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Problems of Observation and Interpretation

The observables that can be used to discriminate between different cosmological models are: the red-shift; the apparent magnitude of galaxies or the flux-density of radio sources; the angular diameters of galaxies, clusters of galaxies, or of radio sources; and counts of galaxies or radio sources. A recent detailed discussion by Sandage [1] shows that optical observations must be extended to the limit of the 200-inch telescope to reach decisive conclusions. Since radio sources can be observed at distances beyond the reach of the 200-inch telescope, their use for cosmological studies seems attractive. The available data, however, serve more to show the difficulties of this approach than to lead to definite conclusions.

The relation between red-shift ($z = d\lambda/\lambda_0$) and apparent magnitude offers most promise for a test of cosmological models, but even at $z = 0.5$ a difference of only $0\overset{\mathrm{m}}{.}8$ separates the steady-state theory and the nearest evolving model of the relativistic cosmology with the cosmological constant $\Lambda = 0$. Observations of clusters of galaxies offer a way to overcome the effects of the large dispersion of absolute magnitudes of galaxies. In clusters, the brightest end of the luminosity function of galaxies can be sampled. If this part of the luminosity function is identical in all rich clusters, the magnitudes of a chosen number of the brightest galaxies may be used to determine the statistically expected magnitude of the first brightest galaxy. Humason, Mayall, and Sandage (HMS) [2] use magnitudes of the first brightest galaxy on the system of the ten brightest galaxies.

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Magnitudes observed in a fixed band of wave lengths, such as *P* or *V*, must be corrected for the photometric effect of red-shifting the intensity distribution of galaxies through the band [2]. The corrected magnitudes are bolometric, except for an irrelevant additive constant. Multicolor photoelectric photometry [3] determines the red-shifted intensity distribution. The area under the distribution curve provides a measure of a bolometric magnitude. The photoelectric magnitudes have so much higher weight that it seems justified to disregard entirely the results obtained in HMS.

Only if evolutionary changes of the intensity distribution are negligible, can z be determined from the shift of the intensity distribution. The photometrically determined values can, therefore, be considered reliable only as far as they can be checked by spectroscopic measures of the red-shift, and such a check becomes increasingly important as the measures are extended to larger red-shifts. It is important, consequently, to extend the spectroscopic observations to the largest red-shifts.

Humason was able to reach $z = 0.2$ with spectroscopic observations. At that red-shift, even the brightest galaxies in clusters are so faint that the night sky veils the spectrum. Attempts to extend the observations to larger red-shifts have failed because absorption lines become indistinguishable, but emission lines of great strength can be expected to remain observable at very large red-shifts [2].

Clusters with $z = 0.2$ are still above the limit of the 48-inch Schmidt telescope. For clusters at the limit, red-shifts of 0.29 and 0.35 were observed by Baum [3]. To extend the observations to still larger red-shifts, it is necessary to find clusters of galaxies beyond the limit of the 48-inch telescope; to permit spectroscopic measures of z, one of the brightest galaxies should have strong emission lines.

To find a cluster with these specifications by random search with the 200-inch telescope is a hopeless task. The search for identifications of radio sources seemed to offer a way to solve the problem. The strongest extra-galactic source, Cygnus A, is the brightest member of a rich cluster at $z = 0.056$ [4, 5]. Its spectrum is dominated by strong emission lines. The object has been interpreted as a collision of two galaxies; in that case, membership in a cluster is a necessary condition. In any case, an association of strong sources with clusters seems to exist [6, 7, 8]. Sources similar to Cygnus A, but at a larger distance, are the objects that are needed.

When a source about 70 times fainter than Cygnus A was shown to have a diameter about 10 times smaller [9], it was earmarked as a likely object, but an attempt to identify the source failed at that time; the best available position in the 2C catalog [10] was not accurate enough. The attempt was repeated when a precise position had been determined by Elsmore, Ryle, and Leslie [11] and the right ascension had been confirmed by Bolton. The source 3C295 [12] was then identified as the brightest member of a rich cluster [13]. Spectra obtained at the prime focus of the 200-inch telescope with a dispersion of 750 A/mm showed one strong emission line at λ 5446.8; a large fraction, perhaps more than one-half, of the energy in the visual range is contained in this line. To judge from spectra of galaxies in general, and, in particular, from the spectrum of Cygnus A, the line is to be interpreted as [O II] λ 3726/9; any other interpretation would require the presence of additional lines that are not observed. The red-shift then is $z = 0.4614 \pm 0.0002$.

This brings the observation close to the limit of the 200-inch telescope. The brightest galaxy in the cluster is of visual magnitude 21.1. A cluster fainter by one magnitude presents probably the limit to which the observations could be pushed. If a suitable cluster can be found, it may be possible to extend the observations as far as $z = 0.6$. One curious difficulty may be mentioned. For $z = 0.5$, the red-shifted line λ 3727 coincides with the strong night sky line [O I] λ 5577 and cannot be used.

The identification of more distant sources analogous to Cygnus A and 3C 295 should permit the discovery of more distant clusters, but it is difficult to estimate how frequent such sources are. Their space density is of the order 5×10^{-9} Mpc^{-3} [14]. The number in reach of the 200-inch – about 1 magnitude fainter than 3C 295 – is unlikely to exceed 10 in the whole sky.

The photometric observations, which Baum will report in the following paper, show a linear relation between z and the apparent bolometric magnitude. Taken at face value, the linear relation contradicts the steady-state theory. But even at $z = 0.46$, only $0^{m}\!.8$ separate the linear relation from the relation of the steady-state theory. It is difficult to say whether systematic errors of the magnitudes can be large enough to save the steady-state theory. The magnitudes would have to be too bright by amounts increasing systematically with z to bring the observations into agreement with the steady-state theory.

Two systematic errors are to be considered: (1) the aperture effect in measuring apparent magnitudes; (2) the effect of observational selection considered by Miss Scott [15].

The possibility of a systematic aperture error arises from the fact that galaxies are not sharply bounded and that, therefore, magnitudes must be defined by selecting an isophote to which they extend. Since the surface brightness varies with $(1+z)^{-4}$, isophotes of a given surface brightness shrink towards the center of a galaxy as z increases. The isophote containing a given fraction of the total light shifts to lower surface brightness as z increases. If this effect is disregarded, magnitudes are obtained that are too faint. The importance of this effect for the determination of magnitudes has been noted by Stock and Schücking [16]. The size of the aperture that contains the same fraction of the total light of a nearby and of a distant galaxy depends on the cosmological model. The selection of the model, however, depends on properly measured magnitudes. A process of successive approximations must be performed. The possible size of the aperture error depends on the distribution of surface brightness. If the computation of the aperture error for NGC 3379 in HSM, Figure A3, gives a representative result, it is virtually impossible to choose a diaphragm so much too large that the magnitudes become too bright by more than $0^m\!.1$.

The "Scott effect" would arise if more distant clusters selected on a photograph are always clusters with greater population, and if, with increasing population, the magnitude of the brightest galaxy becomes brighter. The data listed in Table XI of HMS suggest, however, that the luminosity of galaxies in clusters has an upper limit that is reached already in groups with 30 members. In that case, the Scott effect would not exist. Moreover, the most distant cluster now known has not been found by inspection of photographs, and there is no obvious reason why this cluster should be the richest of all clusters used in the determination of the red-shift-magnitude relation.

Even in the absence of systematic errors the observations do not necessarily contradict the steady-state theory. In this theory, galaxies of all ages coexist. If the luminosity of a galaxy depends on its age, the relation between the magnitude of the first brightest galaxy and the red-shift is not a line, but a band in which the galaxies are distributed according to the luminosity distribution which arises for a given value of z from the age distribution and the evolutionary change of luminosity. If a large number of observations were available, they should, therefore, show considerable scatter; the absence of scatter could actually provide an argument against the steady-state theory. With the small number of observed points available at this time, the observed linear relation could conceivably be not more than the result of a statistical accident, all points being contained in the band admitted by the age distribution and the evolutionary effects. Knowledge of the evolutionary changes is needed before it is possible to decide whether such an interpretation is acceptable. The importance of increasing the number of observations is clear.

If the linear relation is accepted as sound evidence, the interpretation then depends on the size of evolutionary changes of the luminosity of galaxies. If, during the light travel time for $z = 0.46$, the luminosity does not change, the linear relation indicates a deceleration parameter $q_0 = 1$. This means a pulsating model universe in which space is closed and spherical.

For an evolving model, the mean space density ρ_0 can be derived from a relation that is a direct consequence of Einstein's field equations

$$\rho_0 = \frac{3H_0^2 q_0}{4\pi G}, \tag{1}$$

where H_0 is the Hubble constant. With $H_0 = 75$ km/sec/Mpc [17], one obtains

$$\rho_0 = 2.06 \times 10^{-29} \; q_0 \text{ g/cm}^3. \tag{2}$$

The value $q_0 = +1$ thus leads to $\rho_0 = 2.06 \times 10^{-29}$, a value that is 70 times higher than the value 3×10^{-31} g/cm^3 derived by Oort [18] from the space density of galaxies in our neighborhood. A factor 10 is admitted by Oort as the uncertainty of his value, but even then the density from $q_0 = +1$ can be reconciled with the observed density only by assuming that the largest fraction of matter in the universe is not observed.

The time-scale of the expansion offers a further test. For evolving models, a singular point exists in the finite past when the radius of the universe was zero. The time t_0 elapsed from the beginning of the expansion should be smaller than the age of the oldest stars. The time t_0 is a function of H_0 and q_0. For $q_0 = +1$ and $H_0 = 75$ km/sec/Mpc, we have $t_0 = 7.4 \times 10^9$ years. The ages of the oldest stars derived with the latest stellar models, however, are larger than 20×10^9 years [19].*

Both the discordances of density and time-scale would be reduced if H_0 were substantially smaller than the accepted value. The dating of the age of stars by stellar models undoubtedly has some uncertainty. It is perhaps not quite impossible that in the end the solution $q_0 = +1$ may turn out to be acceptable. But it should not be overlooked that this solution is obtained with the assumption that evolutionary changes of galaxies play no role at $z = 0.46$. The travel time of light for $z = 0.46$ and $q_0 = +1$ is 3.4×10^9 years with $H_0 = 75$ km/sec/Mpc. It seems quite possible that evolutionary changes during such a time cannot be disregarded.

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*Added in proof; arguments presented by N. J. Woolf (*Ap. J.* 135, 644, 1962) show that the ages of the oldest stars may have been overestimated by a factor of the order 2.

We may try what happens if we assume that the density discordance is due entirely to the disregard of evolutionary effects and Eq. (1) with Oort's value for ρ_0 gives the correct value for q_0. In that case, q_0 is of the order 10^{-2}. This indicates an open hyperbolic model in which the rate of expansion remains almost unchanged. The age of the universe now is only slightly smaller than $1/H_0$, close to 13×10^9 years. The age discordance, thus, is greatly reduced and perhaps is no longer serious. The solution might appear satisfactory if the evolutionary changes that have to be assumed are acceptable.

The travel time of light for $z = 0.46$ with $q_0 \sim 10^{-2}$ is 3.9×10^9 years. At $z = 0.46$, a difference of $0\overset{m}{.}4$ separates the relation for $q_0 = 0$ from that for $q_0 = +1$. Thus, if the luminosity of galaxies changes by $0\overset{m}{.}4$ in 3.9×10^9 years, the observed red shift-magnitude relation would actually correspond to $q_0 = 0$. We may compare this change with the time change computed by Crampin and Hoyle [20] for an M 67 population. One result of these computations is that rather small color changes accompany the changes of luminosity. The absence of an observable color change in Baum's results, therefore, does not contradict an observable change of luminosity. The computed luminosities can be represented by

$$m_1 - m_2 = 1.92 \log(t_1/t_2) \ , \tag{3}$$

where m_1 and m_2 are the magnitudes at times t_1 and t_2. For

$$t_2 = 13 \times 10^9 \text{ yr. and } t_1 = 9.1 \times 10^9 \text{ yr. ,}$$

the change of magnitudes is $0\overset{m}{.}52$, actually larger than required to make $q_0 = 0$. It cannot be assumed, however, that the luminosity of the brightest galaxies is determined entirely by the stars of an M 67 population. The true change of luminosity depends on the contributions of other type of stars and may well be too small to make $q_0 \sim 10^{-2}$ an acceptable solution. But it is clear that knowledge of the evolutionary changes of galaxies is a prerequisite to the solution of the cosmological problem.

If none of the models with $\Lambda = 0$, discussed in detail by Sandage, can be fitted to the observations, it is not likely that a model with $\Lambda \neq 0$ will turn out to be the solution. With $\Lambda < 0$, the density discordance becomes smaller, but the age discordance larger. The case $\Lambda > 0$ which would reduce the age discordance can probably be ruled out. It leads to a red-shift-magnitude relation that becomes similar to that for the steady-state theory and, therefore, is ruled out by the observed linear relation. The final conclusion may well turn out to be that none of the uniform model universes that were discussed is applicable to the observations.

It is doubtful whether the relations between diameters and red-shift are of practical value for the selection of a cosmological model. At $z = 0.5$, the ratio of a metric diameter for $q_0 = +1$ to that for $q_0 = -1$ is 1.6; the corresponding ratio of isophotic diameters is 1.5. It is questionable whether these differences are large enough to give significant results since the intrinsic dispersion of diameters of individual galaxies or of clusters of galaxies is large. Clusters of galaxies are obviously the more promising objects.

The difficulty is well illustrated by the data on the structural index of clusters found by Zwicky [21]. The structural index α is a metric diameter defined as $1.75\,\gamma$, where γ is the angle at which the number distribution of galaxies in a cluster as a function of the angular distance from the center shows an inflection. Zwicky represents his results by a relation $z = 1 + \exp{(b/\alpha)}$; the value of the constant b, taken from Figure 1 in [21], is 0.070.

Such a relation fits a stationary model universe in which the redshift occurs as a decrease of the energy of a light quantum during the light travel time. As it happens, the diameter from Zwicky's relation is only 8 per cent smaller at $z = 0.46$ than that from the steady-state theory. The observations could be considered as proof for the steady-state theory, but, in reality, the data do not permit any decision since the linear structural indices for four nearby clusters [22] have a relative range from 1 to 2.3. This is the scatter that has to be overcome.

The metric angular diameter for all evolving models reaches a minimum value at a finite z, but decreases asymptotically to a finite value as $z \to \infty$ for the steady-state model. This was first pointed out by Hoyle [23] who suggested that the existence or non-existence of a minimum might serve to discriminate between the steady-state model and the evolving models. The minimum is beyond the reach of the 200-inch telescope, but observations of radio sources could conceivably lead to such a test. It is now established, however, through the identification of the source 3C 48 with a stellar object [24] that galactic sources of stellar dimensions occur at high galactic latitudes. Before the existence of very small sources is used as a cosmological argument, it must be shown that they are not stellar sources. Moreover, radio sources differ greatly in linear size and in structure. It is not established that there is a minimum linear size for extra-galactic sources; if such a minimum does not exist, a minimum of the metric angular diameter cannot be easily established.

Counts of galaxies brighter than a given apparent magnitude were once considered as a promising criterion. The differences between the models that have to be considered are so small even at the limit of the 200-inch telescope that it seems hopeless to determine a model from counts of galaxies [2].

The relation between number and flux-density of radio sources is the analogue of counts of galaxies. In an isotropic and homogeneous space, the number N of sources that are stronger than a given flux-density S is proportional to $S^{-3/2}$ as long as $z << 1$. The spectrum of a radio source can be represented over a wide range of frequencies f by the power law $S \sim f^{x}$; for most extra-galactic sources the spectral index x is in the narrow range from -0.6 to -1.0. It is easy to compute for sources of a given intrinsic strength the expected number versus flux-density relation for various cosmological models, using a reasonable mean value of the spectral index. The luminosity function of sources can be determined [6, 14, 25]. It is, thus, possible to compute the number versus flux-density relation for sources of different intrinsic strengths and, by adding the numbers, to obtain the expected number versus flux-density relation for various models.

It turns out that, at high flux-densities where $z << 1$ for all sources, the intrinsically strong sources are more frequent than intrinsically faint sources; at a given flux-density, the intrinsically strongest sources are, of course, most distant. As one proceeds to lesser flux-densities, i.e. larger z, cosmological effects become noticeable first for the strongest sources, while the number of weak sources still continues to increase with $S^{-1.5}$. Thus, the presence of the weaker sources reduces the cosmological deviation from the $S^{-1.5}$ relation. To achieve discrimination between different models, counts must be extended to numbers of the order 10^5 per steradian [26] and correspondingly low flux-density. The mere existence of the cosmological effects should be noticeable much sooner, at numbers of the order 20 per steradian where the exponent $-3/2$ should have dropped to a value larger than -1.4.

The latest source counts by Mills [6, 27] and by Ryle [28] still lead to somewhat discordant results. Mills obtains $N \simeq S^{-1.5}$, but Ryle finds $N \simeq S^{-1.8}$ from the first results of the aperture-synthesis survey. Space does not permit a discussion of the possible reasons for the discordance. Mills' counts extend to roughly 200 sources per steradian, where, on the basis of the present knowledge of the luminosity function, the exponent should be distinctly larger than -1.5, but if strong sources are less frequent than assumed, the value -1.5 would still be no contradiction to the expected results. An exponent of -1.8 contradicts any cosmological theory if the space distribution of sources is uniform; the red-shift must inexorably lead to a slope less steep than -1.5. A steeper slope can only indicate that the space density of sources increases with increasing distance.

We might be located accidentally in the center of a region with abnormally low space density of sources. Such a situation might even arise in the steady-state universe as a consequence of a statistical fluctuation in the age distribution of galaxies if the intrinsic strength of the radio emission depends on the age of a galaxy [29]. It should be noted, however, that the source counts extend to very large distances and that any interpretation of this type implies that a large inhomogeneity of the space density of sources extends almost to the limits of the observable universe. In the frame of evolutionary cosmologies, the outward increase of the space density of sources could be an evolutionary effect due to the presence of more numerous, or more powerful, sources at the time billions of years ago when the radiation from the most distant sources was emitted. An evolutionary effect of this kind would make it very difficult to use source counts for the selection of a cosmological model. But it is clear that it is of great importance to obtain reliable and definitive source counts.

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DISCUSSION

McVittie: The case $q_0 \simeq 0, t_0 = 1/H$ is Milne's model. The relation for the structural indices of clusters applies just as well in the de Sitter universe of general relativity as in the steady-state theory. For this prediction, the two theories are indistinguishable.

E. Scott: I am not sure that there is no selection although it would be nice if there were none. Referring to Dr. Minkowski's lecture:

1). If there is an evolution effect as sketched by Dr. Minkowski then there is a selection effect.

2). We considered the possibility that there is a cutoff in absolute magnitude of galaxies. The computations published in our paper show that there is still a selection effect in this case although, of course, less.

3). The diagram and the data given in the paper by Humason, Mayall, and Sandage are entirely consistent with a normal distribution (no cutoff) of luminosity.

When looking at the diagram, one must keep in mind that the dispersion in luminosity of the brightest, or of the fifth brightest, and so on, galaxy is much less than the general dispersion. It is this decrease that the astronomer wants to use but he should not be sure that it is zero.

Zwicky: The structural indices of the clusters Coma, Hydra I, Cancer, and Perseus vary from 1 to 2.3 because these clusters are not morphologically identical. Morphologically identical clusters give much less dispersion in structural index.

My dissatisfaction with the steady-state theory is that it is not science at all because it disregards Einstein's dictum, *"Der Herrgott ist zwar raffiniert aber er ist nicht boshaft."*

Lemaître: With a positive value for the cosmological constant, large fluctuations of densities and velocities must be expected. It may be that the observations just discussed serve rather to measure the degree of lack of homogeneity in the universe than to decide between cosmologies. I may recall in this connection the attitude of Eddington as expressed in a letter to Miss Douglas.

Abell: My observations of the luminosity functions of six rich clusters show that the absolute magnitudes of the 10th brightest members of different clusters range over at least $0^{m}.5$.

King: Evolutionary effects depend on the age that one assigns to the population contributing the major part of the light of elliptical galaxies. If the main-sequence turnoff point is lower, the evolutionary changes are slower. The critical point is the age that one assigns to typical nearby ellipticals.

Minkowski: For the M 67 population considered by Crampin and Hoyle, the evolutionary change depends only on the ratio of the ages, i.e., on the ratio of the age of the universe t_0 and the age $(t_0 - t)$ of a population at the light travel time t. For a more realistic model, the situation could be more complicated.

Hoyle: If one goes back to the curves you showed for the various values of q_0, the effect of evolution as you have used it moves the observed curve about half of the way to the steady-state curve.

Minkowski: That is true.

Von Hoerner: You showed very clearly that the evolutionary effects would play an important role as far as magnitudes are concerned. I would like to raise the question whether this might be better if one uses diameters (defined in some way) instead of magnitudes?

Minkowski: It seems possible that evolutionary effects of diameters of clusters of galaxies might present lesser difficulties, but one may question whether this advantage would be great enough to make the use of diameters more attractive. At this moment, the dynamics of clusters seems to be an open question. I have great doubts whether it would be feasible to use diameters of galaxies.

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Photoelectric Magnitudes and Red-Shifts

In a recent paper, Sandage [1] has reviewed four observational tests of world models, namely,

1. The relation between red-shifts and magnitudes of galaxies:
2. The relation between counts and magnitudes of galaxies:
3. The relation between red-shifts and angular diameters of galaxies:
4. The compatibility of age estimates.

Among optical observations, the red-shift-magnitude relation provides the most powerful test, and this paper is concerned with the present photoelectric attack on that problem.

There are two points that should be emphasized at the outset concerning the optical test described here and the radio test described by Ryle [2]. First, the radio test, which pertains to the count-magnitude relation, is unable to provide an unambiguous choice among the usual world models, whereas the red-shift-magnitude relation can lead to a single conclusion. The second point is that radiation from very distant galaxies started on its way when those galaxies were younger. We do not know how to cope with evolutionary changes in the radio magnitudes, but we do have some hope of estimating evolutionary corrections for optical magnitudes.

Apart from effects due to evolution or obscuration, the red-shift-magnitude relation depends upon world models as shown in Figure 1. Curve D represents the relation predicted by the steady-state

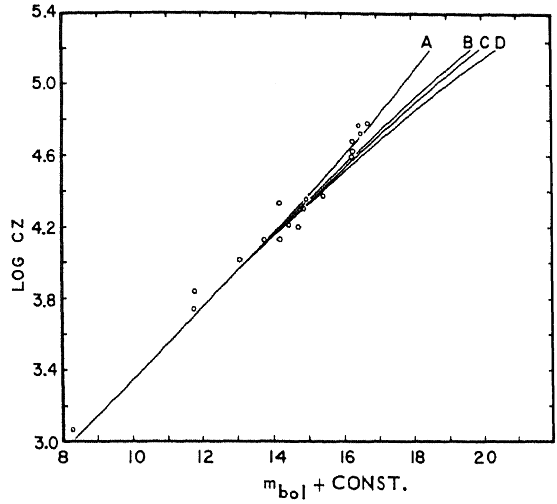

Figure 1. Theoretical curves for the red-shift-magnitude relations predicted by various world models. Adapted from Hoyle and Sandage [3].

model. Curve C is the limiting case for an exploding model with no significant gravitational deceleration; Curve B is the Euclidean case; and Curve A lies in the region of exploding universes which will collapse again. The points representing earlier photographic data [4] show the range of red-shifts ordinarily within the reach of the conventional spectographic method. The main reason for undertaking the photoelectric method [5] was to obtain points nearer the top of the diagram where the various world models are much more clearly distinguished. The photoelectric method is also less vulnerable to systematic errors in the bolometric magnitudes.

In the program described here, both the red-shift and the bolometric magnitude of a galaxy are measured photoelectrically. The light from each galaxy is measured in a number of colors. When the results are plotted on a true energy scale, they yield a spectral-energy distribution curve like that shown in Figure 2. This particular curve represents the mean of six bright elliptical galaxies in the Virgo Cluster. A curve for similar ellipticals in another cluster at a greater distance will have about the same shape, but will be displaced toward the right and will fall at fainter magnitudes. The horizontal displacement yields the difference in red-shift, while the vertical displacement, after a log $(1 + z)$ correction due to the logarithmic abscissa, yields the difference in bolometric magnitude.

This photoelectric program [5], which is still in progress, was undertaken at Palomar six years ago. At that time, it became possible to extend photoelectric observations of the infrared out to the

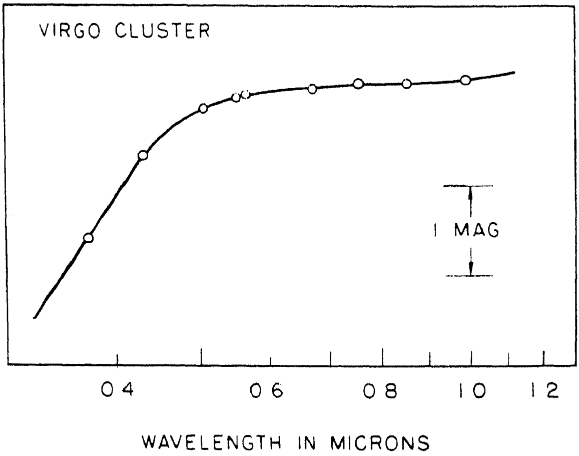

Figure 2. Mean spectral energy distribution curve for six bright elliptical galaxies in the Virgo Cluster. The ordinate represents relative energy per unit wave length expressed on a magnitude scale.

faintest objects which can be photographed. The possibility of measuring very faint objects photoelectrically in the infrared, as well as at shorter wave lengths, provided the opportunity of undertaking a completely photoelectric attack on the red-shift-magnitude problem. In addition to red-shifts and bolometric magnitudes, the photoelectric observations have also yielded a recipe for the stellar content of ellipticals [6] and some data on the intensity profiles of ellipticals.

After a cluster of galaxies has been chosen for study, plates in two or more colors are compared, and a few of the brighter elliptical galaxies in the cluster are selected for photoelectric measurement. In nearby clusters, ellipticals can be identified both by the appearance of the images and by their consistent color index. In distant clusters, one must depend mainly upon the color index to identify ellipticals and to exclude possible foreground galaxies. The photoelectric photometer is set onto faint galaxies by coordinate offsets and the data are obtained by the pulse-counting method. In the infrared, photocurrents as small as 20 electrons per minute must be measured against sky backgrounds which are many times larger.

Most of the photoelectric observations have been made in the nine colors identified in Table 1. In a few cases, two additional colors have been used to provide points in the yellow and orange regions. Effective wave lengths of the various colors in Table 1 were derived by multiplying together the transmission curves of the filters, the spectral sensitivity curves of the photocathodes, the

TABLE 1

Colors Observed

Color	*Photocathode*	*Filter*	*Effective λ for ellipticals ($z=0$)*	*Approximate band-width*
Ultraviolet *U*	Sb Cs	C9863(3) + C7740(2)	3730	500
Violet *V*	Sb Cs	BG12(3) + GG13(2)	4335	740
Blue *B*	Sb Cs	Wide-band interference	5065	430
Green *Ga*	Sb Cs	Wide-band interference	5525	470
Green *Gc*	CsO_2(Ag)	BG18(2) + OG1(2)	5645	700
Red *R*	CsO_2(Ag)	Wide-band interference	6705	850
Infrared *I*	CsO_2(Ag)	Wide-band interference	7525	600
Infrared *J*	CsO_2(Ag)	Wide-band interference + RG10(3)	8520	800
Infrared *K*	CsO_2(Ag)	Heimann No. 205	9875	1100

spectral transmission of the earth's atmosphere (for 1.2 air masses), and the approximate spectral energy distributions of the objects. Indeed, the effective wave lengths of some of the colors have a second-order dependence upon the red-shift of the galaxy being observed, and that effect has been taken into account.

There are two methods by which multicolor magnitudes with arbitrary zero points can be adjusted to a true energy scale. One method is to measure a reference object having a calibrated spectral energy distribution. Another method is a "bootstrap" process wherein data for two clusters of known red-shift, one nearby and the other far away, are compared with one another, and the problem is solved in reverse to derive the shape of the spectral energy curve which yields the correct difference in red-shift. Both methods have been investigated and they yield consistent results. The energy distribution curves in the present paper are based upon observations of the 7th magnitude AO star HD 182487, which has been calibrated against Vega by Oke [7].

Figure 3 shows the spectral energy distribution curve for a cluster of galaxies with a red-shift of $z = 0.19$ or $cz = 57{,}000$ km/sec. The points represent the mean results for three bright ellipticals in the cluster. Since these objects are brighter than 20th magnitude, the observational errors are relatively small. An unshifted energy curve is sketched at the left for comparison. This cluster lies approximately at the limit of the usual range of conventional spectrography, and the red-shifts estimated by the two methods agree within the errors of observation.

Beginning in 1955, I started observing clusters beyond the usual range of the spectrograph. Figure 4 shows the kind of results obtained for a faint, but populous, cluster visible in the October sky.

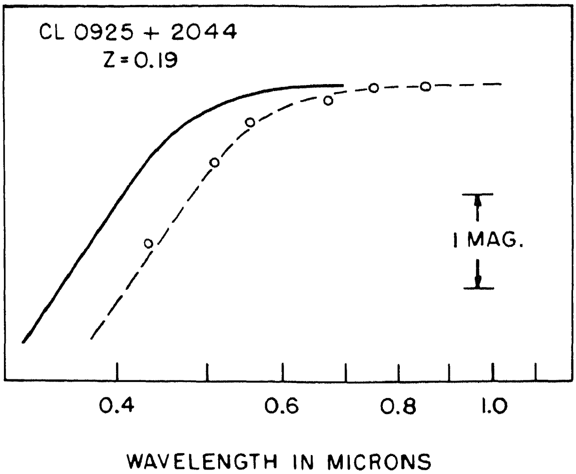

Figure 3. Mean spectral energy distribution for three elliptical galaxies in a cluster with a red-shift of $z = 0.19$. For comparison, the Virgo curve of Figure 2 is sketched here at the left of the points; the vertical displacement has been removed, and the horizontal displacement yields the difference in red-shifts.

The brightest members are around 20th magnitude, and the red-shift indicated here is $z = 0.29$ or $cz = 87{,}000$ km/sec. Each point is the mean for four galaxies. Some of the colors were also observed several times In each color, the process of pulse-counting was continued until enough statistical accuracy had been accumulated to reduce the probable error to the amount indicated by the short bar

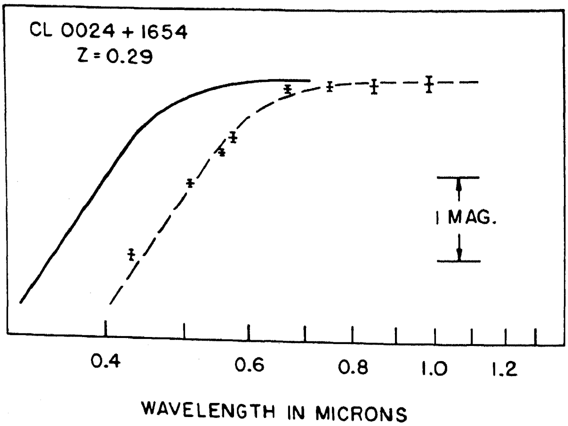

Figure 4. Mean spectral energy distribution for four elliptical galaxies in a cluster at $0^h\ 24^m$, $+16°\ 54'$. These results show the red-shift to be 0.29. The vertical bars through the observed points represent the individual probable errors.

in the diagram. One can obtain any accuracy desired simply by having the patience to continue the pulse-counting long enough; if twice the accuracy were desired, the duration of counting would need to be four times longer. Because of other sources of uncertainty, however, there is little reason to press for higher precision than is shown here.

Figure 5 shows my photoelectric observations for the cluster found by Minkowski [8] in the position of the radio source 3C 295. The points represent the mean results for two galaxies selected on the basis of some preliminary photographic photometry. Bluer-than-average objects which might be foreground galaxies or which might include the radio source itself were carefully avoided. The two galaxies selected for photoelectric observation were similar in color and appearance to the majority of the galaxies in the cluster and were located favorably for photometry. There can be no question that these data truly represent the cluster. The red-shift thus obtained was $z = 0.44 \pm 0.03$ or $cz = 132{,}000$ km/sec.

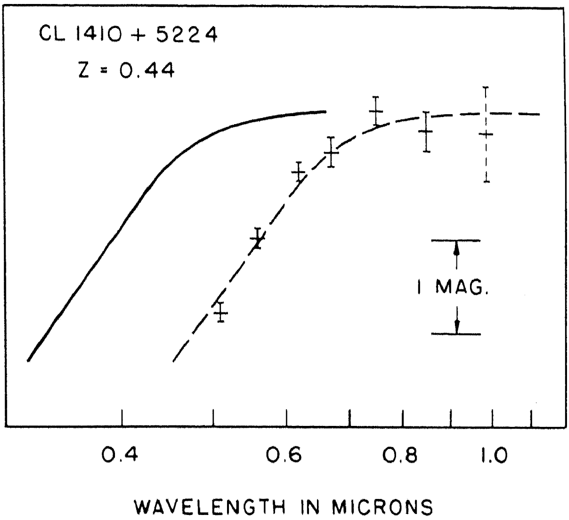

Figure 5. Mean spectral energy distribution for two elliptical galaxies in the cluster to which the Cambridge radio source 3C 295 evidently belongs. These results yield a red-shift of 0.44 ± 0.03.

One of the bluer galaxies, thought to be a likely candidate for the radio source, was found by Minkowski to have a single strong emission line at λ5448. On the reasonable assumption that the line is [O II] 3727, the inferred red-shift ($z = 0.46$) agrees closely with the photoelectric value. This agreement [9] means two things: 1) It strongly reduces any doubt about the membership of Minkowski's object in the cluster and about the identification of the emission line with [O II] 3727; 2) it provides a check, at a very large distance, on the photoelectric method of measuring red-shifts.

For a Hubble constant of 75 km/sec/Mpc, the corrected bolometric distance of this cluster is more than 10^3 Mpc.

TABLE 2

Clusters Observed Photoelectrically for the Red-shift-Magnitude Relation

Cluster	Galaxies Observed	Red-shift z	Probable Error in z
Virgo	6	0.0038	5.2%
Coma	3	0.022	1.6
Cor. Bor.	3	0.072	0.8
UMa 2	2	0.135	0.9
0925	3	0.192	0.9
0024	4	0.29	2.5
1448	2	0.36	4.0
1410	2	0.44	7.1

The red-shifts for eight clusters are listed in Table 2. The probable errors for the nearby clusters are based upon the intrinsic spread of velocities inside clusters, whereas the errors for the distant clusters are based upon the uncertainty in shifting the spectral energy distribution curves to fit the photoelectrically observed points.

The photoelectric red-shift-magnitude relation is plotted in Figure 6. As in Figure 1, the ordinate is the logarithm of the red-shift and the abscissa is a corrected bolometric magnitude. The zero-point of the abscissa scale, however, is not the same. Each cluster of galaxies is represented by a single point. The lowest point at the left represents the Virgo Cluster, and the highest point at the right represents Cluster 1410, which contains the radio source 3C 295. All except the Virgo Cluster are clusters of large membership.

Uncertainties in the magnitudes are greater than those in the red-shifts. The magnitudes require normalization to a standard intrinsic diameter; they require the matching of the luminosity functions of the various clusters to one another; and they are more vulnerable than the red-shift to any evolutionary effect. The normalization of magnitudes to a standard diameter, by means of an intensity profile, requires knowing the relationship between metric diameter and red-shift. This, in turn, depends upon the world model, but the problem can be handled by first assuming a simple $1/z$ relationship and by treating any difference as a correction. The matching of luminosity

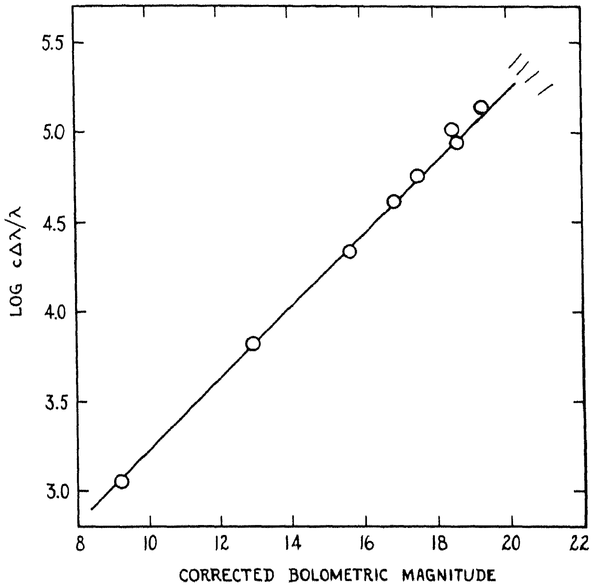

Figure 6. Red-shift-magnitude relation for the eight clusters investigated photoelectrically. Adjustments and corrections to the magnitudes are currently being reexamined. Pending those, the results continue to approximate a straight line, $m(\text{bol.}) = \mathit{constant} + 5 \log z$.

functions is, by far, the weakest link in the chain. The previous procedure of extrapolating to a brightest magnitude is not adequate, but further photometry is needed before a more sophisticated adjustment can be made. The dependence upon evolution also remains to be introduced. This dependence can be estimated from a population model for elliptical galaxies which has been derived [6] in connection with the present program.

Pending the readjustment of magnitudes, a roughly linear relation (m, $\log z$)·is suggested by the eight clusters thus far investigated. The data fall in the range of exploding universes that collapse again after a long time. It is premature to quote a specific value of $R\ddot{R}/\dot{R}^2$, but it is, perhaps, of interest to note that the top end of the curve in Figure 6 must be bent to the right one whole magnitude if it is to approach compatibility with the steady-state universe, and that seems unlikely at present.

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DISCUSSION

Hoyle: I would like to comment on two points. First, that we can say little about the evolutionary variation of the magnitudes of elliptical galaxies. It is not a question of the type of star involved, or of the evolutionary behaviors of individual stars, but of the number of evolving stars. This depends on the initial main-sequence luminosity function, particularly for the range +6 to +3. Since we do not know this for ellipticals, the evolutionary situation for such galaxies is quite uncertain (although one can be more hopeful about the evolutionary variation of *color*).

My second point concerns the difference between the integrated magnitude of a galaxy and its surface brightness. Surface brightness is quite independent of the cosmological theories, while the integrated magnitude does depend on the theory in question. This circumstance arises because angular diameter differs in different theories. Hence, any discussion of apparent magnitude versus red-shift in relation to different theories is really a discussion of angular diameter versus red-shift.

Baum: We do have a reasonable population model for an elliptical galaxy on which to base a first effort at estimating an evolutionary effect. I agree that the luminosity function can, at best, be only crudely estimated, but feel that a crude correction will be better than none.

Bondi: The agreement of the spectroscopic and the photometric measures of red-shift to $z = 0.46$ is an astounding achievement showing the absence of evolutionary effects to this range. How much effect should be expected on an evolutionary picture is in doubt, as stated before. However, the disproof of the Stebbins-Whitford effect is now remarkably complete.

Zwicky: I doubt the value of the red-shift-magnitude test because: *(a)*the identity of the two clusters has not been established; and *(b)*I doubt, unless spectra and spectral red-shifts are available, that it can be ascertained whether or not the galaxies are members of the clusters.

As an alternative, I suggest the use of structural indices of clusters which have proved to be identical by morphological criteria.

Spinrad: Do you think that the photometric synthesis of an elliptical galaxy is unique? There seems to be an equally acceptable model, using colors alone. That would be M 67 plus *many red dwarfs*. Such a synthesis fits the six-color Stebbins-Whitford colors quite well. The spectroscopic data in the visual region indicates that K and M dwarfs do contribute materially to the integrated luminosity—at least in the massive and luminous systems. M 32, however, has a spectrum indicating that giants probably give almost all of the light, even in the red.

Baum: Two synthetic models for the stellar population of elliptical galaxies were examined several years ago by means of six-color data. These two models can be described as Old Disk population and Halo population. To the extent permitted by each model, the luminosity function was left free to be varied. The results clearly favored the Old Disk population. When Tifft's eight-color data became available, they provided a better hold on this problem than the six-color data had, and a mixed model was examined. A model consisting of 20 per cent Halo population (referred to 6000 A) plus 80 per cent Old Disk population fits closer than 1 per cent in all colors from ultraviolet to infrared. I do not agree that a dwarf-enriched M 67 population is equally acceptable, nor can I agree with the analysis of Roberts, who compared no alternatives. There is a paper on this subject in the 1959 *P.A.S.P.*

Schücking: The source 3C 295 should show a variable Doppler shift of $(1+z)\cos b \times 30$ km/sec. With the mentioned precision, one could test the hypothesis that the red-shifts are caused by velocities.

Schmidt: Since angular diameters have been introduced in the discussion, I should like to note that the intrinsic diameter of a galaxy, defined in some way from the light distribution, is quite likely to be subject to evolution effects. These evolution effects would be due, not so much to the relaxation time, but mainly to a variation of distribution of bright stars as a function of time. Since in the case of 3C 295, we look back halfway in time, this could well be an important effect.

Baum: Since the dynamical relaxation time for stars in a galaxy is very long in comparison with most age estimates, an evolutionary change in the diameters or profiles of elliptical galaxies would have to arise, for instance, because the kind of stars near the center might evolve differently than the kind of stars farther out. A gradual trend of color index from the nucleus outward does exist, but it is not large.

Gold: How much error is introduced by a wrong selection of the size of the diaphragm? Can you give the actual change that would result from a different choice?

Baum: It is not the size of the actual diaphragm which matters so much as the diameter to which the data are subsequently adjusted. This adjustment is based upon an intensity profile, scaled in size to a function of the red-shift z. The uncertainty in this procedure ought not to exceed $0^{m}.1$. It is, therefore, a less difficult problem than, for instance, the matching of luminosity functions.

Neyman: The difference in the general appearance of Dr. Baum's and Dr. Sandage's diagrams showing the red-shift-magnitude relation is striking. Dr. Baum's diagram shows much less spread and indicates strict linearity. Presumably, the accuracy of the observations was improved. But was there any attempt made to avoid the effect of selection for brightness?

Oke: If one looks at Baum's energy distribution below $\lambda_0 = 6000$ A the slope is very steep. An intrinsic change of color with red-shift could be quite substantial but still not affect Baum's red-shift appreciably. In this spectral region, the question of the effective wave length which is applicable for each filter band is rather uncertain. This has been pointed out by Whitford already.

Vandererkhove: I want to congratulate Dr. Baum for his difficult and beautiful work which continues the measures of Stebbins and Whitford to the faintest galaxies. May I remind you that twenty-two years ago, I showed theoretically that the gradient of a galaxy should increase when the radial velocity was increasing and I proposed then the measure of the gradient of a galaxy in the spectral region of 5000 A as a criterion of the red-shift deduced from the spectral lines. This has been done, and well done. Now the time is coming for some refinements: effects of population, red giants, evolution, absorption. I think of the beautiful spectra of Dr. Oke. Personally, I can now measure gradients on spectra of galaxies obtained with the prism-objective of 84 cm. clear aperture of the Uccle Observatory.

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A Systematic Effect in Measured Red-Shifts

An analysis of the Humason-Mayall red-shift catalogue [1] leads to the conclusion that the measured red-shifts suffer from a systematic effect that is related to the amount of light reaching the spectrograph plate (or to the average plate density of the spectral region measured). This report gives a summary of the results obtained in a statistical investigation of the observational data, a full account being published elsewhere [2].

The first part of the investigation is based on the Virgo Cluster. A superficial examination of the red-shifts measured for 76 members reveals that the red-shift is correlated with the nebular type; as the type changes from E-SO to Sa-Sb to Sc, the mean red-shift increases systematically from about 1000 to about 1700 km/sec. The most reasonable explanation of the effect seems to be offered by the hypothesis that the measured displacements of the spectral lines are, directly or indirectly, affected by a magnitude equation of some kind. Since the red-shift observations in practically all cases refer to the central regions of the nebulae, the nuclear surface magnitude should thus be an important parameter. For statistical purposes, the central luminosity corresponding to the effective aperture of the average spectrograph slit may, for a given type of nebula, be assumed to be a constant fraction of the total luminosity. The following average relations are then obtained from available photometric data, $m' = m_{tot}$ for type E-SO, $m' = m_{tot} + 1$ for type Sa-Sb, $m' = m_{tot} + 4$ for type Sc, where m', apart from a constant (not needed in the analysis), is supposed to represent the sought nuclear magnitude.

A plot (76 cluster members) of the measured red-shift, V, against the nuclear magnitude, m', shows that there is an increase in the mean red-shift of about 700 km/sec as the magnitude m' changes from 10 to 16. A least-squares solution gives the regression line, $V = 905 + 114(m' - 10)$. The systematic effect, as far as it can be revealed by this analysis, is represented by the inclination coefficient, $\Delta V/\Delta m' = +114 \pm 31$. Since the numerical value of the coefficient is almost four times the mean error, it is presumably significant.

Confirmation of the above result is obtained from an analysis of the groups of nebulae listed by Sandage [1, 144]. A least-squares solution, based on all those groups where red-shifts are available for at least three members, gives an inclination coefficient of the same order of magnitude, $\Delta V/\Delta m' = +86 \pm 26$. The investigation may also be extended to double systems. An analysis of the red-shifts of the members of 23 close pairs in the general field (pairs in the Virgo Cluster and in the Sandage groups not included) leads to an inclination coefficient, $\Delta V/\Delta m' = +50 \pm 32$. It may be noted that in this case the orbital motions are a serious disturbance, the estimated orbital velocities being numerically larger than the sought systematic effect.

It would naturally be possible to include also single field nebulae in the investigation, provided that the individual distance moduli could be estimated with any degree of accuracy. If homogeneous photometric data are available as regards surface magnitudes, such estimates seem to be possible in the case of spiral systems [3]. A least-squares solution including 90 spiral nebulae measured for red-shift, and based on the relation

$$V = Hr + (\Delta V/\Delta m')(m' - 10),$$

gives a result that is in good agreement with the previous ones: $\Delta V/\Delta m' = +79 \pm 29$. It seems possible to arrive at a significant result even if we confine ourselves to the apparent magnitudes as distance indicators, that is, if we assume that the absolute magnitude is constant for a given type of nebula. In the Humason-Mayall [1] catalogue, the numerical relations between log red-shift and apparent magnitude have been determined by Sandage for the various types of field nebulae. In a way similar to the foregoing, the basic equation used by Sandage may be changed to include a correction term, $(\Delta V/\Delta m')(m' - 10)$. Using Sandage's numerical results as a starting point, we now find a mean systematic effect, $\Delta V/\Delta m'$, of +77 km/sec per magnitude.

In the part of the red-shift catalogue referring to the Lick observations, valuable information has been supplied by Mayall as regards the individual results of repeated red-shift observations of the same nebulae. The conclusions reached above are very nicely confirmed by an analysis of these data. A systematic relation is found between measured red-shift and the logarithm of the exposure time (same object; same type of plate), the latter quantity in this case representing the logarithm of the luminosity reaching the plate; an increase in the log (exposure time) by 1.0 corresponds, on an average, to a decrease in red-shift by 140 km/sec. If tripling the exposure is supposed to be equivalent (as regards plate density) to an increase by one magnitude in the intensity of the light source, the result may also be written in the form, $\Delta V/\Delta m' = +67$ km/sec per magnitude. The result points directly to the plate density as a primary factor in the analysis of the red-shift deviations.

In conclusion, it should be mentioned that systematic deviations appear also in the rotational red-shifts measured for NGC 224 and 598. The difference optical red-shift *minus* red-shift (21-cm. line) is always positive, and increases continuously as the distance from the nucleus gets larger and the luminosity of the measured patches fainter. Since no magnitude data are available, it is, however, not possible to compare the results numerically with the previous ones.

According to the Table 1 summary, the systematic increase in measured red-shift per magnitude ranges from 50 to 114 km/sec, the mean of the six independent determinations being +79 ±12. Without doubt, the final result is statistically significant. On the

TABLE 1

Summary of Systematic Magnitude Effect in the Measured Red-Shifts (km/sec per magnitude)

Material	$\Delta V/\Delta m$
Virgo Cluster	+114 ±31
Sandage groups	+86 ±26
Double systems	+50 ±32
Field neb.; spirals (M from surf. magn.)	+79 ±29
Field neb.; all types (M const. for each type)	+77 ±35
Repeated red-shift obs. (Mayall)	+67 ±25
Mean:	+79 ±12

other hand, the systematic deviation is not necessarily a *linear* function of the nuclear magnitude, a simplification that has been introduced in order to facilitate the statistical analysis.

A detailed study based directly on the spectrographic plate material, which is being undertaken, may be expected to yield more information about the nature of the systematic red-shift deviations, and more definite results as regards practical correction formulae.

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DISCUSSION

Mayall: In the first place, I should like to state that systematic errors of the order of 600 to 700 km/sec in the published Mount Wilson and Lick Observatory lists of red-shifts are, in my opinion, nonexistent. At the very beginning of the programs at both Observatories, Humason and I made all the usual tests to insure that the velocities were free of large systematic error from instrumental sources. To indicate the degree of success, I should like to mention some subsequent results, in so far as they refer to those based on the Crossley nebular spectrograms.

1. In the program of radial velocity determinations of the galactic globular clusters, a dozen brighter ones were also observed with the 36-inch Lick refractor. No large systematic differences were found. Later, Joy made some spectroscopic observations of variables in some of these clusters, and he obtained radial velocities for them that agreed satisfactorily with the velocities from integrated light. Finally, Kinman reobserved some of the northern clusters in his comprehensive program that included many of the unobserved southern globular clusters. Again, results for objects in common agreed within the estimated errors of both series of observations.

2. In the comparison of the Mount Wilson and Lick Observatory red-shifts for common objects, differences rarely amounted to as much as 200 km/sec, and in most cases were 100 km/sec or less.

3. Page's velocities of double galaxies, obtained with a grating spectrograph, agreed closely with those in the red-shift catalogues.

4. In their work on the rotations and mass distributions of galaxies, the Burbidges, using the same spectrograph as Page, obtained systematic velocities in satisfactory agreement with the Mount Wilson and Lick previous results.

As regards an effect of exposure time systematically affecting he measured shift, this possibility was investigated in a more comprehensive manner by Dr. A. Kruzchevsky, of the Astronomical nstitute, Warsaw University. He used all the Crossley data, including the results for individual lines, exposure, emulsion, different measurers, and slit widths. Although this investigation is unpublished, its results include evaluation of systematic errors attributable to such factors, and, in all cases, they were found to be smaller than 200 km/sec within the range of the parameters studied.

In reply to Dr. Holmberg's reference to my published statement about the "normal" wave lengths of some of the spectral features being variable by several angstroms, I should like to emphasize that my remark refers to blends, particularly, those shortward of H and K. These were not often used, and, when they were, they entered into the average with small weight. The wave lengths of H, K, and the G band, which were the usual ones measured in absorption, were found to be highly stable.

In conclusion, I should like to state that I do not doubt the reality of the red-shift-magnitude correlations in the data used by Dr. Holmberg. My main objection is their use to correct the catalogued red-shifts by amounts up to 700 km/sec.

Holmberg: I wish to thank Dr. Mayall for this very detailed statement. I would like to point out that, if we accept the final mean result obtained by me, the maximum differential systematic red-shift effect on the Virgo Cluster would be ±250 km/sec. Furthermore, the systematic effect is not necessarily a systematic error; this interpretation has only been a suggestion on my part; the results of the statistical analyses do not give any definite clues as to the primary cause (causes) of the effect. Whereas the red-shift-magnitude correlation must be taken into account when determining the red-shift dispersions in clusters and groups, a computation of individual corrections to the catalogued red-shifts has not been contemplated, or been deemed advisable.

De Vaucouleurs:

1. If, to avoid introducing bias in the correlations by introducing arbitrary corrections depending on type to derive "nuclear" magnitudes m', we limit ourselves to E and SO galaxies in the Virgo Cluster proper and plot the log $V - m_T$ correlation for all objects within 6° from $12^h\ 26^m\!.5$, $+13^\circ\!.2$ (1950), there is no indication of a positive correlation between $\overline{V}$ and m_T over the magnitude range 9 to 14 where Dr. Holmberg expects a velocity differential of 400 km/sec.*

2. In the Fornax Cluster where 14 velocities are now available from Lick spectra, there is an apparent correlation opposite to that predicted by Dr. Holmberg's correlation, from about +1700 km/sec

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*See *Ap. J. Suppl.,* **6**, 213, 1961.

at m_T = 9.5 to +1300 km/sec at m_T = 12.5. The sample is, however, small.

3. Between 1956 and 1960, Dr. Mayall secured 100 additional spectra of bright galaxies with the Crossley spectrograph; 50 are reobservations of old velocities, 50 are previously unobserved objects. These spectra have been recently measured by Mrs. de Vaucouleurs and me. Some preliminary tests for accidental and systematic errors lead to the following results:

(*a*) the internal standard deviation of velocities derived from individual lines (emission and absorption) in the same spectrum is 90 km/sec for unit weight; the total weight per plate is usually 1 to 5.

(*b*) The standard deviation derived from comparisons of velocities derived from duplicate plates of the same object is 100 km/sec for unit weight (mean weight per plate 1.75).

(*c*) The systematic difference between the new velocities and old velocities (in the HMS catalogue) for 48 objects in common is −7 km/sec; the average deviation is ±96 km/sec and the standard deviation of the old and new velocities (assumed of equal weight) is 86 km/sec.

(*d*) The systematic difference between emission and absorption lines for 31 objects which have both types of lines in their spectra is +61 ± 17 (p.e.) km/sec. Even this difference is largely fictitious because the residual line errors are essentially a common function of wave lengths ranging from +50 km/sec at $\lambda 3700$ to −40 km/sec at $\lambda > 4300$A; emission and absorption lines are thoroughly intermingled in the scatter diagram.

4. The suggestion that systematic errors of 5 to 10 A (or 10 to 20 microns) are made in settings of diffuse or complex lines or bands (G, H, K) does not apply to the sharp emission lines 3727 and the H lines, N_1, N_2. Occasional checks from higher dispersion spectra taken at Mount Stromlo† do not indicate the presence of large systematic errors in the Lick and Mount Wilson spectra.

5. Further checks will be made by measuring the apparent radial velocity of the mercury emission lines $\lambda 3650$, 4047, 4358 visible in many of the Lick spectra. I understand from Dr. Mayall that such tests made by him in the past do not indicate significant systematic errors.

Holmberg: My answers to Dr. de Vaucouleurs' various points would be:

1. The determination of nuclear magnitude does not depend on an "arbitrary" correction. As regards galaxies of type E and SO in the Virgo Cluster, we find from the physical members accepted by me a slight positive correlation between red-shift and total magnitude, the inclination of the relevant regression line being +24 ± 45 (m.e.) km/sec per magnitude. On account of the reduced number of

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†See *Mem. R.A.S.* 68, 3, 1961.

objects and the rather small magnitude range, we do not get a statistically significant result. If, however, the E and SO objects in the Virgo Cluster are combined with those in the Sandage groups and in the double systems, we arrive at a result that seems to be significant, the inclination of the regression line being +69 ± 32 (m.e.) km/sec per magnitude.

2. With regard to the high dispersion in the red-shifts, the sample is too small, and the range in magnitude too limited, to allow us to reach a statistically significant result in this case. Dr. de Vaucouleurs does not tell us what method he has used to ascertain the physical membership of the selected galaxies; the results of the analysis may be radically changed by the inclusion of a few optical members.

3*c*. I have so far not advanced the opinion that the systematic effect indicated in the red-shifts represents a personal equation, that is, that the person measuring the plates is responsible for the effect.

3*d*. The systematic difference between red-shifts based on emission lines and on absorption features depends, of course, on the normal (unshifted) wave lengths used for the reduction. It is interesting to note that the normal wave lengths adopted by Mayall differ from those used by Humason.

4. According to the final mean result, the maximum systematic effect in the red-shifts would amount to about ±250 km/sec, or ±3 A. The interpretation of the effect as a systematic error is only a suggestion on my part; there are other possible interpretations.

E. M. Burbidge: McDonald grating spectrograms are available for some spectra in Humason, Mayall, and Sandage's list. Various nebular types have been observed, but most are of type Sc, the type with which most of the discussion here has been concerned. Most are nearby galaxies studied for rotation, but some more distant objects have been observed. I have three points to make; first, the systematic accuracy of the McDonald observations; second, the comparison between emission- and absorption-line measures; and third, comparison with the Humason, Mayall, and Sandage list.

First, our spectra extend from the ultraviolet into the red, and systematic velocities of emission lines can be checked from the three night-sky lines in the green and red (λλ5577, 6300, 6363). Errors of 25 to 30 km/sec on single plates are found. Wave lengths of Fraunhofer lines in the moonlit or twilight sky, with varying densities of exposure, have been measured and a large scatter, ±50 to 60 km/sec, is found, but no sign of systematic errors.

Second, measures of emission and absorption lines in galaxies studied for rotation with several spectra of varying exposures show no systematic deviations between emission-line and absorption-line measures nor any systematic effects depending on photographic density on the plate, i.e., exposure time for any one galaxy. Third, as Dr. Mayall has said, comparison of the results with the Humason, Mayall, and Sandage velocities show no systematic discrepancies and the deviations appear to be random and within the published probable errors.

Fehrenbach: At the Observatoire de Haute Provence, it was necessary to test the precision and the absence of systematic errors in the determination of the radial velocities of galaxies taken with a spectrograph associated with Lallemand's electronic camera. The following table gives the preliminary results obtained by Mme. Duflot-Augarde on some galaxies with emission lines of H and [O III].

TABLE 2

Radial Velocities and their Errors in km/sec.

Galaxy	*Humason*	*Mayall-Sandage*	*Burbidge*	*Spectro E O.H.P.*	*Electronic Camera O.H.P..*
NGC 1068	1020 ±40	1121	1131	1079±17	1073 ±27
NGC 2782	2517 ±20			2566 ±15	2526 ±12
NGC 4051	627 ±20				677 ±23
NGC 4151	960 ± 8	934			952 ±10
NGC 4631	591 ±65			646	692 ±12
NGC 4656		721			826

In most cases the agreement between the measurements is very good. In some cases, like NGC 4631, the differences come from the fact that this galaxy contains large emission objects with large motions.

From our experience there are no systematic differences nor larger errors than in classical determinations. It was a necessary step in our work which is now continued.

The measurements for emission nebulae in galaxies made by Dr. G. Courtès in his interferometric work are also in good agreement with Dr. Mayall's results.

Holmberg: The Humason-Mayall red-shifts for the objects listed are based mainly on emission lines. The systematic magnitude equation to be expected would, thus, be small.

Minkowski: It should not be forgotten that there is a very serious problem here. If the systematic error does not exist, there exists a situation in the Virgo Cluster that may be even more peculiar than Holmberg's data show. For instance, the mean radial velocity for eight Sc galaxies with emission lines in Mayall's list is 1100 (±150 m.e.) km/sec, but for ten Sc galaxies without emission lines, 1800 (±200 m.e.) km/sec.

While a test for correlation with nuclear magnitudes is the only possible one without inspection of the plates, it is not realistic.

The observer sees the nuclear region and adjusts his exposure accordingly. The correlation with the photographic density of the spectra is what should be tested.

Holmberg: I agree with Dr. Minkowski. However, I feel certain that there is a residual correlation between the luminosity of the nuclear region and plate density, in spite of the fact that the observer makes adjustments. The importance of replacing nuclear magnitude by plate density has been stressed in my paper.

Page: Instead of including spirals with ellipticals in your analysis of systematic errors in V, I think you should look for a systematic difference between V_e from emission-line spectra, and V_a from absorption lines. Within the V_a group, you may well have a correlation with m, but it is *not* justified to extrapolate the regression to fainter magnitudes (where V is based on emission lines almost entirely). Also, *density on the spectrogram* would be the ideal parameter to correlate with V_a.

Holmberg: I think that you are entirely right. The statistical analysis is based exclusively on the data contained in the red-shift catalogue; it is, however, suggested in my paper that the plate density would be a more suitable parameter. The systematic difference between red-shifts based on emission lines and red-shifts based on absorption lines has also been noted. Thus, among the 76 Virgo Cluster members, there are about a dozen cases where the red-shifts are based mainly on emission lines; these red-shifts do not show any correlation with nuclear magnitude. On account of the limited amount of observational information given in the Mount Wilson-Palomar part of the red-shift catalogue, it has seemed advisable to postpone a more detailed analysis until it can be based directly on the original plate material.

Roberts: I would like to call attention to the 21-cm. measures of radial velocities of extra-galactic nebulae which have been made at Harvard. We have data for 17 galaxies which have a range of optically determined velocities of about −350 to +500 km/sec. The total range of differences between optical and radio determinations is about 250 km/sec; the average value of these differences is close to zero. In the sense optical minus radio velocities, there are eight positive values, six negative values and three in which the difference is essentially zero.

I should point out that there *may* be some bias in the above data, since we generally search for H I radiation over a range of about ±150 km/sec around the optical value for the radial velocity.

Holmberg: The optical red-shifts of these nebulae are mainly based on emission lines, and the systematic magnitude equation to be expected would, thus, be small.

D. S. Evans: Although only few results are involved, it is of interest to point out that the extra-galactic velocities determined in the south, often at high altitudes, agree well with observations of the same systems from the north at low altitudes. One can imagine difficulties arising with small objects which do not completely fill the spectrograph slit, but it is difficult to think that the nuclear magnitude correction can apply to large systems where, for example, the nucleus of an Sc system may be identical as far as observation goes with an E galaxy filling the whole slit.

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Distribution of Galaxies in Space

GALAXIES AND CLUSTERS OF GALAXIES

1. The structure of the near metagalaxy. The fact that the distribution of the galaxies in our neighbourhood is very uneven is made abundantly clear by the results of the census of bright galaxies in the Shapley-Ames Catalogue [1]. Figure 1, which is reproduced from the catalogue, shows the distribution of the galaxies brighter than the 13th magnitude on the two hemispheres. We mention only some of the most important features: the Virgo Cluster; the groupings in Coma, Canes Venatici, and Ursa Major which are more or less arranged in a band; the "southern extension of Virgo" descending in the direction of Centaurus; the groupings in Leo; the several southern groupings, roughly arranged in two belts; the general deficiency of the southern galactic hemisphere as compared to the northern; and the near emptiness in both hemispheres of the sector near 0° longitude.

We also show the distribution of the galaxies with measured radial velocities less than 1500 km/sec. The data are taken from the list of Humason, Mayall, and Sandage [2]. The southern boundary of their survey at 40° southern declination is indicated. Members of the Local Group are indicated by crosses; galaxies with a velocity of less than 500 km/sec by open circles. This diagram is very similar to the other one, especially for the northern hemisphere.

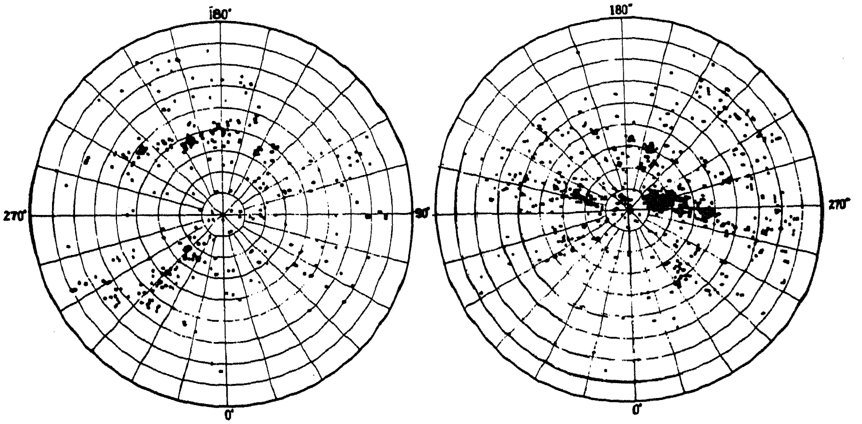

Figure 1. Distribution of galaxies brighter than the 13th magnitude.

If the distribution of galaxies on the sphere is studied in more detail, we often find within the larger areas smaller groups of galaxies which stand out as more or less separate units. Some of these smaller groups may even be seen on the accompanying diagrams, notably in Leo. The impression that these groups are real subunits, is confirmed by a study of the radial velocities. So we find for the UMa and CVn areas in general, after the exclusion of some nearby groups, a velocity dispersion of about 220 km/sec. If we take the subgroups separately, we find a dispersion of only 140 km/sec. In the Leo region, this division into subgroups is much more pronounded. We may add that, according to de Vaucou-

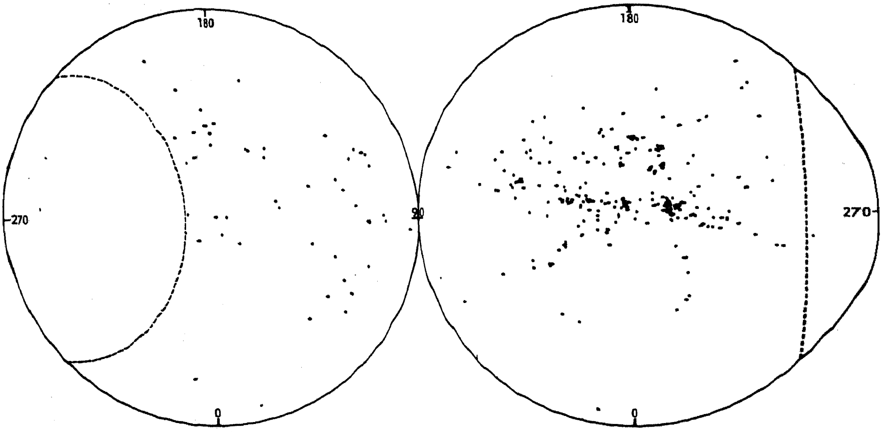

Figure 2. Distribution of galaxies with radial velocities less than 1500 km/sec.

leurs [3], the Virgo Cluster also seems to consist of a number of subgroups.

It should not be concluded, however, that the groups represent a well-defined intermediate step in a hierarchy from galaxy to cluster, and that they always stand out clearly by themselves. If we try to list the members of a group, we often find a few galaxies clustering together, some other galaxies spread more or less loosely around this centre, and still some others farther out, which may or may not be considered as associated with the group. The division between neighbouring groups may not always be clear, and there may be other galaxies in the same general area which one would not like to associate with either group, but which seem to be members of the general field of the larger aggregate to which the groups belong. So one would like to describe the groups as more or less distinct condensations in a larger complex. The looser the structure of this complex is, the easier the groups are recognizable as separate units. Apparently, some groups are more or less isolated in space.

We will not attempt to go into any detail with respect to the distribution in space of the nearby galaxies. For the present, only the main trends have to be considered. We, therefore, direct our attention now to the possible existence of a large-scale organization.

During the last few years, the hypothesis of the existence of a supergalactic system has been revived and considerably extended by de Vaucouleurs [4, 5]. De Vaucouleurs considers a highly flattened system with its centre in the direction of Virgo. Counts of galaxies in "supergalactic longitude and latitude" reveal the shape and the orientation of this system. The "supergalactic disc" is projected on the sky in the form of the belt of bright galaxies passing near the two galactic poles; the second southern belt which passes through Fornax is interpreted as a second supergalactic system seen edgewise. The faintness of the southern supergalactic belt in our own system is explained because it is in the direction of the anticentre.

A very significant part of the argument regarding the supergalactic system is derived from radial velocities. De Vaucouleurs argues that the high flattening of the system suggests that it may be rotating. Evidence for a rotation is then found in the variation of the average radial velocity with supergalactic longitude for various magnitude intervals. In Figure 3 we reproduce the relevant curves as published by de Vaucouleurs [5]. The theoretical curves which are drawn in the figure have been calculated on the assump-

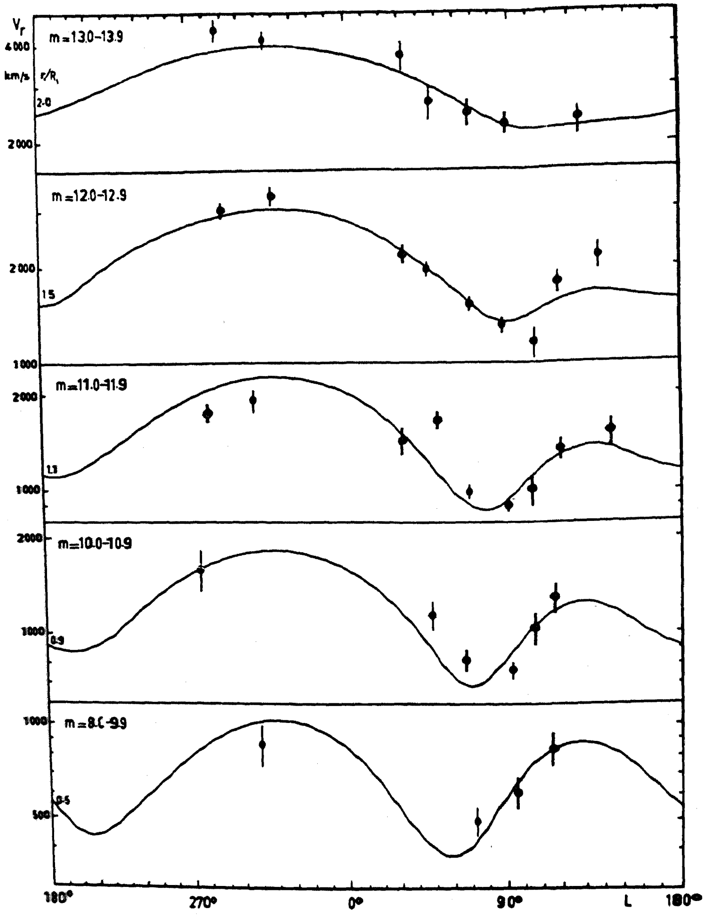

Figure 3. Variation of the average radial velocity with supergalactic longitude for various magnitude intervals.

tion of a combined rotation and expansion of the supergalactic system. The laws of rotation and expansion are assumed: the rotation is supposed to be rapid in the centre of the system and slow in the outer parts, whereas the opposite is true for the expansion. Each of the two laws contain one free parameter, which is determined from the observations.

De Vaucouleurs' hypothesis is of very great interest, and deserves a critical examination. Before they can be used, his (*m*, log *V*) curves have to be translated into a velocity-distance relation. This would be a simple matter if all galaxies had the same absolute magnitude.

In order to obtain a nearly homogeneous sample, de Vaucouleurs applies his morphological classification and rejects those systems which are classified as low luminosity types. Nevertheless, the dispersion in absolute magnitude of the remaining sample cannot be neglected entirely. Using de Vaucouleurs' [5] figures for the mean value and the dispersion of the absolute magnitudes of the various types of galaxies, we find that the dispersion in M for the remaining mixture should be about $0\overset{\mathrm{m}}{.}76$.

This remaining dispersion has a considerable effect on the results obtained. As an example, we choose one particular point from the set of (m, log V) curves, for which a mean velocity was found of (1355 ± 90) km/sec, based on nine velocity determinations. The probable error of 90 km/sec refers to the internal consistency in the velocities themselves. In order to use this point in a velocity-distance relation, we must know the average absolute magnitude of the nine galaxies which constitute this sample. Because of the dispersion in M, this mean absolute magnitude has a probable error not less than $0\overset{\mathrm{m}}{.}17$, corresponding to an 8 per cent error in distance. If the error is transferred to the velocity at a standard distance, we find an additional probable error of 108 km/sec. Hence, the total probable error of the radial velocity at standard distance becomes 140 km/sec. If similar corrections to the probable errors of other points in the diagram are applied, we find that the agreement between the theoretical curves and the empirical points is much better than may be expected from the intrinsic accuracy of the latter. This would seem to indicate that the theoretical model contains too many free parameters.

The effect of the dispersion in absolute magnitude becomes much more serious, however, if selection effects come into play, which is certainly the case because of the tendency of the galaxies to be grouped in clusters. As an example, we take the longitude range from 100° to 110°. Within this range, practically all galaxies belong to the Virgo Cluster. If the galaxies in this longitude interval are grouped according to apparent magnitude, they are grouped primarily according to absolute magnitude and only to a very small degree according to distance. This would still be so, even if the Virgo Cluster itself were a complex, provided this complex was not cigar-shaped. The mean radial velocities for the 10th, 11th, and 12th magnitude intervals at this longitude thus refer to only slightly different distances. It is quite understandable, therefore, that these velocities turn out to be nearly equal and are very close to the radial velocity of the Virgo Cluster itself. In de Vaucouleurs' interpretation, these points are treated as belonging to different distance groups. As a result, it is concluded that at a considerable distance from its centre, the expansion of the supergalactic system is still very small. Since our Galaxy is moving away from the centre at a speed of 1100 km/sec, this effect produces the notion of a differential expansion.

If we now consider the longitude range from 60° to 100°, we find many large and small groups of galaxies with rather small radial velocities. Figure 4 presents a rough map of our environment, based on the conventional assumption that radial velocity is a measure of distance. The map is a projection on the supergalactic plane of all groups with a supergalactic latitude less than 30° and a radial velocity less than 3000 km/sec for which at least four radial velocities have been measured. The areas of the black circles are proportional to the number of radial velocities measured.

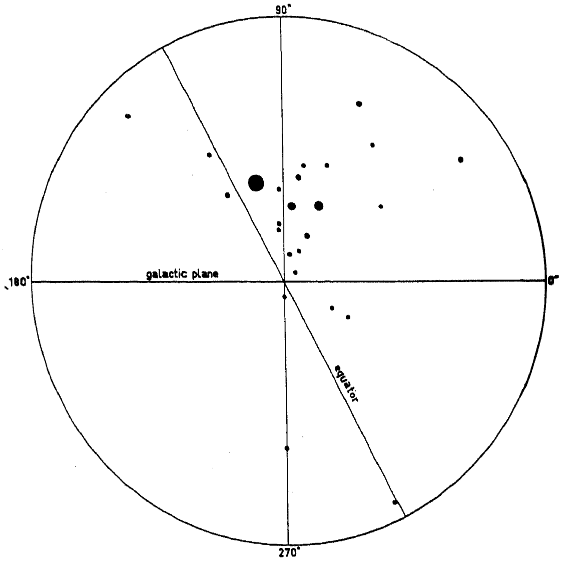

Figure 4. Map showing the projections on the supergalactic plane of all groups of galaxies with a supergalactic latitude of less than 30 ° and a radial velocity of less than 3000 km/sec.

Unless the reality of these groups is questioned, it must be assumed that two galaxies belonging to the same group are at approximately the same distance from us, regardless of whether they are bright or faint. To find the distances of these galaxies, we should not rely primarily on their apparent magnitudes, but on their group membership. There is no reason to suppose, even if the proportionality between velocity and distance is not accepted, that the distribution of the galaxies in depth is the same in the

60° to 100° interval as between 260° and 300°. An average 11th magnitude galaxy in the first interval will not be at the same distance as an average 11th magnitude galaxy in the second region, but presumably it will be much nearer. This effect almost certainly causes the dip in the (m, log V) curves in the 60° to 100° interval. If this effect is not taken into account in the interpretation of the results, one arrives at the conclusion that the system is rotating in the sense indicated by de Vaucouleurs. It does not seem that this interpretation is well founded.

One remark about the kinematic and dynamic aspects of de Vaucouleurs' model may be added. The assumed law of rotation

$$\omega = \omega_0 e^{-(R/R_1)^2}, \tag{1}$$

where R_1 is the distance from our Galaxy to the centre of the system, implies that for $R > R_1$, the angular momentum decreases outwards. This assumption seems very improbable from the dynamical point of view. Among other things, one wonders how galaxies with a small angular momentum ever reached these remote regions. From the velocities of expansion and rotation at the position of the sun we conclude that our Galaxy must have passed the centre of the system at a distance of less than 4 Mpc and at a time less than 8×10^9 years ago. At this epoch, the sun would have had a velocity of at least 1200 km/sec, whereas, the local velocity of rotation is at most 500 km/sec. If such large deviations from circular motion occurred in this direction, why not in other directions also? It seems that the model of de Vaucouleurs cannot be understood as a natural development of a generally acceptable initial state, but that its continuation into the past leads to rather artificial constructions.

In concluding this discussion, we may say that a study of the mutual relationships of the groups, swarms, and clusters of galaxies in our neighbourhood is of the greatest importance. We should be careful, however, not to proceed from preconceived notions with regard to the spatial arrangement of these galaxies or the kinematic phenomena. We should also realise that the methods of stellar statistics which are used in galactic research cannot be applied to the metagalaxy for the evaluation of space densities or systematic motions because the predominance of clustering requires a different approach. The determination of local deviations from the strict proportionality between velocity and distance will contribute greatly to our understanding. Such a determination would, however, be extremely difficult, because distance determinations of very great accuracy and independent of the radial velocity measurements would be required.

2. The more remote regions. The study of the remote regions of the metagalaxy can be attacked only with statistical methods. At the faint magnitudes, so many clusters appear that an extremely great amount of labour in photometry and in the determination of radial velocities would be necessary to achieve a substantial separation. Even the question as to what extent the concentrations of galaxies which are seen on the plates can be trusted to be real clusters is debated.

For future investigations, the material contained in Zwicky's catalogue, which will give the magnitudes of all galaxies down to the 15th magnitude as far as they can be observed from Palomar, will be of very great importance. At the present time, we may refer primarily to the equally ambitious program of counts of galaxies to the 18th magnitude which is being carried out at Lick, because this material has already been partially analyzed.

Neyman, Scott, and Shane [6] have made an analysis of counts of galaxies in an area at high galactic latitude. This analysis is based on theoretical considerations contained in an earlier paper by Neyman and Scott [7]. The region is divided into squares with a side of 1°, and counts of galaxies in squares situated at various distances from each other are compared. Technically, this is done through the determination of a series of quasi-correlations $Q(k\Delta)$ between the counts in squares separated by a distance $k\Delta$. Here Δ indicates the size of the side of a square and k is a positive integer. From the shape of the function $Q(k\Delta)$, the properties of the distribution of galaxies must be derived.

The analysis by Neyman, Scott, and Shane is based on a number of assumptions, of which we mention the following: (a) galaxies occur only in clusters; (b) the cluster centres have a random distribution in space; (c) the density distribution has the same shape and the same scale parameter for all clusters; (d) the luminosity function of the galaxies is the same everywhere; (e) the clusters of galaxies are fixed in space, i.e., the universe is not expanding; and clusters of galaxies do not evolve.

It is clear, that several of these assumptions have been chosen on the basis of simplicity rather than probability. Nevertheless, the theory derived on the basis of these assumptions is an important first step.

On the basis of these assumptions, it can be shown that the function $Q(k\Delta)$ depends only on the internal structure of the clusters and on the luminosity function of the galaxies. If some of the assumptions are dropped, the function Q may depend also on other factors, for instance, on the expansion of the universe [8].

In [6] it has been assumed that the distribution of galaxies within a cluster can be described by a Gaussian function. With this assumption, the size of the cluster, characterized by the dispersion σ of the density distribution function, can be determined if the luminosity function of the galaxies is known.

Neyman, Scott, and Shane show that it is possible in principle to determine the parameters M_0 and σ_M of the luminosity function—which they assume to be Gaussian—simultaneously with σ from the $Q(k\Delta)$ curves. The determination essentially depends on the relative visibility of the remotest clusters. The effects are so small that a simultaneous determination of all parameters has little value, unless we have a complete trust in the assumptions underlying the theory. It would be preferable to determine the luminosity function independently. Such a determination has been made for clusters by several authors. Neyman and Scott [8] also determined the luminosity function for "field galaxies," i.e., for galaxies which are not clearly associated with large clusters. The determination rests on the study of the distribution of red-shifts for galaxies of equal apparent magnitude and the same type. The results, obtained by Neyman and Scott for "field galaxies" do not agree with results obtained by other authors for cluster members [9].

If we take $M_0 = -18$, $\sigma_M = 1.0$, and assume a value 18.5 for the limiting magnitude of the counts, the resulting value of σ is about 3 Mpc.

In later work, Neyman, Scott, and Shane [10] have found that the value of σ obtained in their analysis depends strongly on the length of the sides of the elementary squares. Using squares of 10′ side length, instead of 1°, the value of σ was reduced by a factor of 0.27.

The discrepancy between the results of the two determinations shows that something must be wrong with the assumptions underlying the analysis. Neyman, Scott, and Shane suspect especially the following: (a) the assumption that the universe is static; (b) the assumption of simple (not multiple) clustering; and (c) the assumptions underlying the correction of the actual counts to "ideal" counts to a fixed apparent magnitude.

In this connection, we would like to add some remarks. First, it is quite possible that not all clusters are of the same size. Thus, there might not exist a unique scale parameter. Second, the assumed Gaussian density distribution seems quite remote from the structure of actual clusters. If the density distribution should be of the form given by Wallenquist [11], who found that ρ is roughly proportional to r^{-2}, even the definition of a scale parameter might present difficulties. Third, it is possible that some clusters have a complicated structure consisting of a relatively small and dense nucleus surrounded by an extended corona.

As an experiment, we have applied an analysis similar to that by Neyman, Scott, and Shane to the survey of galaxies in the Shapley-Ames Catalogue. For this purpose, we have used a strip 50° wide extending along the celestial equator, and remaining at least 30° from the galactic plane. The strip contains the Virgo region and part of Leo, but not the areas in Coma, CVn, and UMa. The statistics were limited to galaxies brighter than $m = 13.0$, for which the catalogue is supposed to be substantially complete. The counts in squares with sides of 1° and 5°, respectively, were combined to form two series of quasi-correlations. We found exactly the same discrepancies between the two determinations of $Q(k\lambda)$ as were mentioned by Neyman, Scott, and Shane. In this case, neither "counting errors," nor the expansion of the universe, nor the evolution of clusters and galaxies could be held responsible. The most likely explanation now seems to be the existence of clusters and groups of widely differing linear extent. This does not exclude, of course, the possibility that other factors contribute to the discrepancies found in the analysis of the counts of faint galaxies.

As is pointed out by Neyman and Scott, counts of galaxies will be extremely valuable if they are made in a consistent way with various limiting magnitudes. They expect that counts in special regions, down to very faint magnitudes, may contribute to the solution of the cosmological problem.

The mathematical apparatus constructed by Neyman and Scott forms a very powerful tool for the interpretation of these counts. In order to be effective, however, it should be freed as much as possible from unrealistic assumptions. Clearly the counts of galaxies by themselves cannot give an answer to all problems simultaneously, and it seems advisable to incorporate in the theory as much information from other sources as possible. Thus, the luminosity function of the galaxies may be studied in our own neighbourhood and considerable knowledge of the structure of clusters of galaxies may be gained from the study of individual cases. If the unknowns relating to these functions are eliminated from the theory, the analysis of counts may yield its maximum profit in the investigation of problems which cannot be attacked by a study of our immediate environment.

A problem of considerable importance, which has been emphasized very strongly by Zwicky [12, 13], is the possible existence of inter-galactic obscuring matter. According to Zwicky, the absorbing matter is heavily concentrated in the clusters of galaxies. One of Zwicky's arguments, which rests on a study of the "dispersion ratio"—a quantity which measures the deviation from a Poisson distribution—has been seriously questioned by Neyman, Scott, and Shane [14]. Zwicky's findings—that areas in the direction of conspicuous nearby clusters are deficient in faint galaxies and distant clusters—deserve a thorough study. If inter-galactic obscuration exists, it will have far-reaching consequences for all our studies of remote regions of space.

The distribution of rich clusters of galaxies has been studied by Abell [15]. He found that the distribution of cluster centres is non-random. Abell infers the existence of superclusters, with a diameter of the order of 24 Mpc if $H = 180$ km/sec/Mpc.

DYNAMICAL THEORY OF CLUSTERING

In this section, we will be concerned with the formation and the evolution of clusters of galaxies under the influence of gravitational forces. Ultimately, we would have to seek an answer to the question: can the present distribution of galaxies in space be explained as the result of "gravitational amplification" of statistical fluctuations in a uniform initial distribution? In practice, we have to limit ourselves to a much less ambitious program.

The development of an initial disturbance with perfect spherical symmetry poses a less complicated problem. We mention some of the results which were obtained in a preliminary investigation [16].

Gravitational instability is effective only in regions whose linear dimensions exceed a certain limit. Because of the expansion of the universe, the relative minimum dimensions of such regions steadily decrease. During the early stages of the expansion of the universe, the development of instabilities may have been impeded by radiation pressure. This would explain the absence of large-scale inhomogeneities in the universe. The "age" of the universe at the time when gravitational instability first became effective may be estimated at 10^8 years. The estimate is based on the relativistic formulae, without the cosmological term.

After this period, clusters began to form in regions of sufficiently large size which, at the critical epoch, had an excess density. An excess density of only a few per cent would be sufficient for the development of a cluster. The smaller the excess density, the slower the development of the cluster. Ultimately, the cluster stands out clearly against the background, mainly because the radius of the universe, during the time of development of the cluster, has increased by a large factor. Thus, the expansion of the universe is essential to the theory.

Because clusters originating from disturbances with a small amplitude (not wave length) develop more slowly, the universe has more time to expand during the formation of these clusters. This gives rise to the interesting possibility that clusters originating from disturbances of smaller amplitude may ultimately reach higher relative densities. The remark may be of importance for rich clusters. Originating in a region of greater relative dimensions, their development would start earlier, but proceed more slowly if the amplitude of the original disturbance was small, as is to be expected from their large population. At the present epoch, they may have a higher density than clusters which began as a stronger disturbance in a smaller region.

The structure of the cluster itself during the early stages of development is very smooth. Over a large region the density curve may be represented reasonably well by Wallenquist's [11] interpolation formula. Also Zwicky's [17] isothermal model may give a reasonable approximation. Nevertheless, the cluster is not in equilibrium and the velocity distribution of the galaxies is far from "isothermal."

A series of numerical integrations has been carried out at the Mathematical Centre at Amsterdam. A more detailed report of the results has been presented elsewhere [18]. Here we will mention only some of the main results.

In the later stages of evolution, the cluster develops a nucleus which stands out sharply against the corona. The structure of the corona itself remains very similar to that in the earlier stages of evolution. The nucleus itself is almost homogeneous, but there are indications of a tendency to form local condensations. It is not clear at present how much of the density fluctuations which are found are introduced by shortcomings in the computing technique and how much is real.

Figure 5 illustrates the density profiles at an early, and at a more advanced, stage of evolution. The quantity $(k\xi)^{-3}$ is proportional to the corresponding "age" of the universe. If this age is equal to the present age, the absolute linear dimensions of the cluster may be found from the length of the 1 Mpc line, to within a factor of two or so.

During the later evolutionary stages, the nucleus steadily contracts, but galaxies are streaming in from the corona, so that accretion of the nucleus occurs at the same time. In this process, the mean density of the nucleus increases approximately as the square of its mass. Gradually, the inner parts of the corona are

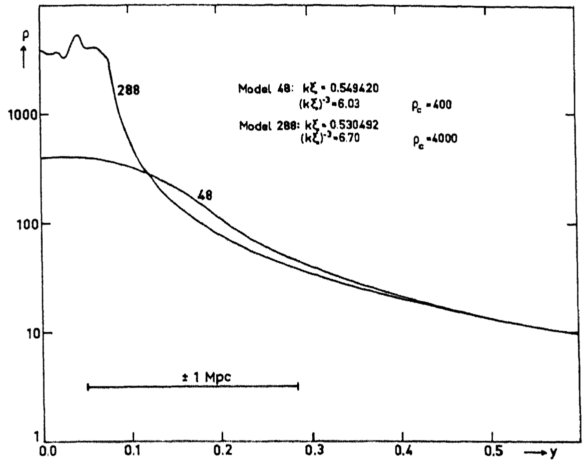

Figure 5. Density profiles for the evolution of a cluster of galaxies.

consumed, but because galaxies from more distant regions keep coming in, the structure of the corona as a whole does not change considerably.

The galaxies in the corona acquire large velocities while they are falling towards the centre of the cluster, but the dispersion in their radial motions is very small. This is understandable because a fixed value of the dispersion in the kinetic energy will cause a decreasing dispersion in the velocity when the absolute value of the velocity increases. The dispersion in the velocities perpendicular to the direction towards the centre is greater. Superimposed on the systematic motion of contraction, there is, thus, a highly flattened velocity distribution. The velocity of contraction, however, considerably exceeds the velocity dispersion in all directions.

The resultant velocity distribution is illustrated in Figure 6. For the nuclear region, where the computations indicate very strong local fluctuations in the velocity distribution, only the average values of the dispersion are shown.

It is interesting, to compare this theoretical diagram with the observational data obtained by Mayall [19] (Figure 7). The similarity between the two diagrams is obvious. Also the size of the nucleus—which is about the same as in Figure 4—and the velocities themselves—which are shown in units of about 50 km/sec—are of the right order of magnitude.

If we can trust these models, we also may try to answer the question: what error would be produced in the determination of the mass of the cluster if the virial theorem is applied in the conventional way? In ideal circumstances, i.e., if the density distribution in the cluster is accurately known, if measures of radial

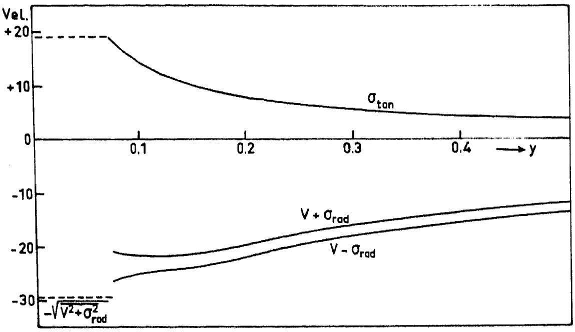

Figure 6. Calculated velocity distribution of galaxies in a cluster.

velocities are complete and exact, and if there is no confusion with foreground or background galaxies, the mass would be estimated too high by a factor ranging from 1.5 to 3 if the virial theorem were applied to the nuclear region alone, and by a factor between 0.5 and 2 if a substantial part of the corona were included. The more advanced the state of evolution of the cluster, the higher this factor will be. We must stress the point that none of our models represents a very far advanced stage: only a small fraction of the corona has been absorbed by the nucleus, even in our latest models.

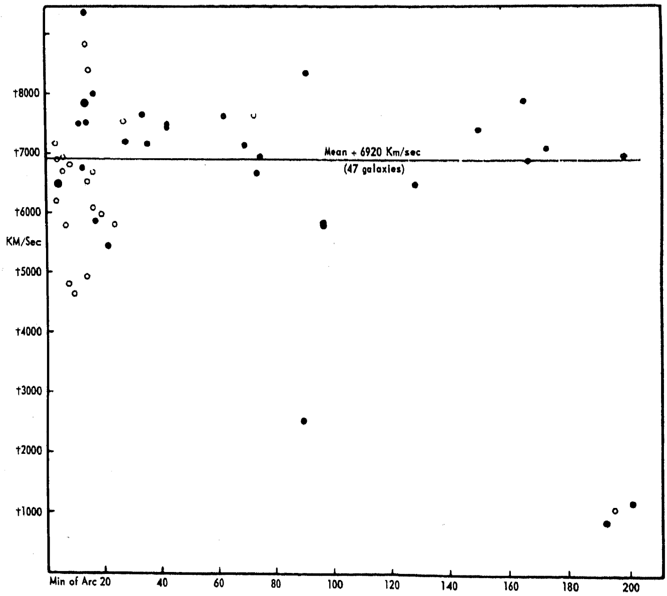

Figure 7. Observed velocity distribution of galaxies.

It does not seem probable that the virial factor will assume very high values in more advanced models. Nevertheless, its gradual increase with time supports the hypothesis that the clusters will ultimately disintegrate.

The theory which has been used in these calculations is still in a rather primitive state. Besides the assumption of perfect spherical symmetry, other simplifying assumptions about the velocity distribution of the galaxies have been used. In principle, it is possible to reach a considerably higher degree of accuracy in the solution of the problem. It does not seem, however, that the results would be changed fundamentally, although the quantitative correction may be of importance.

It is still not possible to study theoretically complicated structures like the aggregates in our own neighbourhood although we might be inclined to believe that a process of disintegration is going on. Nevertheless, the comparison of the theoretical results obtained up to now with the structure of regular clusters like Coma is very promising. It does seem possible at present, that the problem mentioned in the beginning of this section is capable of a solution, at least in principle.

A few words must be added concerning the relation of this investigation to similar studies by Bonnor [20], Lifshitz [21], and Lemaître [22, 23]. The work of these authors refers to the initial stages of instability where deviations from homogeneity are small. Though these authors start from different physical assumptions, the mathematical parts of their investigations, carried out with different methods, all lead to the same results when they overlap. Bonnor studies gravitational instability in a gaseous medium and concludes that the formation of a condensation of galaxy size in an expanding universe is not possible starting from an excess density of the order of $10^{-34}\rho$, the expected statistical fluctuation in the number of constituent gas particles. Lifshitz gives a complete relativistic treatment but, at a certain stage of the calculation, limits himself to a universe of zero pressure. In this case, the relativistic solution indicates the same rate of growth for the perturbation as for the corresponding solution deduced from Newtonian mechanics. Lifshitz's solution is in agreement with the result obtained in [16], but the fact that it predicted an instability was overlooked. He obtained two solutions for the amplitude of the perturbation as a function of time; a linear combination of these two solutions demonstrates the instability.

In an expanding universe, the strong development of a cluster is to be understood in a relative sense; the high relative densities are due primarily to the increase of the radius of the universe by a factor of the order of 40. In Lemaître's theory, the clusters are formed during an almost Einsteinian stage in the development of the universe and this hypothesis permits only a moderate increase of the radius of the universe since the time when the clusters were formed. It seems somewhat doubtful whether, under these circumstances, the clusters would reach the observed high relative densities. An additional difficulty is that this theory does not seem to account for the upper limit in the population of clusters which probably exists. It is of some interest to note the intimate connection between the phenomenon of clustering and cosmology.

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DISCUSSION

Bonnor: The work of Lifshitz, and later of myself, investigated the formation of condensations in a model of uniform gas disturbed only by small fluctuations permitted by statistical mechanics. It showed that condensations can form, but at an extremely slow rate. In the case of cosmological models with vanishing cosmological constant, this rate is insufficient for the formation of galaxies in the time available; in the Lemaître model, however, galaxies can form by gravitational instability alone provided that a rather long time-scale is used.

Lemaitre: If I have understood correctly, Dr. van Albada has said that the development of a cluster would depend on the expansion of the universe. This seems to be impossible on the hypothesis that no exchange of matter would occur between the cluster and the surrounding field.

Van Albada: The exchange of galaxies between clusters and field is part of the theory. The increase in density of the cluster is mainly relative to the surrounding universe. In a region of the expanding universe which had an excess density, the expansion would be slowed down and, at the later stages, would change into a contraction. At the latest stages which have been considered, the central density of the cluster is still smaller than that of the universe at the time when the development started. To reach the high relative densities which have been observed, an increase in the radius of the universe by a factor 40 or more during the development of the cluster is required.

McCrea: The only quantity of dimension time associated with a density ρ is $(G\rho)^{-1/2}$, where G is the gravitational constant. All theories give something of this order for the Hubble time. The time of gravitational collapse is also of this order. So condensations cannot be formed sufficiently quickly by gravitation alone. Either we must postulate some other force or we must start with a different initial state.

Van Albada: The density of the universe decreases by a factor of the order 10^{-5} during the development of the cluster, hence, the characteristic time increases correspondingly. The time of collapse in the region of excess density does not increase as fast, because the expansion of the region is retarded. Therefore, the collapse is accelerated, at least relatively. The mathematical theory developed in [16] and the numerical work reported in [18] prove that the process works. The assumption that the radius of the universe has increased by a large factor during the process is essential.

E. Scott: First, a rather trivial remark. We did not plan to use counts to estimate the luminosity function of galaxies. In 1951, we could not find any luminosity function on which astronomers would agree. Therefore, we considered several in order to cover the range of possibilities. It seems doubtful that we could find agreement about the luminosity among astronomers today.

As regards the possibility that clusters have extended "wings," we have made computations with the structure being the sum of several Gaussian components with which we can achieve wings as large as we please. No appreciable change in the theoretical serial correlation occurs from that corresponding to the average of the σ's of the components. We also considered models with the structure depending on the luminosity, since some astronomers suggest that the more luminous galaxies have a different distribution from the fainter galaxies. No appreciable change results.

However, we have not been able to have σ a function of the number of galaxies. This is a more difficult problem. There is little evidence whether or not this is necessary. The point is, that if a cluster has many galaxies following a normal distribution, then it will appear to be very compact. But if a cluster has but few members following the *same* normal distribution, it will appear to be loose. This is illustrated in our 1953 paper.

King: Can you make it clear just what sort of model you used in your calculations of the contraction of a cluster? Was it a Monte-Carlo model with individual mass points, or a smoothed-out density distribution, or what?

Van Albada: A smoothed-out model was used. The changes in the density and in the velocity distribution were calculated from Liouville's equation. This was done through relations connecting the moments of the velocity distribution. The diffusion terms, connected with the moments of order three, were neglected in this computation. This is probably one of the most serious shortcomings in the present stage of this work.

Neyman: With reference to certain doubts expressed regarding the model of cluster formation by van Albada, I expect that van Albada does not consider his problem finally solved. However, I find it interesting that this promising work, whatever may be its weakness, predicted something which we found as a puzzling result of empirical studies using our own, admittedly very imperfect, model of the distribution of galaxies. Studying seven clusters and fifteen groups of galaxies we found indications of their instability and, specifically, that some of them (Ursa Major) appear to expand and, at the same time, some others (Coma) appear to contract. To begin with, this latter finding made us uncomfortable. But then Dr. van Albada's paper arrived and we were cheered by his finding that at some stage of development the inner parts of clusters should contract.

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General Review of Cosmological Theories

In this report many questions of detail must perforce be considered as of secondary importance. But even so, speaking for 25 minutes about those aspects which could perhaps be considered as of primary importance I must yet leave the subject in a state of unavoidable sketchiness.

A theory can be, and should be, judged not merely from an empirical standpoint. Its motivation, its freedom from inner contradictions, the wideness of its scope, its connection with other fields of science—generally, but vaguely, speaking—its style, contribute to the esteem it deserves.

We want to compare the cosmologies of Newton, Einstein, Jordan, Hoyle, Bondi and Gold, and finally, Milne. The first two names designate the founders of the basic apparatus commonly used. The other five names give the authors of their respective cosmologies.

I

Newtonian and Einsteinian cosmologies have much in common. They both use as a framework the conservation laws of mass (or energy) and momentum, and a law of gravitation. The frame consists of certain sets of differential equations describing the mechanical behavior of compressible continuous material systems.

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I

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Both cosmologies fill their frames with the same idea: that cosmology demands the treatment of the total material and energy content of the universe in its possible motions. We know today that Newton's theory is not so much a first approximation of Einstein's theory as it is a rudimentary form of the latter. We should not be astonished to find many common features in both cosmologies. Eventually, the simpler, and in a sense poorer, Newtonian theory can be useful as indicating specific problems in the field of Einstein's theory.

If the red-shift of the nebulae had been discovered 50 years earlier, the Newtonian cosmology could have been developed without difficulty. The central problem was an alteration of the boundary conditions at infinity of the gravitational potential. This question has been cleared up a few years ago [1]. Newton's theory is adequate to treat infinite homogeneous systems in infinite space. But the treatment of finite, and yet unlimited, spaces should also be possible [2]. We have in the Newtonian theory a good instrument for the representation of more or less local phenomena. The dynamics of a single nebula, of multiple systems, or of whole nebular clusters is its proper domain. Van Albada gave a typical example, but much remains to be done. Naturally one is free to introduce, if necessary, hydromagnetic forces or a Λ term into classical mechanics. One main problem deserves a careful study, namely the occurrence of different kinds of singularities in the solutions.

But we have to confess that this classical treatment of cosmological problems has its inherent limits. There occur inevitably systems with relative accelerations of any amount, and classical optics has no adequate means with which to treat this situation. In addition, there occur relative velocities of any amount either in microscopic motions, if matter assumes states of high compression, or in macroscopic motions, as in the case of the nebular recession. We know that these difficulties have been overcome by Einstein's general theory of relativity. So, if we approach limits of velocity or of compression, or if we want to handle large-scale optical problems, we should prefer to use Einstein's theory. But, once again, there is no reason to look down on the use of Newtonian mechanics in cosmology. On the contrary, it has its special merits as it indicated new problems in the realm of Einstein's theory.

II

Einstein's theory was applied to cosmological problems 17 years earlier than Newton's. Today, I need not explain the mutual effects of theory and observation on each other between 1920 and 1940. There exist good text books and reviews which cover the field [3]. We have the complete set of the homogeneous and isotropic world models, named after Friedman and Lemaître. Optics in these models is fully developed. The main achievement of this cosmological theory is a very conservative one. It states: We observe an almost homogeneous universe in an almost isotropic linear expansion. This is a possible behaviour of a continuous system. It obeys the laws of mechanics. It is even the only type of motion which—in the cases of homogeneity and isotropy—could be expected at all. There is nothing especially puzzling in the phenomenon. And it is in this sense that we can say we "understand" the phenomenon; we have reduced it to normal mechanics, as we reduce orbital motions in the planetary system to ordinary mechanics; we have succeeded in reducing the unknown to the known. I believe this a *great* achievement because at the start it was not by any means clear that such a simple reduction could be carried out.

But in the frame of the theory as just described there remained many problematic details. I think the major point is the following: everybody agrees about the high degree of idealization contained in the homogeneous and isotropic models. Nevertheless, we extrapolate these models and arrive at the superdense explosive start of the universe. It has not only been considered as a singularity in our museum of mathematical models, but as a cosmogonically highly significant instant in the history of the universe. Nobody would attempt to extrapolate our knowledge of the planetary orbits by means of celestial mechanics in order to gain much insight into the history of the solar system. But, in the case of the whole observable universe we scarcely hesitate, seduced by the extreme simplicity of the formal side of the problem, to extrapolate cheerfully. And then we stand wondering at the mess we have created!

This result is intimately connected with a world-wide attitude among astronomers interested in cosmogony: they like to assume that early states of matter might easily be supposed to be homogeneous and isotropic. In our present context, a special problem was seen in deriving the conspicuous irregularities of matter out of earlier, or even primeval, homogeneous distributions. In reality, the problem was just the reverse one, namely to make an early homogeneity understandable at all. We have indications today that early homogeneity is a doubtful assumption [4]. During the process of expansion it is highly probable that irregularities will smooth out—as long as gravitational effects are considered. During a contraction, irregularities are more easily built up. The consequence is that the irregularities we see today have probably been much larger in the earlier phases of the universe. We actually do not know what the state of highest compression looks like. Perhaps there is no singularity at all; perhaps the character of the singularity is very complicated; perhaps it is broken up so that a set of isolated singularities arises [5]. Who knows? The problem is of the highest importance and deserves much more attention than it has received up to now. In any case, it is by no means certain that we have to worry about an age of a cluster larger than the so-called age of the universe as derived from Hubble's constant. If, moreover, we take a positive Λ term into account, we have Lemaître's way of reconciling the time-scales and of building up inhomogeneities. The important problem of the synthesis of elements has been connected in various ways with the possible singularity of the cosmos. On the other hand, the synthesis is also possible in stars. Today, there seems to be no unambiguous relation between cosmology and the origin of the elements.

Sometimes it is believed that the question of the Λ term or the finiteness or infinity of space are problems. I think in a sense they are not because:

1. The Λ term is unavoidable. The belief in its necessity has to be exactly as strong as the belief in Einstein's field equations. To base the Λ term on its apparent cosmological usefulness [6] was as weak an argument as to discard it because of its apparent cosmological unimportance [7]. We know of only one exact derivation of Einstein's field equations. This derivation leads to the Λ term [8].

2. Finiteness or infinity of space is not a matter of spatial curvature. The mathematicians know of finite and unlimited Euclidean and hyperbolic spaces [2]. But, more important, our models are only meant as a spatial interpolation of the observable part of the universe. Why is it necessary to talk about the unobservable and extrapolated part of space? If a parabola happens to fit some observed points in a two-dimensional array, it would be risky to infer that, beyond the observed points, there exists something *extending to infinity* apart from the parabola itself. Talking about parabolas is nice; it is equivalent to talking about world-models. Investigating their mathematical properties is useful and necessary. But it is a different thing to talk about their truth in the sense of their coinciding with the real world. This coincidence can be established only in the observable part of the world. Finiteness of space could only be proved by any experiment which is equivalent to seeing around the world. Infinity can never be proved.

III

Jordan was the only theorist who considered as promising Dirac's cosmological interpretation of the large numbers of the order 10^{40} [9]. Dirac had tentatively indicated, that the gravitational constant, the total mass of the closed universe and its density could possibly be simple powers of the age t of the universe. Then the radius of spatial curvature had to be $R \sim t$. This result was the starting point of Jordan's theory of gravitation which led to a very elaborate cosmology. Jordan brought projective Riemannian geometry into a general shape so as to comprise Einstein's old theory of 1916-1917 together with the field equations of electromagnetism and a new scalar field quantity. The latter replaced the old gravitational constant. The leading motive in Jordan's search for the right field equations was always the desire to have an expanding space with $R \sim t$ as a special cosmological solution.

A similar attitude was shown by the earlier cosmologists, Neumann and Seeliger, and, later, by Einstein [6], when they changed the law of gravitation by a Λ term in order to establish a world picture corresponding to their needs. They changed the field equations so as to guarantee a predetermined solution. Fortunately, in relativity, there is a much better reason for taking a Λ term into account; consequently, Einstein's attitude about that term need not be taken seriously. But in Jordan's problem the field equations have a much higher degree of arbitrariness. The postulated condition that solutions with $R \sim t$ should exist is, therefore, really decisive. Naturally, Jordan is very interested in having his theory tested empirically. But, on the one hand, as was shown by Pauli [10] and by Fierz [11], the self-interpretation of Jordan's theory is not unambiguous. On the other hand, Jordan's attempts [12] to combine his theory of a diminishing constant of gravitation with other evolutionary hypotheses about the earth, the moon, the planets, and the double stars, do not, in my opinion, strengthen a general confidence in his theory.

Jordan's additional idea, that the creation of mass becomes visible in the sudden birth of whole stars, is not a necessary consequence of his theory but a supposition for which there is scarcely any independent evidence.

A new relativistic theory of gravitation was developed by Brans and Dicke [13]. Formally, it has much in common with Jordan's theory. It is made to fulfill Mach's principle which, briefly, states that a completely empty space cannot have any physical properties of geometry and inertia. It leads to a variable "constant" of gravitation which decreases with time. Dicke has applied this theory [14] to a revision of the galactic evolution rates. He considers his main success to be that he has brought the ages of the oldest stars in the Galaxy, of the Galaxy itself, of an evolutionary universe, and the age of the heavy elements, into close agreement. This cosmology is the very opposite of those we are going to discuss below.

IV

Hoyle's form of the steady-state theory [15] has a certain affinity in style to Jordan's theory. Once more, a certain picture of the universe, its homogeneity in space, and its over-all steadiness in time, is the basic a priori concept. Only later have systems—one such system in 1948, a second in 1958, and a third in 1960—of field equations been developed under the viewpoint that the steady-state universe must be a special solution. The reasons for the introduction of the steady-state idea are not easy to reconstruct. But, certainly, the singularity of the simple relativistic models was felt to be so full of physical and cosmological problems that the idea of a radical escape became plausible. For some time, the synthesis of chemical elements was thought of as being intimately connected with the singularity of relativistic models. Therefore, desiring to get rid of this singularity, the steady-state cosmology was compelled to develop other methods of "cooking" elements. This was done with remarkable success [16], and in this respect the steady-state theory deserves our high esteem. However, one has to admit that the element-synthesis in stars is by no means a steady-state privilege; rather, it is a general possibility which could be welcomed by any cosmology whatsoever.

The singularity in the simple relativistic models had another consequence. For some time, the idea was commonly held that the major stellar systems, the galaxies, were all of one age. The stellar populations in a single galaxy, however, were considered as old, medium, and young. The steady-state hypothesis has introduced instead the idea of galaxies of different ages. This idea was even offered as a test feature of the steady-state universe. Finding extremely old galaxies, older than the average, was considered as a test in favour of the steady-state model. Naturally the number density of old galaxies must be much lower than that of the young ones. The old galaxies have already expanded away to a large extent. At present, it is considered decisive to detect a large number of very young galaxies. But here, again, the statement that relativistic cosmology demands that all galaxies be of the same age would be much too narrow. On the contrary, even the notion of a maximum age for galaxies would be unnecessary if the character of the relativistic singularities in an inhomogeneous universe were better understood.

The number-brightness and the red-shift-brightness relations seem to offer a more reliable decision between Einstein's and Hoyle's theories. There seems to be an indication [17] that the relativistic models represent the observations slightly better than the steady-state models. But, as the effects are near the limit of our observations, it is also impossible to convince a sceptic by pointing to the empirical side of the problem.

V

Hoyle's cosmology is almost identical with the steady-state cosmology developed slightly earlier by Bondi and Gold [18] since both give the same picture of the universe. But Hoyle, on the one hand, considered it desirable to add a set of field equations which he tried to fit into the frame of normal physics, whereas Bondi and Gold, on the other hand, intentionally dispensed with any field equations at all. The following line of thought is well known: if a set of empirical data is given as a basis of logical deductions constructed by mathematical reasoning, it is clear that, eventually, the last conclusions will suffer from the unavoidable uncertainties in the empirical data. The conclusions may gradually become so uncertain that they almost lose their connection with the empirical origin of the argument. This is a common feature of all scientific deductions. And it is especially true in cosmology where extrapolations sometimes seem to be unlimited.

The above mentioned difficulty can be partially diminished by improving the empirical side of the argument. But a second source of error is considered more essential by Bondi and Gold. The mathematical structure used in the chain of deductions, the so-called basic theory (in our case, any set of classical or relativistic field equations) has uncertainties in itself; it has an approximate validity only. Even if all mathematical steps are rigourous, the representation of reality by the basic concepts and axioms must remain uncertain. The degree of uncertainty is unknown to some extent. The cosmological extrapolations, therefore, have this second uncertainty which is propagated through the whole line of reasoning. And it is because of this uncertainty that Bondi and Gold purposely avoid the use of field equations. They consider the inherent uncertainties of all our present theories of gravitation as being so large that their application to very extensive portions of the universe becomes completely futile. Instead they make, so to say, a new experimental picture of the universe without recourse to local field equations of any kind. This picture of the universe then, when compared with observation, serves to indicate improvements of both empirical and theoretical investigations. The picture is a sort of working hypothesis.

But precisely because I wholeheartedly agree with the statement that, after a long series of arguments, our definite conclusions contain the uncertainties of both the basic empirical data and the only approximately true basic theory, I doubt the usefulness of Bondi's and Gold's treatment of the cosmological problem. If one cuts off the connection between local physical laws and the interpretation of observations at remote distances, it is impossible to gain improvements to the basic theory. Why should we not rather consider relativistic cosmology as an experimental hypothesis, reasonably well founded in local experience, but still unexplored to a large extent? It provides an everywhere controlable connection between our vicinity and the largest distances. And exactly because we carefully preserve this connection, it allows us to reconsider any and every part of a long argument if our experience and our observations make it necessary. The steady-state theory in the form of Bondi and Gold cannot have, by its very structure, this same instrumental character because it does not provide for the means of correcting either our basic physical assumptions or any step in the course of our long deductions.

VI

We come to a short review of the cosmological ideas of the late E. A. Milne [19]. He discovered as early as 1931 that if one adopts a definite continuous group of transformations and demands a world model to be invariant with respect to this group, then the laws governing the motion of the so-called fundamental observers and of free particles are fixed to a large extent. Milne chose the Lorentz group as a starting point. He found new ways of deriving it from a certain set of axioms, it is true. But he considered his main achievement to be a world-model which was constructed "more arithmetico" as he, himself, said and which he claimed to coincide with nature. The basic group was the rigid frame into which everything had to fit, and exactly this rigidity made it impossible to get away from a completely smooth distribution of matter. No clustering, no individual forms, could be described in the theory without inner contradictions to the basic invariance. Milne gave the proof of this impossibility in 1935; but he, nevertheless, later offered a theory of spiral structure based on the time dependence of his constant of gravitation. He offered two time-scales and ascribed each of them to definite microscopic and macroscopic phenomena.

But he did not see that his two time-scales were possible in relativistic cosmology also, and that their separate connection with two definite phenomena was a dream. A very artistic and elaborate mathematical building was erected, but its relation to nature was nowhere understood. As far as I know, scarcely any additional paper about Milne's theory has appeared in the last decade.

VII

If I am correct, it was Max Planck who once remarked: "A theory is never removed by arguments. Its supporters have to die out." If we apply this sceptical statement to our present cosmological controversies, I find it difficult to prophesy. If, on the other hand, we are less pessimistic and believe that, to a large extent, we still speak the same language, it should be possible to come to a better understanding than we have today.

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For a different approach with other results compare:

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DISCUSSION

Bondi: This was a magnificent review of cosmological theories, of amazing clarity, conciseness, and impartiality. I very much appreciate the spirit in which it was given.

There are minor comments I would like to make. First, it seems to me that evolutionary theories involving a contracting phase of the universe do not, as yet, take the thermodynamic difficulties of contraction sufficiently seriously. On the basis of Olbers' paradox or of the Wheeler–Feynman–Hogarth absorber theory, conditions in a contracting universe would be so unimaginably different from what we know that the applicability of our physics must be very much in doubt. Therefore, any real avoidance of a "big bang" would appear to raise difficulties at least as great as those implied in a "big bang".

Secondly, as a protagonist of the steady-state theory, I can challenge those who believe in an earlier very dense state of the universe to show me its fossil remains. Lemaître is, as far as I know, the only worker on such theories who has seriously faced this challenge.

Finally, I am perhaps not quite as hostile to a field theory of the steady-state as Dr. Heckmann suggested. Clearly, a field theory will be very difficult to construct, but fortunately the steady-state theory makes enough predictions to enable it to be checked astronomically independently of any field theory. If the theory survives these tests, then the effort required to construct a field theory will be justified and, if the theory does not survive the tests, then the effort will not be needed.

Lemaître: As a comment on Bondi's remarks, I may say that, in my theory, the cosmic rays are precisely the remnants of the primeval phenomenon connected with the singularity.

I would like to say that I am attracted by the introduction of the anisotrophy proposed by Heckmann and Schücking, not in order to destroy the initial singularity, but to make it less awful. Perhaps, it will be possible to arrive at some such thing as the "spin of the primeval atom."

Van Albada: If clusters of galaxies collect their material from the surrounding regions, their gravitational field is effectively screened. If they had time to wander far from the places where they were formed, their gravitational interactions might have created large space velocities. This consideration might favor near-homogeneity at some early stage.

Ambartsumian: Accepting the usual interpretation of the energy-momentum tensor, a theorem concerning the existence of solutions of Einstein's field equations with any arbitrary behavior of the volume of the comoving space is proved by A. L. Zelmanov.* In particular, the possibility is shown of the passage of the comoving space through a regular finite minimum of volume instead of a singularity. It is also possible to have a combination of an expansion of the volume of the comoving space in one region with a contraction in another region.

Page: After the report by Ambartsumian concerning the elimination of the singularity in normal relativistic models, it is worth remarking again that any evolutionary model *without* a very high degree of compression could not account for the "reorganization" of matter necessary. Otherwise, if there had been infinite time before, the results of irreversible processes would be infinite (white dwarfs, etc.), and if previous time was *not* infinite, at what time do you start?

Heckmann: I agree that simply removing the singularity of the homogeneous and isotropic models produces the problem you mention. But, as I said, the character of more general singularities may be much more complicated; the introduction of rotation and shear, for example, may produce a much longer time-scale without removing a singularity. The singularity, however, need not occur simultaneously for all regions of space. If Zelmanov has shown that relativistic rotating models may be without singularity, we learn how sensitive the old idea of the primeval explosion is against small changes in the initial conditions. Therefore, I am not

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*See *C. R. Acad. Sci. U.R.S.S.*, **135**, No. 6, 1960.

disposed to agree that we have to start at a certain moment in a universal time.

Hoyle: It seems to me that good grounds exist for the postulates of large-scale homogeneity and isotropy of the distribution of matter. The kinematical reduction of the line element leads immediately to a first-order derivation of Hubble's law. Moreover, Mach's significant point that the inverse square law of Newtonian gravitation applies only in the frame of reference that is nonrotating with respect to the distant universe (also the Schwarzschild metric of relativity theory) follows directly from homogeneity and isotropy.

I would of course agree that in an early condensed phase of the universe there might have been significant departures from homogeneity and isotropy, departures that have been greatly weakened by expansion from the condensed phase. But such a view relies heavily on the fact that we do not observe the conditions of an early phase.

Whitrow: Milne's theory was originally introduced to give a simple explanation of the expansion of the universe but became a complicated theory because of the introduction of two time-scales. The object of the latter was to explain the existence of inertial frames. Recently, it has occurred to me that the reason for introducing a second time-scale to explain inertial frames was due to the form Milne obtained (not in an entirely convincing way) for the equation of motion of a free particle and that, in fact, we can adopt a simpler equation of motion of a free particle in a uniformly expanding universe which automatically accounts for the existence of inertial frames without having to introduce a second time-scale. I was very interested to hear that Dr. Minkowski finds good agreement with the observed density of the universe if the deceleration parameter is nearly zero, that is, close to the value it has in a uniformly expanding model such as Milne's.

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Galaxies as Members of the Universe: Summary

We all deplore the reason which causes me to address you in place of Dr. H. P. Robertson: as you have heard he was involved in a motor-car accident a few days ago and is in the hospital. We all wish him a quick and complete recovery.*

In Part III, Miss Scott has described the curious and unexpected difficulties which confront the astronomer who tries to count the images of galaxies to the limit of a photographic plate. That these difficulties exist is most unfortunate because, as you know, the march of the number of galaxies per unit area of the celestial sphere versus limiting apparent magnitude is an item of great cosmological interest. It is one of the criteria which would help to distinguish between one uniform model of the universe and another or, as I would prefer to say, to select one type of uniform motion of expansion from the large number that theory presents to us. We all hope that the problems of counting will be satisfactorily solved some day.

The stochastic models of the universe that were described by Neyman seek to combine uniformity in the large- with small-scale irregularities. Thus, they are of the greatest value in cosmology.

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*Unhappily, Dr. Robertson did not recover from the effects of the injuries he received. He died on 26 August 1961.—Ed.

However, I noticed that the terms: "time," "distance," and "volume," were employed without precise definitions. Were the absolute time, distance, and volume of a classical universe meant? Or did they refer to a relativistic universe? In the first case, time, distance, and volume are independent of the material content of the universe. In the second case, we have to consider a group of objects in motion, the group, as a whole, defining what is to be meant by time, distance, and volume. In this connection, I may also refer to my prejudice against the term "cosmological effect" to describe any allowance that may be made for the red-shift. It seems to me that this expression is used whenever an investigator has carried out some analysis on the assumption that all galaxies are at mutual rest in a classical universe and then wants to combine his conclusions with deductions based on the opposite assumption that all galaxies are in relative motion in a relativistic universe. This procedure may be, in fact, correct but it always makes me a little uneasy because it is not self-evident that the results of the two assumptions are superposable in this way.

Neyman and Zwicky both referred to the possibility that the red-shift might not be a Doppler effect. The implication seemed to be that the Doppler principle was not a "physical" effect. Personally, I do not know how else I would describe the Doppler effect. In any case, it is not sufficient to say that the red-shift might be due to some cause other than relative velocity. The proposed alternative physical process must be described exactly and all its implications should be stated. It is probably well-known to you that many such alternative physical processes have been suggested to account for the red-shift. They have been abandoned because they have, so far, always led either to contradictions with observation or have proved to be *ad hoc* interpretations more puzzling than the red-shift phenomenon itself.

This consideration leads me back to Neyman's suggestion that the stochastic models of the universe may serve to "check" the interpretation of the red-shift as a Doppler effect. I have the uneasy feeling that the check may, in fact, prove to be a demonstration of the adequacy or inadequacy of the definitions of time, distance, and volume used in constructing the model, rather than an elucidation of the Doppler interpretation.

New results on the angular diameters of radio sources were the subject of the contributions by Palmer and by Moffet. A remarkable feature was that so many of the sources are now thought to have two centres of emission. One wonders why there should be only two centres, and not three, four, or more, in each source. Is it conceivable that the duplicity might be due to some instrumental effect or to some feature in the analysis of the observational record obtained for a source? At any rate, the angular diameter of a source of radiation, be it of a radio source, of the optical image of a galaxy, or of a cluster of galaxies, or be it Zwicky's structural index of a cluster, is a matter of great cosmological importance. If, in a statistical sense, some type of angular diameter can be regarded as constant for all sources, we would have an additional datum in cosmology. We should arrive at this datum, of course, by determining the variation of this angular diameter with respect to some other observable such as flux-density, apparent magnitude, or red-shift.

Ryle has spoken to us about his observations of radio sources, and he should be congratulated on the great task which he has accomplished. Everyone knows how few are the data relevant to the cosmological problem and how hard it is to obtain them. Ryle argues that all but about 1 per cent of Class II radio sources are indeed extra-galactic and he obtains a slope of -1.8 for his log N versus log S plot. Here, of course, N refers to the number of sources of flux-density not less than S. In this connection, I might point out that a slope of -1.5 for the log N - log S line would pose as great a cosmological problem as does a slope of -1.8. The reason is this: the Class II radio sources presumably share in the motion of expansion evidenced by the red-shift and they are also probably very distant. Under these circumstances, one can prove a general theorem to the effect that the number of sources per unit volume must have been more numerous in the past than they are now if the slope of the log N - log S curve is either -1.5, or has some value which is more negative than this. A special case of this result has been found by Priester who has interpreted most Class II sources as the result of galaxy collisions. I suspect that Hanbury Brown's interpretation, as presented by Palmer, is an alternative expression of the same theorem. Because the result could also be stated as a deficiency of sources in our neighborhood, if one remembers that distance from ourselves is also equivalent to remoteness in time. One wonders if we should go on trying to interpret the $N - S$ data in terms of a relationship in which N is proportional to some negative power of S. Would it not be better to use the formulas predicted by the uniform models of the universe, formulas which give N as a far more complicated function of S than a simple power law?

Zwicky gave us an account of his views on clusters of galaxies. The subsequent discussion revealed that no final judgment on this work can be reached until the data are available. These are in course of publication in Zwicky's catalogue. We are all immensely interested in the supernova search which he directs, and wish him continued success in this most important task.

Sandage's discussion of the cosmological distance problem revealed a return to the Cepheid variables as the touchstone. The other distance indicators—RR Lyrae stars, novae, H II regions, and brightest stars—are regarded as providing subsidiary evidence concerning the distances of the nearby galaxies. The calibration of the Cepheid distance criterion is still made through a very small number of objects in our Galaxy. These are five Cepheids in galactic clusters. Unfortunately, these Cepheids are all of about the same period. A similar weakness occurs in the calibration of the distance scale from the novae of M 31. Eleven galactic novae were used by Schmidt for this purpose and I am told by those experts on novae whom I have consulted that the distances of these objects are very far from being accurately known. There is also a problem connected with the brightest star criterion of distance. When I consult experts on the bright stars of our Galaxy, I am told that the upper limit of luminosity corresponds to an absolute magnitude of -7.5 to -7.7. Yet in the nearby galaxies, with the distances now proposed, the brightest stars have absolute magnitudes up to -9 and beyond. Does this arise because our Galaxy is unusually poor in bright stars compared with its neighbours, or is it because the distance scale is too large?

It seems to me that our present method of pinning our faith to one or another of the various distance indicators in turn, and thus producing sudden changes in the distance scale is an unfortunate one. Surely it would be more satisfactory to use each distance indicator independently, (Cepheids, RR Lyrae, globular clusters, novae, brightest stars, H II regions, planetary nebulae, etc.) to determine the distance of, say, M 31? The results would be discordant, of course. Nevertheless, we could assign a weight to each method and thus arrive at a distance for this galaxy. As information accumulated, the relative weights of the different methods could be altered. In 1958, when the distance modulus of M 31 had been raised from 24.25 to 24.6, I had the curiosity to apply this method to the 11 determinations of the distance modulus that I could find in the literature. These values ranged from 23.8 to 24.8. I was interested to observe that by assigning probable weights in two different ways to the determinations, I found in both cases a distance modulus of 24.3.

The value of about 100 km/sec/Mpc for the Hubble constant deduced by Sandage is in agreement with the determination from the red-shifts of clusters of galaxies if the average brightest member of each cluster has an absolute magnitude of about -21.

Turning now to the red-shifts, I would like to emphasize the importance of Minkowski's and Bolton's work on the interplay of radio and optical astronomy. By studying, in this way, the radio source 3C 295 they have been able to measure a red-shift of 0.46. Baum has also described the photoelectric methods by which he observed the red-shift of the cluster of galaxies to which 3C 295 belongs and of seven other clusters. On the question of the retardation of the expansion, one may say that evolutionary effects might vitiate current determinations of the acceleration factor q_0. This possibility should be recognized, but I am not prepared to draw the conclusion that current values of q_0 are, therefore, worthless. It is one thing to realize that some effect might change our views; it is illogical to change them before the magnitude of the effect has been determined. For aught we know, the amount of the effect might be quite unimportant for the purpose in view.

I gather that Minkowski and Sandage now want to reject the photographic magnitudes for the brighter members of the clusters which have been used [1] to determine the Hubble constant and the acceleration factor. Baum's photoelectric results should be used instead. I seem to detect here the same tendency that I remarked on with regard to the distance scale: the weight to be assigned to an observational method is either 1 or 0, nothing in between is acceptable! Suppose that we try to compare the methods of Sandage and of Baum with regard to the determination of the acceleration factor q_0 [2]. Sandage has 18 clusters with photographically determined apparent magnitudes for the "brightest cluster members." Baum has eight clusters with corresponding photoelectric determinations. Five of Sandage's clusters (Virgo, Coma, Cor. Bor., 1055 + 5702, 0925 + 2044) also occur among Baum's eight clusters, and the red-shifts for these are common to both investigations. The important theoretical variable is $z = m - 5 \log c\delta$ where m is the apparent magnitude, corrected for the red-shift, of the average brightest cluster member, and δ ($= d\lambda/\lambda$) is the red-shift for the cluster. The Sandage and Baum values of z for the five common clusters can be calibrated by taking the Coma Cluster to have identical z in the two magnitude systems. This also gives identity for the Virgo Cluster z values; for the remaining three clusters, it follows that Sandage's apparent magnitudes are fainter than Baum's by not more than $0^m.4$.

The theory of the model universes shows that $z = a + b\delta$, to the first order in δ, where a, b are constants of which b gives the acceleration factor q_0. If we plot z against δ, the following points emerge: (i) the value $q_0 = 2.8$ is found when all Sandage's 18 clusters are used to find a and b. The predicted z for 3C 295 then differs from Baum's measured value by an amount that corresponds to a reduction in his apparent magnitude by $0^m.25$. This lies well within his errors of measurement. (ii) The value $q_0 = 1$ found by Baum appears to arise by an effect of selection, because he has only five clusters in the region of red-shifts less than 0.2. Sandage has 18 clusters in this region and it is this greater number of observations which accounts for the larger value of q_0.

If Sandage's photographic magnitudes are rejected, it must be explained why his results lie so close to Baum's. A second point that should be cleared up is the question of the z values for all Sandage's clusters; it is not sufficient to select five out of the 18 for photoelectric observation.

A reflection of a different kind is suggested by Minkowski's paper. I deprecate the identification of the cosmology of general relativity with the special cases in which the cosmical constant Λ is zero and the pressure is zero also. It is only if we make this quite arbitrary preliminary selection that we can agree with Baum that the red-shift apparent-magnitude relation leads to a single conclusion regarding the model universe, or that we can accept most of Minkowski's numerical results. The restriction to these models out of an infinity of possibilities is equivalent to asserting that the problem of the nature of the universe has already been solved, except for the relatively minor detail of selecting one out of a few very similar alternatives. This procedure appears to be in complete contradiction to the assertions of the observers that there are hardly any reliable data from which the nature of the universe can be deduced—to their satisfaction at least!

I agree with Heckmann that any mathematically sound derivation of Einstein's gravitational equations shows that Λ is present. The theory cannot determine the value of Λ, but we can now see how it could be found from observation. In any model of the universe in which the pressure is zero, let H be the Hubble parameter, q_0 the acceleration factor, kc^2/R_0^2 the quantity proportional to the space curvature, and let

$$\sigma_0 = 4\pi G \rho_0 / H^2,$$

where ρ_0 is the average density of matter in the universe in our, cosmically speaking, neighbourhood. Then Einstein's equations can be expressed as

$$\Lambda = -(3q_0 - \sigma_0) H^2,$$

$$kc^2/R_0^2 = -(q_0 + 1 - \sigma_0) H^2.$$

For ρ_0 I adopt Oort's [3] value of 3×10^{-31} gr/cm^3. Then it can be shown that σ_0 is independent of the value of H and $\sigma_0 = 0.044$. Since q_0 probably lies in the range 1 to 3 it follows that Λ is negative and the space curvature is also negative (hyperbolic space). On the other hand, if we assume on aesthetic grounds that $\Lambda = 0$, we are compelled to multiply the average density of matter given by Oort by a factor of 100. The assumption $\Lambda = 0$ also forces a closed spherical universe onto us.

I should also like to say a word about comparisons between the general relativity models and the steady-state theory model. The second model is identical with the de Sitter universe of general relativity in so far as optical relations are concerned. Examples of such relations are: the march of red-shift with apparent magnitude or with flux-density (radio source); the variation of angular diameter with red-shift, and so on. The de Sitter universe and the steady-state model give different predictions only when the numbers of galaxies, or the average density of matter in space, are in question. Only this type of prediction can be used to distinguish between the steady-state theory and the de Sitter universe of general relativity.

Holmberg discussed an investigation he has carried out in which he believes that he has found a systematic dependence of the red-shift on the type of galaxy and/or its apparent magnitude. It is important that these conclusions should be checked in the calm of one's study. If Holmberg's results are not due to errors of measurement, then we are faced with a new physical effect which will have to be elucidated. Of course, this remark applies only if Holmberg's work can be substantiated.

Van Albada analyzed the notion of a supergalaxy and warned us of the pitfalls involved in the discussion of the spatial arrangement of groups, swarms, and clusters of galaxies. He then passed on to the question of the formation of clusters of galaxies out of a kind of uniform expanding distribution of galaxies.

I do not think that there is anything useful I can add to Heckmann's admirable summary of cosmological theory. I do sometimes wonder why we stick so rigidly to the idea that one model, and one only, should be regarded as adequate to describe the entire history of the observed universe. We often run into time-scale difficulties when we stick closely to this idea; so why not take a leaf out of the theory of stellar structure and investigate the fitting of one model onto another? In any case, we need more realistic models of the observed universe. I will therefore end with a question: What is the solution of Einstein's gravitational equations that corresponds to a universe populated by a random

distribution of material objects separated by relatively large empty regions?

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[1] M. L. Humason, N. U. Mayall, and A. R. Sandage. *A. J.* **61**, 57 (1956) Tab. XIII.
[2] G. C. McVittie. *Fact and Theory in Cosmology*. London: Eyre and Spottiswoode, Ltd., 1961; Ch. 6.
[3] J. H. Oort. *La structure et l'évolution de l'univers*. Brussels: Stoops, 1958; p. 163.

DISCUSSION

Bondi: I feel that a discussion of the cosmical constant is unnecessary. While there clearly are arguments in favor of the Λ term, they are not accepted by every student of relativity. Indeed, Einstein himself was very much opposed to the Λ term.

Gold: Professor McVittie discussed the question of the mean density of the universe. It is my impression that there is no case to be made for the assumption of any particular value for this density within the wide range allowed by the observational evidence. We have, therefore, attempted to see what these limits are from all observational points of view: 10^{-31} gr/cm^3 is implied by the sum of shining stars in galaxies, and is, therefore, a firm lower limit: 10^{-27} would have dynamical effects on the gas masses in galaxies greater than are observed, and this is, therefore, an upper limit. All other considerations of which I am aware do not limit the quantity more closely.

I would welcome any observations or considerations that would allow this range to be narrowed, but, at present, I can see no reason for considering any value as more probable than any other within the range. It is as much an assumption to consider most matter to be luminous as to consider most matter to be non-luminous.

Bok: With one exception, we have stressed that clusters of galaxies are not stable aggregates. The argument is that the estimated total masses of clusters of galaxies are too small by a factor of ten or more if we ask for stability on the basis of the virial theorem and the observed dispersions in linear velocity. Without much question, we seem to accept the conclusion that clusters of galaxies are systems of positive energy (Ambartsumian); hence, that clusters are unstable aggregates and often of rather recent origin. I hope that we shall not accept this far-reaching conclusion lightly, certainly not until all sorts of other approaches will have been explored. The problems that are created by the requirement of recent birth and rapid evolution of the wide variety of individual galaxies in clusters, including dwarf galaxies, are tremendous ones, not readily amenable to solution. My suggestion is that we should continue to search for the matter required to satisfy stability and here we turn, naturally, to inter-galactic matter. The search must be pressed for inter-galactic hydrogen, be it in molecular, neutral atomic, or ionized atomic form.

Oort: There may be a slight misunderstanding in connection with the value of 3×10^{-31} gr/cm^3 for the average density in the universe which Professor McVittie has quoted. In calculating this density, I used a Hubble constant of 75 and rather large values for M/L (viz., 50 for E and SO galaxies, 20 for Sa and Sb, and 7 for Sc spirals). With such values of M/L the requirement of the virial theorem is roughly fulfilled in clusters like Coma, but *not* in the Virgo Cluster where a 10 or 20 times higher mass is required. Therefore, on the basis of the virial theorem, a factor of, at most, about 10 might have to be applied to the density of 3×10^{-31} quoted.

However, it is conceivable that there would be much unobservable matter in the universe which does not partake in the clustering tendency displayed so strongly by the intrinsically bright galaxies. Unless this matter were concentrated in galaxies, it would never reveal itself to observation, neither directly, nor indirectly via the virial theorem.

McVittie: In the past, there were no observational data available from which the value of the cosmical constant could be deduced. Therefore, acceptance or rejection of the constant could perhaps be justified on the basis of personal predilection. Today, we are within sight of the data that will give us the value of the cosmical constant.

Gold's argument for the presence of a high average density of matter appears to rest on the notion that, if we can find no evidence *against* a density of, say, 10^{-29} gr/cm^3, we are, therefore, entitled to suppose that such a value is possible. This seems to me to run very close to the old fallacy in logic called the "argument from ignorance." Oort has provided positive direct evidence for a density of from 3×10^{-31} to 3×10^{-30} gr/cm^3. Either of these values, combined with Einstein's gravitational equations shows that the cosmical constant is negative.

I am unwilling to take account, in our cosmological theories, of large quantities of matter which can never reveal themselves to observation. This would turn cosmology into the study of an imagined universe, instead of an investigation of the universe which we observe.

Bonnor: I would like to say that I believe it to be a matter of urgency that the steady-state theory should be provided with field equations and a dynamical theory. In spite of a number of interesting attempts in this direction by Hoyle, a satisfactory theoretical backing is not available.* The tendency has been to try to solve problems in the steady-state theory (e.g., that of the formation of galaxies) by using general relativity or Newtonian mechanics.

I have recently shown that such attempts have no validity since the steady-state model corresponds to a degenerate case of general relativity and of Newtonian theory.† As it stands at present the steady-state theory cannot properly deal with any detailed cosmological problems.

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*Hoyle, F., M.N.R.A.S., **122**, 381, 1961.
†Bonnor, W. B., M.N.R.A.S., **121**, 475, 1960.